Optimization of the Dynamic Behavior of a Heat Exchanger Subject to Fouling Comparison of Three Optimization Models[*]

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Abstract

This paper proposes a dynamic analysis of fouling of a heat exchanger. The used criteria is mean thermal power exchanged over time. The proposed results are relative to co-current configuration and tubular geometry of the heat exchanger, but other cases have been explored (countercurrent, plane geometry). An optimum time is determined, sensitivity analysis of the corresponding value to three cases of flow regimes (constant mass flow rate, constant flow velocity and constant pumping power) and various kinetics of fouling has been performed. The time of stop of the installation for cleaning is the main parameter. All the results are proposed in nondimensional form.

Key words: dynamic optimization, fouling, heat exchanger, kinetic of fouling, flow regime

1. Introduction

The industrial use of heat exchangers does not generate particular problems in the case where heat transfer surfaces remain clean during all the duration of the machine operation. However, in numerous applications the flow of one of the two fluids is accompanied by the occurrence of a deposit and usually the thickness of this deposit grows with time. The presence of deposits reduces the thermal flow and prevents realization of the desired operation. Therefore, it is mandatory to proceed to a periodic cleaning of circuits.

Considering this phenomenon of fouling, it is possible to determine an optimal value of the operating time, associated with the maximum of average exchanged power during a cycle.

We will focus more particularly on elaborating an optimization model of the exchanged power. Having in mind that the major goal for thermal processing is principally related to the exchanged heat power, and that this power has a tendency to decrease markedly with the appearance of a fouling deposit, it seems judicious to optimize the average exchanged power during a complete (on - off) cycle of a heat exchanger. In this model, we will introduce the notion of foul-

ing kinetics. We will consider the fouling fluid flow rate as constant, the velocity of this same fluid constant, or the pumping power constant. We will compare the different approaches.

2. Method

Among a number of possible operating cases, we will present only one; it will concern a cylindrical geometry exchanger, in the co-current configuration and laminar flow. All other cases are combinations of two different geometries, two configurations and two types of flow ; they are available in internal reports (L.Schaal, 1996-1997). We will limit this study to only one fouling fluid (the internal fluid); this fluid will also be the limiting one, that is the calorific product ($\dot{m}.Cp$) characterizing it will be the smallest of the two fluids concerned in the heat exchange. In this case, the evolution of the deposit, considered as homogeneous and uniform, corresponds to the thermal resistance:

$$R_f(t) = \frac{\ln\left(\dfrac{ri}{ri - \delta(t)}\right)}{2.\pi.\lambda_d.L} \times Su_{i0} \qquad (1)$$

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The evolution deposit is:

$$\delta(t) = ri \times \left\{ 1 - \cfrac{1}{\exp\left(\cfrac{\lambda_d}{ri} \times R_f(t)\right)} \right\} \qquad (2)$$

In the literature, we have noted four fouling kinetics, $R_f(t)$.

The asymptotic kinetics model in which the resistance evolves until a limit value is developed by Kearn and Seaton (1959). This model is representative of the particulate fouling (L. M. Chamra and R. L. Webb, 1994), the expression of this kinetics is the following:

$$R_f(t) = R_f^* \times \left[1 - \exp\left(-\frac{t}{\tau_e} \right) \right] \qquad (3)$$

This evolution will be considered as a reference; that is, we will express the other kinetics in terms of the τ_e and R_f^* magnitudes appearing in this model. τ_e represents the time constant of this type of kinetics and R_f^* the asymptotic value of the fouling resistance.

The proportional kinetics model of A. Bejan (1994). This deposit growth would be representative of frost, notably on surfaces of evaporators. It is expressed as:

$$R_f(t) = \frac{R_f^*}{\tau_e} \times t \qquad (4)$$

The root squared kinetics model. This third type of evolution is characteristic of the fouling by solidification or crystallization (H. W. Schneider, 1978). Its expression is the following:

$$R_f(t) = \frac{R_f^*}{\sqrt{\tau_e}} \times \sqrt{t} \qquad (5)$$

The squared kinetics model. This is representative of the evolution of the fouling resistance by dairy products during pasteurization or sterilization (F. Delplace, 1995).

$$R_f(t) = \frac{R_f^*}{\tau_e^2} \times t^2 \qquad (6)$$

However, in the following calculations, to simplify expressions, we will keep the general form of the kinetic, that is:

$$R_f(t) = R_f^* \times f(t/\tau_e)$$

Next, we define necessary geometrical quantities for the study.

The exchange area is
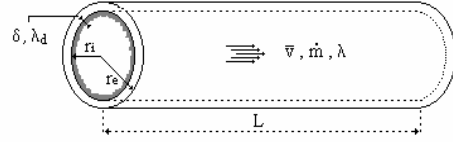
$$Su_{i0} = \pi \, Di_0 \, L$$

Figure 1. Representation of a tube, component of heat exchanger in cylindrical geometry.

$$Su(t) = Su_{i0} \times [\exp(-B \times f(t/\tau_e))] \qquad (7)$$

where,

$$B = \frac{\lambda_d \times R_f^*}{ri} \qquad (8)$$

The cross-section area is

$$Se_{i0} = \pi.ri^2$$

$$Se(t) = Se_{i0} \times [\exp(-B \times f(t/\tau_e))]^2 \qquad (9)$$

The tube inside diameter is

$$Di(t) = Di_0 \times [\exp(-B \times f(t/\tau_e))] \qquad (10)$$

Then, we introduce quantities characterizing the laminar flow in a circular section.

The Prandtl number

$$Pr = \frac{\mu.Cp}{\lambda} \qquad (11)$$

The Reynolds number

$$Re = \frac{\dot{m} \times Di}{\mu \times Se} \qquad (12)$$

The Nusselt number for Sieder and Tate (J. F. Sacadura, 1980):

$$Nu_i(t) = 1.86 \left(\frac{Di(t)}{L} Re(t) \, Pr \right)^{1/3} \mu^* \qquad (13)$$

The heat transfer coefficient

$$hi(t) = \frac{Nu_i(t) \times \lambda}{Di(t)} \qquad (14)$$

We have chosen to optimize the average exchanged heat power during a complete operation cycle:

$$\bar{q} = \frac{1}{t_1 + t_2} \int_0^{t_1+t_2} \dot{q}(t) \, dt \qquad (15)$$

We seek the maximum of this average exchanged power:

$$\max(\bar{q}) = \max\left(\frac{1}{t_1 + t_2} \int_0^{t_1+t_2} \dot{q}(t) \, dt \right) \qquad (16)$$

This maximum corresponds to the time t_1^* for which the derivative vanishes:

$$\frac{\partial \bar{\dot{q}}}{\partial t_1} = 0 \Leftrightarrow \frac{(t_1 + t_2) \times \frac{\partial}{\partial t_1}\left(\int_0^{t_1+t_2} \dot{q}(t) \times dt\right) - \int_0^{t_1+t_2} \dot{q}(t) \times dt}{(t_1 + t_2)^2} = 0$$

This gives the following optimization expression

$$(t_1 + t_2) \times \frac{\partial}{\partial t_1}\left(\int_0^{t_1+t_2} \dot{q}(t) \times dt\right) - \int_0^{t_1+t_2} \dot{q}(t) \times dt = 0 \qquad (17)$$

In this expression, the first term represents the derivative of an integral with variable limit. This derivative can be placed in the form (J. Bass, 1977) :

$$\frac{\partial}{\partial t_1}\left(\int_0^{t_1+t_2} \dot{q}(t) \times dt\right) = \dot{q}(t_1)$$

It results in the final form of the optimum condition:

$$\dot{q}(t_1)(t_1 + t_2) - \int_0^{t_1+t_2} \dot{q}(t) \times dt = 0 \qquad (18)$$

By solving this equation, we determine the optimal operating duration (t_1^*) corresponding to maximal average exchanged power. This is a dynamic optimization.

It remains thus to express the exchanged power, $\dot{q}(t)$. For this purpose, three major directions considered : constant mass flow rate, constant flow velocity and pumping power. The details follow.

2.1 Constant mass flow rate

We can consider that the fouling fluid flow rate ($\dot{m}$) is constant during the functioning of the heat exchanger. This translates into:

$$\dot{q}(t) = \dot{m} \times Cp \times \Delta T_{max} \times \varepsilon(t) \qquad (19)$$

The exchanged power depends thus on the time by means of only one term: the effectiveness of the exchanger.

$$\varepsilon(t) = \frac{1 - \exp\left[-(1 + \beta c) \times NTU(t)\right]}{1 + \beta c} \qquad (20)$$

NTU(t) represents the number of transfer units:

$$NTU(t) = \frac{kg(t) \times Su_{ref}}{(\dot{m} \times Cp)_{min}} \qquad (21)$$

With kg(t) the overall heat transfer coefficient:

$$\frac{1}{kg(t) \times Su_{ref}} = \frac{1}{he \times Su_e} + \frac{R_{cd}}{Su_{ref}} + \frac{R_f(t)}{Su_{ref}} + \frac{1}{hi(t) \times Su_i(t)} \qquad (22)$$

By introducing expressions of the surface (7), the diameter(10) and the thickness of the deposit (2) in the function of the fouling kinetic $R_f(t)$, and by introducing nondimensional parameters (noted Xe, A', A", L*, H*), the heat transfer coefficient is:

$$\frac{1}{kg(t) \times Su_{ref}} = \frac{1}{he \times Su_e}\left\{L^* + H^* + A' + A'' \times f(t/\tau_e)\right\}$$

and therefore, the expression for NTU(t) in this case study is

$$NTU(t) = \frac{Xe \times L^*}{L^* + H^* + A' + A'' \times f(t/\tau_e)} \qquad (23)$$

where

$$Xe = \frac{he \times Su_{i0}}{(\dot{m}_0 \times Cp)_{min}} \qquad (24)$$

$$L^* = \frac{ri}{re} \qquad (25)$$

$$H^* = \frac{he}{hi} \qquad (26)$$

$$A' = he \times Rcd \qquad (27)$$

$$A'' = he \times R_f^* \qquad (28)$$

The power to be introduced in the optimum condition:

$$\dot{q}(t) = \frac{\dot{m} \times Cp \times \Delta T_{max}}{1 + \beta c_0} \times \left\{1 - \exp\left[-\frac{(1 + \beta c_0) \times Xe \times L^*}{L^* + H^* + A' + A'' \times f(t/\tau_e)}\right]\right\} \qquad (29)$$

The equation to solve to obtain the optimal operating time corresponding to maximal average power is therefore after some simplifications:

$$(t_1 + t_2) \times \left\{1 - \exp\left[-\frac{(1 + \beta c_0) \times Xe \times L^*}{L^* + H^* + A' + A'' \times f(t_1/\tau_e)}\right]\right\} - \int_0^{t_1} \left\{1 - \exp\left[-\frac{(1 + \beta c_0) \times Xe \times L^*}{L^* + H^* + A' + A'' \times f(t/\tau_e)}\right]\right\} \times dt = 0 \qquad (30)$$

2.2 Constant flow velocity

In this second case, we can consider that the fouling fluid velocity is constant. This implies a modification of the expression of the exchanged power:

$$\dot{q}(t) = \overline{v} \times \rho \times Se(t) \times Cp \times \Delta T_{max} \times \varepsilon(t) \quad (31)$$

The power depends on the operating time by two terms: the efficiency of the heat exchanger and the cross-section area.

The Reynolds number is expressed as:

$$Re_i(t) = Re_{i0} \times [\exp(-B \times f(t/\tau_e))] \quad (32)$$

This allows the heat transfer coefficient to be obtained as:

$$hi(t) = hi_0 \times [\exp(-B \times f(t/\tau_e))]^{-\frac{1}{3}} \quad (33)$$

and the number of transfer units as :

$$NTU(t) = \frac{Xe \times L^* \times [\exp(B \times f((t/\tau_e)))]^2}{L^* + A' + A'' \times f(t/\tau_e) + H^* \times [\exp(B \times f((t/\tau_e)))]^{\frac{2}{3}}} \quad (34)$$

Also, in this case, due to variable mass flow rate, the calorific quotient βc becomes dependent of time as follows:

$$\beta c(t) = \beta c_0 \times [\exp(-B \times f(t/\tau_e))]^2 \quad (35)$$

The power to be introduced in the optimum condition:

$$\dot{q}(t) = \overline{v} \times \rho \times Cp \times \Delta T_{max} \times \frac{1 - \exp[-(1 + \beta c(t)) \times NTU(t)]}{1 + \beta c(t)} \times Se_{i0} \times [\exp(-B \times f(t/\tau_e))]^2 \quad (36)$$

For clarity, we have not inserted expressions of the number of transfer units (34) and of the calorific quotient (35). The equation to solve in order to obtain the optimal functioning time corresponding to maximal average power becomes:

$$\frac{(t_1 + t_2) \times [\exp(-B \times f(t_1/\tau_e))]^2}{1 + \beta c(t_1)} \times \{1 - \exp[-(1 + \beta c(t_1)) \times NTU(t_1)]\}$$

$$- \int_0^{t_1} \frac{[\exp(-B \times f(t/\tau_e))]^2}{1 + \beta c(t)} \times \{1 - \exp[-(1 + \beta c(t)) \times NTU(t)]\} \times dt = 0 \quad (37)$$

2.3 Constant pumping power

Finally, we can suppose that the pumping power is constant; this corresponds to industrial practise. In this third approach, we introduce the mechanical losses, which have not been introduced in the two proceeding approaches mechanical losses will be represented by means of a friction coefficient; this coefficient depends essentially on the geometry, the state of the surface (roughness) and the flow regime.

The pumping power is expressed as:

$$\dot{Pm} = \dot{Qv}(t) \times \frac{dP}{dx}(t) \quad (38)$$

where,

$$\dot{Qv}(t) = \frac{\dot{m}(t)}{\rho} \quad : \text{volumetric fluid flow rate}$$

dP/dx: pressure loss per meter of duct (therefore, we suppose implicitly that singular losses are proportional to regular losses).

$$\frac{dP}{dx}(t) = \frac{2 \times f(t) \times \dot{m}^2(t)}{\rho \times Di(t) \times Se^2(t)} \quad (39)$$

In the laminar flow regime (Re<2000) for a circular duct homogeneous in roughness, the expression of the friction coefficient is given by the Hagen-Poisseuille relationship (I. E. Idelcik, 1986):

$$f(t) = \frac{16}{Re(t)} \quad (40)$$

The Reynolds number associated to the fouling fluid is expressed as:

$$Re_i(t) = \frac{\dot{m}(t) \times Dh(t)}{\mu \times Se(t)}$$

So, we deduce the expression of the flow rate:

$$\dot{m}(t) = \dot{m}_0 \times [\exp(-B.f(t/\tau_e))]^2 \quad (41)$$

These expressions allow us to define the corresponding expression of the flow rate and consequently the power as functions of time as follows:

$$\dot{q}(t) = \varepsilon(t) \times \dot{m}(t) \times Cp \times \Delta T_{max} \qquad (42)$$

In this last expression, the power depends on operating duration by means of the mass flow rate and the efficiency of the exchanger.

As before, we define the Reynolds number:

$$Re(t) = Re_0 \times [\exp(-B \times f(t/\tau_e))] \qquad (43)$$

The heat transfer coefficient:

$$hi(t) = hi_0 \times [\exp(-B \times f(t/\tau_e))]^{-\frac{1}{3}} \qquad (44)$$

The number of transfer units:

$$NTU(t) = \frac{Xe \times L * \times [\exp(B \times f(t/\tau_e))]^2}{L* + A' + A'' \times f(t/\tau_e) + H^* \times [\exp(B \times f(t/\tau_e))]^{\frac{2}{3}}} \qquad (45)$$

and the equation of optimization:

$$\frac{(t_1 + t_2) \times [\exp(-B \times f(t_1/\tau_e))]^2}{1 + \beta c(t_1)} \times \{1 - \exp[-(1 + \beta c(t_1)) \times NTU(t_1)]\}$$

$$- \int_0^{t_1} \frac{[\exp(-B \times f(t/\tau_e))]^2}{1 + \beta c(t)} \times \{1 - \exp[-(1 + \beta c(t)) \times NTU(t)]\} \times dt = 0 \qquad (46)$$

3. Application

From the previously developed equations, we present two kinds of results. First, we will present a comparative study of the three approaches; that is, for a chosen kinetics (squared kinetics) we will study the influence of the considered approach (constant mass flow rate, constant velocity, or constant pumping power) on the evolution of the optimal operating time τ_1^*. Then, we will present the influence of the kinetic on this time τ_1^*; that is, we will consider in the case of the constant pumping power and we will present the four curves corresponding to the four kinetics studied.

A certain number of dimensionless parameters and variables has been introduced in order to work in a nondimensional form. The dimensionless operating time (variable of the optimization study) is defined as:

$$\tau_1 = \frac{t_1}{\tau_e} \qquad (47)$$

This change of variable introduces a new parameter τ_2 that represents the nondimensional duration of rest period at the beginning of which it is necessary to stop the installation in order to clean it. Following results concerning this parameter (τ_2) have been chosen to illustrate the model.

$$\tau_2 = \frac{t_2}{\tau_e} \qquad (48)$$

Indeed, from sensitivity studies it is conclued that among the eight parameters: τ_2, A', A'', B, βc_0, Xe, L*, H*,τ_2 has the greatest influence on the evolution of the optimal operating time. Moreover, this parameter is important for manufacturers because it represents the duration for cleaning. To optimize an installation, it is neces-

sary to find a good compromise between the operating time and the cleaning duration ; because τ_2 is a parameter important for the maintenance. It entails often a stop or a sensitive decline of the production.

This variation range is: $\tau_2 \in [0{,}01 - 1]$.

The other parameters are fixed to a central value:

$$A' = 0.05, \quad A'' = 0.05, \quad B = 0.005,$$

$$\beta c_0 = 0.8, \quad H* = 1, \quad L* = 0.9, \quad Xe = 1$$

Recall that the three expressions of optimization are valid for:

- cylindrical geometry
- co-current and laminar flow
- by considering only one fouling fluid

that is the internal and limiting fluid.

We observe that there is no marked difference between the three approaches at the level of τ_1. Nevertheless, the study at constant pumping power is the most pessimistic for estimating of the optimal operating time. But we are tempted to think that this approach is the most realistic and that it adapts better to demands of manufacturers using heat exchangers that are fouling. Indeed manufacturers confronted with this type of phenomenon, are tempted to work with a constant pumping power rather that a constant flow rate or constant velocity. Moreover, this last model is the only one of the three that takes into account of mechanical irreversibilities linked to the phenomenon of fouling. Effectively, we have introduced in this approach of constant pumping power load losses caused by the deposits inside the tube by means of a friction coefficient ; the mechanical losses added to thermal losses can explain why the optimal operating duration is shorter in this case.
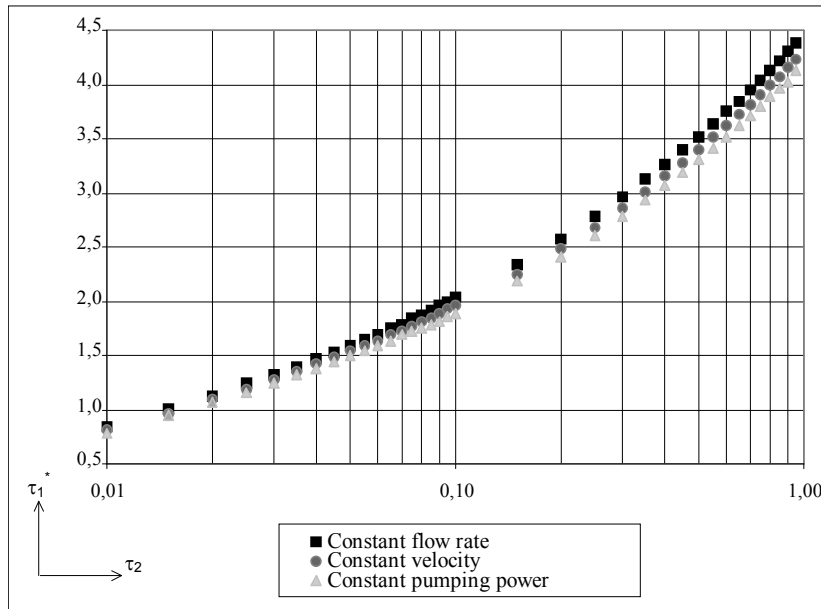
Figure 2. Influence of the type of approach on the evolution of the optimal operating time.
Co - current and laminar flow - cylindrical geometry - squared kinetic

For what concerns the study of the influence of the deposit kinetic, we observe that the optimal operating time τ_1^* is always much long when the evolution of the deposit is slow. This verification seems natural because: an evolution of the rapid fouling resistance translating a marked deposit, (such that the squared kinetics) will entail a shorter operating time, and to the contrary, a slower kinetics (the asymptotic kinetic tends to a threshold value) will entail operating times longer, to see infinite in our time scale (year). The decreasing results obtained for operating times are as follow: the asymptotic kinetics to which are associated longest operating times, followed by squared root kinetics and proportional kinetics, then the squared kinetics to which will be associated the shortest operating time.

By putting the expressions of the fouling kinetics in the form

$$R_f(t) = R_f^* \times \left(\frac{t}{\tau_e}\right)^n \text{ with n=\{0, 0.5, 1, 2\}} \quad (49)$$

we can study the influence of the power index of kinetics, n, on the value of the optimal operating time. In effect, proportional, squared root and squared kinetics can be easily put under this form by considering respectively n = 1, n = 0.5 and n = 2. The asymptotic kinetics is approached by the same relationship with n tending to zero. The extreme case n = 0 corresponds to the threshold value of the fouling resistance.

The graph in *Figure 4*, is only another form to present the influence of the kinetics on the

optimal value of the operating time: τ_1^*. This allows verification of the preceding conclusions. So, we observe that the more rapid the kinetics (n great), the shorter is the optimal operating time τ_1^*.

Then that, to the contrary, for kinetics of evolution a lot slower, close to a constant evolution (n tending to zero) such as the asymptotic kinetics, the optimal operating time is a lot more important.

We notice that the influence of the kinetic is much more pronounced when n is weak; thus the influence is very important for n < 0.5, but clearly less marked for n > 0.5. Therefore, it is important to know perfectly the kinetics if n < 0.5.

4. Conclusion

This article presents an optimization study of the operating time of a heat exchanger under the phenomenon of fouling; the optimization is based on the maximization of the average power exchanged.

Considering the complexity of the phenomenon and the diversity of cases, we have presented an important special case, which concerns a cylindrical geometry exchanger with fouling on the inside of the tube, for fluids in co - current and laminar flow.

To take into account the mode of operation of the exchanger, three different approaches have been developed: we have considered that the mass flow rate of fouling fluid was constant,
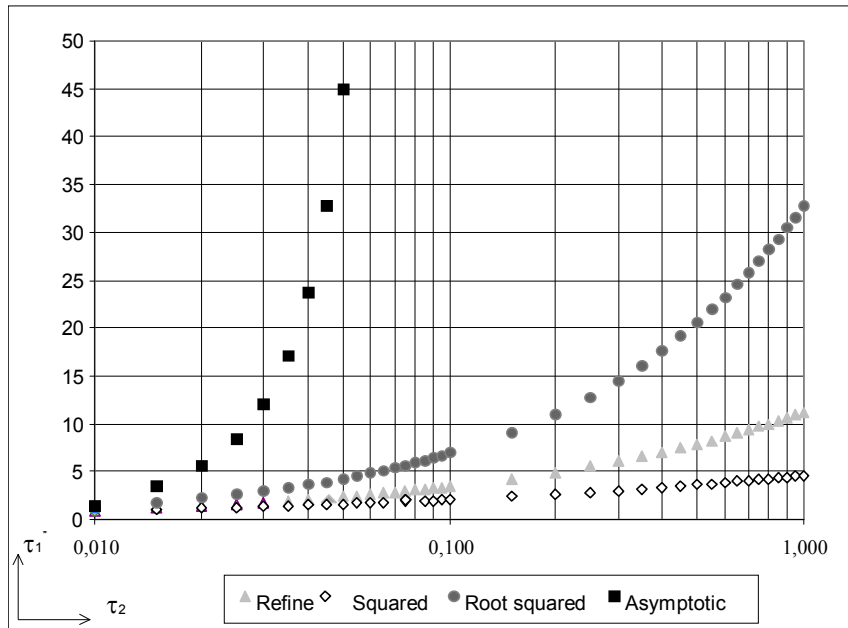
Figure 3. Influence of the kinetics of fouling on the evolution of the optimal operating time. Co-current and laminar flow - cylindrical geometry - constant pumping power.
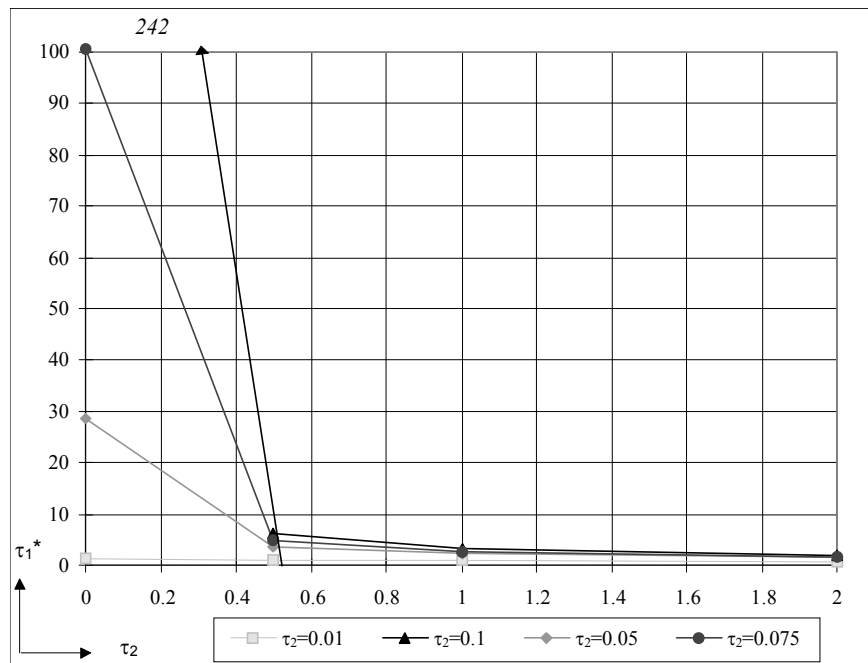

Figure 4. Influence of the kinetics of fouling on the evolution of the optimal operating time. Co - current and laminar flow - cylindrical geometry - constant pumping power

or we have considered that the velocity in the section was constant or again we have considered a the pumping power was fixed. A comparison of results of these three approaches allows us to observe that, even if they don't lead to differences at the level of the optimal functioning time, it seems that the approach to constant pumping power is the most adapted to depict the phenomenon, since in this model we take into account thermal losses and load losses linked to the fouling.

Moreover, we have presented a study with respect to the deposit kinetics that allows a better understanding of the role of the fouling.

The totality of cases (two geometries, two regimes and two natures of flow, the four fouling kinetics, and the three studies) has brought us to

study near a hundred of different cases. For each of these cases, we have taken care to do a sensitivy study around the eight parameters introduced in models; this to be able to judge their influence on the optimal value of the time of functioning τ_2. It emerges from these studies that the parameter τ_2 (proportional to the interuption of the installation for cleaning) has the most marked influence on τ_1^*; it is why results in this article are functions of the parameter τ_1^*. The totality of all the other works is available in internal reports relative to a thesis currently in progress (L. Schaal, 1996-1997).

Nomenclature

A′	dimensionless group defined by Eq.(26)
A″	dimensionless group defined by Eq.(27)
B	dimensionless group defined by Eq.(8)
β_c	calorific quotient
β_{c0}	initial calorific quotient
c_p	specific heat (J.kg^{-1}.K^{-1})
δ	deposit thickness (m)
Di	tube inside diameter (m)
ε	heat exchanger efficiency
f	friction coefficient
ΔT_{max}	maximal temperature difference (°C)
hi	internal heat transfer coefficient (W.m^{-2}.K^{-1})
he	external heat transfer coefficient (W.m^{-2}.K^{-1})
H*	dimensionless group defined by Eq.(25)
kg	overall heat transfer coefficient (W.m^{-2}.K^{-1})
L	tube length (m)
L*	dimensionless group defined by Eq.(24)
λ_d	thermal conductivity of the deposit (W.m^{-2}.K^{-1})
λ	thermal conductivity of the fouling fluid (W.m^{-1}.K^{-1})
$\dot{m}$	mass flow rate (kg.s^{-1})
Nu	Nusselt number.
NTU	number of transfer unit
$\dot{P}m$	Pumping power (W)
Pr	Prandtl number
q	heat transferred (J)
$\dot{q}$	heat power (W)
Qv	volumetric flow rate (m^3.s^{-1})
ρ	density (kg.m^{-3})
Re	Reynolds number
R_f	fouling resistance (m².K.W^{-1})
R_f*	asymptotic fouling resistance (m².K.W^{-1})
ri	inner radius (m)
Se	tube cross section (m²)
Su	exchange area (m²)
t	time (s)
t_1	operating time (s)
t_2	cleaning time (s)
τ_e	time constant (s)
τ_1	dimensionless group defined by Eq.(46)
τ_2	dimensionless group defined by Eq.(47)
μ	absolute viscosity (kg.m^{-1}.s^{-1})
$\overline{v}$	mean velocity in the section (m.s^{-1})
Xe	dimensionless group defined by Eq.(23)

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