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Efficient exponential ratio estimator for estimating the population mean in simple random sampling

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Abstract

This paper proposes, with justification, two exponential ratio estimators of population mean in simple random sampling without replacement. Their biases and mean squared error are derived and compared with existing related ratio estimators. Analytical and numerical results show that at optimal conditions, the proposed ratio estimators are always more efficient than the regression estimator and some existing estimators under review.

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1. Introduction

In Sample Surveys, auxiliary information are always used to improve the precision of estimates of population parameters. This can be done at either estimation or selection stage or both stages. The commonly used estimators, which make use of auxiliary variables, include ratio estimator, regression estimator, product estimator and difference estimator. The classical ratio estimator is preferred when there is a high positive correlation between the variable of interest, Y and the auxiliary variable, X with the regression line passing through the origin. The classical product estimator, on the other hand is mostly preferred when there is a high negative correlation between Y and X while the linear regression estimator is most preferred when there is a high positive correlation between the two variables and the regression line of the study variable on the auxiliary

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variable has intercept on Y axis. Ratio estimation has gained relevance in Estimation theory because of its improved precision in estimating the population parameters. It has been widely applied in Agriculture to estimate the mean yield of crops in a certain area and in Forestry, to estimate with high precision, the mean number of trees or crops in a forest or plantation. Other areas of relevance include Economics and Population studies to estimate the ratio of income to family size.

According to [13], regression estimator, in spite of its lesser practicability, seems to be holding a unique position due to its sound theoretical basis. The classical ratio and product estimators even though considered to be more useful in many practical situation have efficiencies which does not exceed that of the linear regression. As a result of this limitation, most authors have carried out several researches towards the modification of the existing ratio, product or classes of ratio and product estimators of the population mean in simple random sampling without replacement to improve efficiency. Among authors, who have carried out researches in this direction are [9], [10], [11], [25], [14], [15] , [5], [2], [3], [1], [20], [21], [22], [23], [15], [16], [4], [19] and [28].

So far, only the estimators proposed by [17], which is a modification of those of [9] and [10] is more efficient than the linear regression estimator.. This paper therefore proposes ratio estimators using an exponential ratio estimator, whose efficiencies would be better than regression estimator, [5] and compared with other ratio estimators including [17]. Authors like [6], [7],[13] and [18] extended related works of ratio estimators to stratified sampling.

This work reviews some related existing estimators, proposes new improved estimators and derive their properties. Their efficiencies are used to compare with other existing estimators and empirical results used to validate every theoretical claim.

2. Review of some related existing Estimators

Consider a finite population $\Pi = \{\pi_1, \pi_2, \ldots, \pi_N\}$ of size N. Let Y and X be the study and auxiliary variables with population means $\bar{Y}$ and $\bar{X}$ respectively. It is assumed that information on the population mean $\bar{X}$ of the auxiliary variable is known and $Y_i, X_i \geq 0$ (since the survey variables are generally non-negative). Let a sample of size n be drawn by simple random sampling without replacement (SRSWOR) from the population Π and the sample means $\bar{y}$ and $\bar{x}$ of the study and auxiliary variables obtained respectively. Given the above population, a summary of some related existing estimators with their Mean Squared Errors (MSE's) are given below:

Table 1: Existing related estimators with their MSEs

S/N	Estimators	MSE
1	$\bar{y}$, unbiased sample mean	$\bar{Y}^2\lambda C_y^2$
2	$\bar{y}_R = \frac{\bar{y}}{\bar{x}}\bar{X}$, Classical Ratio	$\bar{Y}^2\lambda[C_y^2 - 2\rho C_y C_x + C_x^2]$
3	$\bar{y}_{\Re} = \bar{y}\exp\left[\frac{(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})}\right]$ Bahl and Tuteja [1]	$\bar{Y}^2\lambda[C_y^2 + \frac{C_x^2}{4}(1-4k)]$
4	$\bar{y}_{\text{GS}} = \left[\omega_1^*\bar{y} + \omega_2^*\left(\bar{X}-\bar{x}\right)\right]\left(\frac{\eta\bar{X}+\delta}{\eta\bar{x}+\delta}\right)$ Gupta and Shabbir [5]	$\bar{Y}^2[1-\nu_1]$
5	$\bar{y}_{\text{GS}} = \psi_1^*\bar{y}\left(\frac{\eta\bar{X}+\delta}{\eta\bar{x}+\delta}\right) + \psi_2^*\left(\bar{X}-\bar{x}\right)\left(\frac{\eta\bar{X}+\delta}{\eta\bar{x}+\delta}\right)^2$ Singh and Solanki [17]	$\bar{Y}^2[1-\nu_2]$
6	$\bar{y}_{\text{reg}} = \bar{y} + b\left(\bar{X}-\bar{x}\right)$, Regression Estimator	$\bar{Y}^2\lambda C_y^2(1-\rho^2)$
7	$t_{(\alpha,\zeta)} = \bar{y}\{2 - \left(\frac{\bar{x}}{\bar{X}}\right)^{\alpha}\exp\left[\frac{\zeta\left(\bar{x}-\bar{X}\right)}{\left(\bar{X}+\bar{x}\right)}\right]\}$ Solanki et al [25]	$\bar{Y}^2\lambda\{C_y^2 + \frac{(2\alpha+\zeta)}{4}C_x^2[(2\alpha+\zeta)+4k]\}$

where
$C_x = \frac{S_x}{\bar{X}}$ be the coefficient of variation of the auxiliary variable,
$C_y = \frac{S_y}{\bar{Y}}$ be the coefficient of variation of the study variable,
$\rho = \frac{S_{xy}}{S_x S_y}$ be the correlation coefficient between the auxiliary and study variables
$k = \frac{\rho C_y}{C_x}$ and $f = \frac{n}{N}$, the sampling fraction; where

$$S_x{}^2 = (N-1)^{-1}\sum_{i=1}^{N}\left(x_i - \bar{X}\right)^2,$$

population variance of the auxiliary variable;

$$S_y{}^2 = (N-1)^{-1}\sum_{i=1}^{N}\left(y_i - \bar{Y}\right)^2,$$

population variance of the study variable;

$$S_{xy} = (N-1)^{-1}\sum_{i=1}^{N}\left(x_i - \bar{X}\right)\left(y_i - \bar{Y}\right),$$

population covariance between the auxiliary and study variables;

$$\bar{X} = N^{-1}\sum_{i=1}^{N} x_i, \text{ population mean of the auxiliary variable}$$

$$\bar{Y} = N^{-1}\sum_{i=1}^{N} y_i, \text{ population mean of the study variable}$$

$$\bar{x} = n^{-1}\sum_{i=1}^{n} x_i, \text{ sample mean of the auxiliary variable,}$$

$$\bar{y} = n^{-1}\sum_{i=1}^{n} y_i, \text{ sample mean of the study variable,}$$

$$\alpha_1=\left\{1+\lambda\left[C_y^2+\tau C_x^2\left(3\tau-4k\right)\right]\right\},\alpha_2=\lambda C_x^2,\alpha_3=\lambda C_x^2\left(k-2\tau\right),$$
$$\alpha_4=\left[1-\lambda\tau C_x^2\left(k-\tau\right)\right],\alpha_5=\lambda\tau C_x^2$$
$$\tau=\frac{\eta\bar{X}}{\left(\eta\bar{X}+\delta\right)}.$$
$$A=\left\{1+\lambda\left[C_x^2+\tau C_x^2\left(3\tau-4k\right)\right]\right\},B=\lambda C_x^2,C=\lambda C_x^2\left(3\tau-k\right),$$
$$D=\left[1+\lambda\tau C_x^2\left(\tau-k\right)\right],E=2\lambda\tau C_x^2$$
$$\omega^*=\frac{\left(\alpha_2\alpha_4+\alpha_3\alpha_5\right)}{\left(\alpha_1\alpha_2-\alpha_3^2\right)},\omega^*=\frac{R\left(\alpha_1\alpha_5+\alpha_3\alpha_4\right)}{\left(\alpha_1\alpha_2-\alpha_3^2\right)},$$
$$R=\frac{\bar{Y}}{\bar{X}},\nu_1=\frac{\left(\alpha_2\alpha_4^2+2\alpha_3\alpha_4\alpha_5+\alpha_1\alpha_5^2\right)}{\left(\alpha_1\alpha_2-\alpha_3^2\right)}$$
$$\psi_1^*=\frac{(\mathrm{BD}-\mathrm{CE})}{(\mathrm{AB}-C^2)},\psi_2=\frac{(\mathrm{AE}-\mathrm{CD})}{(\mathrm{AB}-C^2)},\nu_2=\frac{\left(BD^2-2\mathrm{CDE}+AE^2\right)}{(\mathrm{AB}-C^2)}$$

ω_1^*,ω_2^*, ψ_1^* and ψ_2^* are optimum values of ω_1,ω_2, ψ_1 and ψ_2 respectively, η $(\eta\neq 0)$, α, δ and ζ are suitably chosen constants or functions of the known parameters such as standard deviation S_x, moment ratios $\beta_1(x)$, $\beta_2(x)$, Coefficient of Variation, C_x, and Correlation Coefficient $\rho_{Y,X}$ between the variables Y and X, and so on.

[17] made corrections on the Mean Squared Error(MSE) of the class of estimators proposed by [5] to obtain the correct expression of the MSE. The corrected version would be used in this study. They went further to compare the efficiency of the estimators of [5] with those proposed by [9], [10], [11] and found that a class of estimators proposed by [5] was more efficient than those of [9], [10], [11]. [17] proceeded to propose a new class of modified estimators from that of [5]. These estimators were more efficient than those of [5], [9], [10], [15] and the regression estimator. In this paper, two alternative ratio estimators which are more efficient than the linear regression estimators are proposed with justification.

3. Proposed Estimator I

The first ratio estimator is proposed as

$$\bar{y}_{\mathrm{pr1}}=\theta_1\bar{y}+\theta_2\left(\bar{X}-\bar{x}\right)\exp\left[\left(\bar{X}-\bar{x}\right)/\left(\bar{X}+\bar{x}\right)\right] \tag{3.1}$$

θ_1 and θ_2 are suitably chosen scalars, such that $\theta_1>0$ and $-\infty<\theta_2<\infty$.

3.1. The bias and Mean Squared Error of the proposed estimator. The proposed estimator in terms of e's, is expressed as

$$\bar{x}=\bar{X}\left(1+e_x\right)\bar{y}=\bar{Y}\left(1+e_y\right) \tag{3.2}$$

where $e_x=\bar{x}-\bar{X}/\bar{X}\quad e_y=\bar{y}-\bar{Y})/\bar{Y}$.

$$\begin{aligned}&E[e_x]=E[e_y]=0,E[e_x]^2=\frac{1-f}{n}{C_x}^2;E[e_y]^2=\frac{(1-f}{n}{C_y}^2;\\&E[e_xe_y]=\frac{(1-f)}{n}\rho\ C_xC_y=\frac{(1-f)}{n}k{C_x}^2.\end{aligned} \tag{3.3}$$

$$\bar{y}_{\mathrm{pr1}}=\bar{Y}\left[\theta_1+\theta_1e_y-\theta_2\frac{\bar{X}}{\bar{Y}}\left[1-\frac{e_x}{2}\left(1+\frac{e_x}{2}\right)^{-1}+\frac{e_x^2}{2}\left(1+\frac{e_x}{2}\right)^{-2}+\dots\right]\right]. \tag{3.4}$$

It is assumed that $|e_x|<1$; $|e_y|<1$ so that $\left(1+\frac{e_x}{2}\right)^{-1}$ and $\left(1+\frac{e_x}{2}\right)^{-2}$ can be expanded.

Expanding equation (3.4) by Taylor series approximation and neglecting terms of e's having powers greater than two, we have:
$\bar{y}_{\mathrm{pr1}} = \bar{Y}\left[\theta_1 + \theta_1 e_y - \theta_2 m e_x[1 - (e_x/2)\left(1 - e_x/2 + e_x^2/4\right) + e_x^2/8]\right]$
where $m = \bar{X}/\bar{Y}$, leading to

$$\bar{y}_{\mathrm{pr1}} - \bar{Y} = \bar{Y}\left\{(\theta_1 - 1) + \theta_1 e_y - \theta_2 m e_x + \theta_2 m \frac{e_x^2}{2}\right\}. \tag{3.5}$$

Therefore, the Bias of the estimator is given as

$$B(\bar{y})_{pr1} = E[\bar{y}_{\mathrm{pr1}} - \bar{Y}] = \bar{Y}[(\theta_1 - 1) + \theta_2 \mathrm{m}\lambda \frac{{C_x}^2}{2}] \tag{3.6}$$

The MSE of $\bar{y}_{\mathrm{pr1}}$ to first degree approximation is obtained by squaring equation (3.5) and ignoring powers of 'e' greater than two and taking the expectation of the square as follows:
$\left(\bar{y}_{\mathrm{pr1}} - \bar{Y}\right)^2 = \bar{Y}^2\left[(\theta_1 - 1)^2 + \theta_2(\theta_2 - 1) m e_x^2 + \theta_1^2 e_y^2 - 2\theta_1\theta_2 m e_y e_x + \theta_2^2 m^2 e_x^2\right]$
$= \bar{Y}^2\left[\theta_1^2 - 2\theta_1 + 1 + \theta_1\theta_2 m e_x^2 - \theta_2 m e_x^2 + \theta_1^2 e_y^2 - 2\theta_1\theta_2 m e_y e_x + \theta_2^2 m^2 e_x^2\right]$
$= \bar{Y}^2\left[1 + \theta_1^2\left(1 + e_y^2\right) - 2\theta_1 - 2\theta_1\theta_2 m\left(e_y e_x - \frac{e_x^2}{2}\right) - 2\theta_2 m \frac{e_x^2}{2} + \theta_2^2 m^2 e_x^2\right].$

$$\begin{aligned} \mathrm{MSE}\left(\bar{y}_{\mathrm{pr1}}\right) &= E\left(\bar{y}_{\mathrm{pr1}} - \bar{Y}\right)^2 = \bar{Y}^2[1 + \theta_1^2\left(1 + \lambda C_y^2\right) - 2\theta_1 - \\ &- 2\theta_1\theta_2 \mathrm{m}\lambda C_x^2\left(k - \frac{1}{2}\right) - 2\theta_2 \mathrm{m}\lambda \frac{C_x^2}{2} + \theta_2^2 m^2 \lambda C_x^2] \\ &= \bar{Y}^2[1 + \theta_1^2\gamma_1 - 2\theta_1 - 2\theta_1\theta_2 m\gamma_2 - 2\theta_2 m\gamma_3 + \theta_2^2 m^2\gamma_4] \end{aligned} \tag{3.7}$$

where $\gamma_1 = 1 + \lambda C_y^2, \gamma_2 = C_x^2\lambda\left(k - \frac{1}{2}\right), \gamma_3 = \frac{\lambda C_x^2}{2}, \gamma_4 = \lambda C_x^2$

3.2. Optimal conditions for MSE of proposed estimator I. To obtain the optimum values of θ_1 and θ_2 that would minimize the MSE of the estimator, the partial derivative of (3.7) is taken with respect to θ_1 and θ_2 respectively and equated to zero as shown below:

$$\frac{\partial \mathrm{MSE}\left(\bar{y}_{\mathrm{pr1}}\right)}{\partial\theta_1} = 2\theta_1\gamma_1 - 2\theta_2 m\gamma_2 - 2 = 0 \Rightarrow \theta_1\gamma_1 - \theta_2 \mathrm{m}\gamma_2 = 1 \tag{3.8}$$

$$\frac{\partial \mathrm{MSE}\left(\bar{y}_{\mathrm{pr1}}\right)}{\partial\theta_2} = -2\theta_1 m\gamma_2 - 2m\gamma_3 + 2\theta_2 m^2\gamma_4 = 0 \Rightarrow -\theta_1 m\gamma_2 + \theta_2 m^2\gamma_4 = m\gamma_3 \tag{3.9}$$

Solving equations (3.8) and (3.9) simultaneously gives the optimal values of θ_1 and θ_2 as

$$\theta_1^* = (\gamma_4 + \gamma_2\gamma_3)/(\gamma_1\gamma_4 - \gamma_2^2) \tag{3.10}$$

$$\theta_2^* = R(\gamma_2 + \gamma_1\gamma_3)/(\gamma_1\gamma_4 - \gamma_2^2) \tag{3.11}$$

where $R = \bar{Y}/\bar{X}$

Substituting equations (3.10) and (3.11) in (3.7) gives the minimum MSE as:

$$\mathrm{MSE}_{\min}\left(\bar{y}_{\mathrm{pr1}}\right) = \bar{Y}^2\left\{1 - [(\gamma_4 + 2\gamma_2\gamma_3 + \gamma_1\gamma_3^2)/(\gamma_1\gamma_4 - \gamma_2^2)]\right\} \tag{3.12}$$

which leads to

$$\mathrm{MSE}_{\min}\left(\bar{y}_{\mathrm{pr1}}\right) = \bar{Y}^2[1 - q_1] \tag{3.13}$$

where $q_1 = (\gamma_4 + 2\gamma_2\gamma_3 + \gamma_1\gamma_3^2)/(\gamma_1\gamma_4 - \gamma_2^2)$. These results can be summarized in theorem I below:

3.1. Theorem. *If $\theta_1 \to \theta_1^*$ and $\theta_2 \to \theta_2^*$ such that $\theta_1^* > 0$ and $-\infty < \theta_2^* < \infty$ the proposed estimator will have a Mean Squared Error,*
$MSE(\bar{y}_{pr1}) \geq \bar{Y}^2 \left\{1 - [(\gamma_4 + 2\gamma_2\gamma_3 + \gamma_1\gamma_3^2)/(\gamma_1\gamma_4 - \gamma_2^2)]\right\}$,
with strict equality holding if $\theta_1 = \theta_1^*$ and$\theta_2 = \theta_2^*$.

3.3. Some special cases of proposed estimator I.

Case I: When $\theta_1 = 1$. The proposed estimator becomes

$$\bar{y}_{\text{pr11}} = \bar{y} + \theta_2 \left(\bar{X} - \bar{x}\right) \exp\left[(\bar{X} - \bar{x})/\left(\bar{X} + \bar{x}\right)\right], \tag{3.14}$$

which is obtained by setting $\theta_1 = 1$ in (3.7). The optimum value of θ_2 that would make the MSE a minimum is:

$$\theta_2^{'} = \frac{\gamma_2 + \gamma_3}{m\gamma_4} = B \tag{3.15}$$

where B is the regression coefficient. Substitution of equation (3.15) into (3.7) with $\theta_1 = 1$, gives the minimum MSE as

$$\text{MSE}_{\min}\left(\bar{y}_{\text{pr11}}\right) = \lambda\bar{Y}^2 C_y^2 (1 - \rho^2) \tag{3.16}$$

Remark I: It should be noted here that equation (3.16) gives the same expression as the Variance of the linear regression estimator

$$\bar{y}_{\text{reg}} = \bar{y} + b(\bar{X} - \bar{x}) \tag{3.17}$$

where b is the sample regression coefficient. Therefore, when $\theta_1 = 1$ and θ_2 is optimal, the proposed estimator I has the same efficiency as the simple linear regression estimator.

Case II: When $\theta_1 = 1$ and $\theta_2 = 1$. The proposed estimator reduces to

$$\bar{y}_{\text{pr12}} = \bar{y} + \left(\bar{X} - \bar{x}\right) \exp[(\bar{X} - \bar{x})/(\bar{X} + \bar{x})] \tag{3.18}$$

with MSE given as

$$\text{MSE}\left(\bar{y}_{\text{pr12}}\right) = \lambda\bar{Y}^2 [C_y^2 - mC_x^2 \left(2k - m\right)] \tag{3.19}$$

Case III: When $\theta_1 = 1, \theta_2 = 0$ The proposed estimator reduces to unbiased sample mean estimator $\bar{y}$, with Variance given as:

$$V\left(\bar{y}_{\text{pr13}}\right) = \lambda\bar{Y}^2 C_y^2 \tag{3.20}$$

These cases are specific members of the family of the proposed estimator I obtained by varying the values of θ_1 and θ_2. Table 2 gives a summary of some members of this proposed family of estimators.

Table 2: Some members of the family of proposed estimator I and their MSE's.

S/N	Estimator	θ_1	θ_2	MSE
1	$\bar{y} + \theta_2(\bar{X} - \bar{x}) \exp\left[\frac{(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})}\right]$	1	$\frac{\gamma_2+\gamma_3}{m\gamma_4} = b$	$\lambda\bar{Y}^2 C_y^2 (1 - \rho^2)$
2	$\bar{y} + \left(\bar{X} - \bar{x}\right) \exp[\frac{(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})}]$	1	1	$\bar{Y}^2 [C_y^2 - mC_x^2 \left(2k - m\right)]$
3	$\bar{y}$	1	0	$\lambda\bar{Y}^2 C_y^2$
4	$\theta_1^*\bar{y} + \theta_2^*(\bar{X} - \bar{x}) \exp\left[\frac{(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})}\right]$	θ_1^*	θ_2^*	$\bar{Y}^2 \left\{1 - [\frac{\gamma_4+\gamma_1\gamma_3^2+2\gamma_2\gamma_3}{\gamma_1\gamma_4-\gamma_2^2}]\right\}$

4. Proposed estimator II

The second proposed estimator takes the form

$$\bar{y}_{\mathrm{pr}2} = \varphi_1 \bar{y} + \varphi_2 \left(\bar{X} - \bar{x}\right) \exp[2(\bar{X} - \bar{x})/(\bar{X} + \bar{x})] \tag{4.1}$$

Where φ_1 and φ_2 are suitable scalars and $\varphi_1 > 0, -\infty < \varphi_2 < \infty$. Expressing (4.1) in terms of e's gives

$$\bar{y}_{\mathrm{pr}2} = \bar{Y}\left\{\varphi_1 + \varphi_1 e_y - \varphi_2 m[1 - e_1\left(1 + \frac{e_x}{2}\right)^{-1} + \frac{e_x^2}{2}\left(1 + \frac{e_x}{2}\right)^{-2} + \ldots]\right\}. \tag{4.2}$$

The first degree approximation of equation (4.2) is obtained as: $\bar{y}_{\mathrm{pr}2} = \bar{Y}[\varphi_1 + \varphi_1 e_y - \varphi_2 m e_x \left(1 - e_x + e_x^2\right)]$

$$= \bar{Y}[\varphi_1 + \varphi_1 e_y - \varphi_2 m e_x + \varphi_2 m e_x^2]$$

$$\bar{y}_{\mathrm{pr}2} - \bar{Y} = \bar{Y}[(\varphi_1 - 1) + \varphi_1 e_y - \varphi_2 m e_x + \varphi_2 m e_x^2] \tag{4.3}$$

The Bias of $\bar{y}_{\mathrm{pr}2}$ is obtained from equation (4.3) as:

$$B\left(\bar{y}_{\mathrm{pr}2}\right) = E\left(\bar{y}_{\mathrm{pr}2} - \bar{Y}\right) = \bar{Y}\left[(\varphi_1 - 1) + \varphi_2 \mathrm{m}\lambda C_x^2\right]. \tag{4.4}$$

Squaring equation (4.3) and ignoring powers of 'e' greater than two, we have:

$$\begin{aligned}(\bar{y}_{\mathrm{pr}2} - \bar{Y})^2 &= \bar{Y}^2[(\varphi_1 - 1)^2 + 2\varphi_2\left(\varphi_1 - 1\right) m e_x^2 \\ &+ \varphi_2 e_y^2 - 2\varphi_1\varphi_2 m e_y e_x + \varphi_2^2 m^2 e_x^2] \\ &= \bar{Y}^2[1 + \varphi_1^2\left(1 + e_y^2\right) - 2\varphi_1 - 2\varphi_1\varphi_2 m\left(e_y e_x - e_x^2\right) - 2\varphi_2 m e_x^2 + \varphi_2^2 m^2 e_x^2]\end{aligned} \tag{4.5}$$

Its MSE is obtained by taking the expectation of equation (4.5) as shown below:

$$\begin{aligned}\mathrm{MSE}\left(\bar{y}_{\mathrm{pr}2}\right) &= E(\bar{y}_{\mathrm{pr}2} - \bar{Y})^2 = \bar{Y}^2\left[1 + \varphi_1^2\left(1 + \lambda C_y^2\right)\right. \\ &-2\varphi_1 - 2\varphi_1\varphi_2 \mathrm{m}\lambda C_x^2\left(k - 1\right) - \varphi_2 \mathrm{m}\lambda C_x^2 + \varphi_2^2 m^2 \lambda C_x^2\left.\right] \\ &= \bar{Y}^2[\varphi_1^2\gamma_1 - 2\varphi_1 - 2\varphi_1\varphi_2 m\gamma_5 - 2\varphi_2 m\gamma_4 + \varphi_2^2 m^2\gamma_4]\end{aligned} \tag{4.6}$$

where $\gamma_5 = \lambda C_x^2(k - 1)$

4.1. Optimality conditions for estimator II. To investigate the optimal conditions for estimator II, let

$$\frac{\partial \mathrm{MSE}(\bar{y}_{\mathrm{pr}2})}{\partial \varphi_1} = \frac{\partial \mathrm{MSE}(\bar{y}_{\mathrm{pr}2})}{\partial \varphi_2} = 0$$

so that,

$$\varphi_1\gamma_1 - \varphi_2 m\gamma_5 = 1 \tag{4.7}$$

$$-\varphi_1 m\gamma_5 + \varphi_2 m^2\gamma_4 = m\gamma_4. \tag{4.8}$$

Solving equations (4.7) and (4.8) simultaneously give the optimal values of φ_1 and φ_2 as:

$$\varphi_1^\star = (\gamma_4 + \gamma_4\gamma_5)/(\gamma_1\gamma_4 - \gamma_5^2) \tag{4.9}$$

$$\varphi_2^\star = R(\gamma_5 + \gamma_1\gamma_4)/\left(\gamma_1\gamma_4 - \gamma_5^2\right). \tag{4.10}$$

Substituting equations (4.9) and (4.10) in (4.6) yields the minimum MSE of the estimator as:

$$\mathrm{MSE}\left(\bar{y}_{\mathrm{pr}2}\right) = \bar{Y}^2\{1 - \left[(\gamma_4 + 2\gamma_4\gamma_5 + \gamma_1\gamma_4^2)/(\gamma_1\gamma_4 - \gamma_5^2)\right]\} = \bar{Y}^2[1 - q_2] \tag{4.11}$$

where,

$$q_2 = (\gamma_4 + 2\gamma_4\gamma_5 + \gamma_1\gamma_4^2)/(\gamma_1\gamma_4 - \gamma_5^2)$$

These results are summarized in the following theorem.

4.1. Theorem. *If $\varphi_1 \to \varphi_1^\star$ and $\varphi_2 \to \varphi_2^\star$ such that $\varphi_1^\star > 0$ and $-\infty < \varphi_2^\star < \infty$, the proposed estimator will have a Mean Squared Error of*
$MSE(\bar{y}_{pr2}) \geq \bar{Y}^2 \left\{1 - [(\gamma_4 + 2\gamma_4\gamma_5 + \gamma_1\gamma_4^2)/(\gamma_1\gamma_4 - \gamma_5^2)]\right\}$,
with strict equality holding if $\varphi_1 = \varphi_1^\star$ and $\varphi_2 = \varphi_2^\star$.

4.2. Some special cases of proposed estimator II. Some special cases of $\bar{y}_{\mathrm{pr2}}$ with varying values of φ_1 and φ_2 and MSEs are given in Table 3.

Table 3: Some types of estimator II and their MSEs

S/N	Estimator	φ_1	φ_2	MSE
1	$\bar{y}+\varphi_2(\bar{X}-\bar{x})\exp\left[\frac{2(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})}\right]$	1	$\frac{\gamma_4+\gamma_5}{m\gamma_4}=b$	$\lambda\bar{Y}^2C_y^2(1-\rho^2)$
2	$\bar{y}+(\bar{X}-\bar{x})\exp[\frac{2(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})}]$	1	1	$\bar{Y}^2[C_y^2-mC_x^2\,(2k-m)]$
3	$\bar{y}$	1	0	$\lambda\bar{Y}^2C_y^2$
4	$\varphi_1^{\star}\bar{y}+\varphi_2^{\star}(\bar{X}-\bar{x})\exp\left[\frac{2(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})}\right]$	$\varphi_1^{\star}$	$\varphi_2^{\star}$	$\bar{Y}^2\left\{1-[\frac{\gamma_4+\gamma_1\gamma_4^2+2\gamma_4\gamma_5}{\gamma_1\gamma_4-\gamma_5^2}]\right\}$

5. Efficiency Comparison

In this section, the MSE of some existing ratio estimators are compared with the optimal MSE of the proposed estimators.

5.1. Unbiased simple random sample mean, $\bar{y}$. The Variance of the simple random mean expressed in terms of $\gamma's$ is:

(5.1) $$V(\bar{y})=\bar{Y}^2(\gamma_1-1)$$

Therefore, for the proposed estimator I to be more efficient than the simple sample random mean, $\bar{y}$, $V(\bar{y})-\text{MSE}(\bar{y}_{\text{pr1}})>0$

$$\Rightarrow \bar{Y}^2[\gamma_1+q_1-2]>0$$

(5.2) $$\Rightarrow [\gamma_1+q_1-2]>0.$$

Also for $\bar{y}_{\text{pr1}}$ to be more efficient than $\bar{y}$

$$V(\bar{y})-\text{MSE}(\bar{y}_{\text{pr2}})>0$$

$$\Rightarrow \bar{Y}^2[\gamma_1+q_2-2]>0$$

(5.3) $$\Rightarrow [\gamma_1+q_2-2]>0.$$

If equations (5.2) and (5.3) hold, then the proposed estimators would be more efficient than the simple random sample mean.

5.2. Classical ratio estimator, $\bar{y}_R$. The MSE of $\bar{y}_R$ expressed in terms of $\gamma's$ is given by: For estimator I,

(5.4) $$\text{MSE}(\bar{y}_R)=\bar{Y}^2[\gamma_1-2\gamma_2-1]$$

And for estimator II

(5.5) $$\text{MSE}(\bar{y}_R)=\bar{Y}^2[\gamma_1-2\gamma_4-\gamma_5-1]$$

Therefore, for the proposed estimator $\bar{y}_{\text{pr1}}$ to be more efficient than the classical ratio estimator,

$$\text{MSE}(\bar{y}_R)-\text{MSE}(\bar{y}_{\text{pr1}})>0$$

$$\Rightarrow \bar{Y}^2[\gamma_1-2\gamma_2-2+q_1]>0$$

(5.6) $$\Rightarrow [(\gamma_1+q_1)-2(\gamma_2+1)]>0$$

Similarly, for $\bar{y}_{\text{pr2}}$ to be more efficient than $\bar{y}_R$

$$\text{MSE}(\bar{y}_R)-\text{MSE}(\bar{y}_{\text{pr2}})>0$$

$$\Rightarrow [\gamma_1-2\gamma_4-\gamma_5-2+q_2]>0$$

(5.7) $$\Rightarrow [(\gamma_1+q_2)-2(\gamma_4+1)-\gamma_5]>0$$

Therefore, for the proposed estimators to be more efficient than the classical ratio estimator, equations (5.6) and (5.7) must hold.

5.3. Regression Estimator, $\bar{y}_{\mathbf{reg}}$. The Variance of the regression estimator expressed in terms of $\gamma's$ is given as: For estimator I

(5.8) $$V\left(\bar{y}_{\text{reg}}\right)=\bar{Y}^2\{\gamma_1-[(\gamma_2+\gamma_3)^2/\gamma_4]-1\}$$

and for estimator II

(5.9) $$V\left(\bar{y}_{\text{reg}}\right)=\bar{Y}^2[\gamma_1-[(\gamma_4+\gamma_5)^2/\gamma_4]-1]$$

Therefore, for the proposed estimators to be more efficient than the regression estimator,

$$V\left(\bar{y}_{\text{reg}}\right)-\text{MSE}\left(\bar{y}_{\text{pr1}}\right)>0$$
$$\Rightarrow \bar{Y}^2\left[\gamma_1-[(\gamma_2+\gamma_3)^2/\gamma_4]-1-(1-q_1)\right]>0$$
$$\Rightarrow \bar{Y}^2\left[\gamma_1-[(\gamma_2+\gamma_3)^2/\gamma_4]-2+q_1\right]>0$$

(5.10) $$\Rightarrow [\gamma_4\left(\gamma_1-1\right)-\gamma_2(\gamma_2+\gamma_3)]^2/\gamma_4\left(\gamma_1\gamma_4-\gamma_2^2\right)>0$$

(5.10) holds if [$\gamma_4\left(\gamma_1\gamma_4-\gamma_2^2\right)>0$]. Therefore,

$$\gamma_4\left(\gamma_1\gamma_4-\gamma_2^2\right)>0$$
$$\Rightarrow \gamma_1\gamma_4^2-\gamma_2^2\gamma_4>0$$
$$\Rightarrow \lambda^2C_x^4\left(1+\lambda C_y^2\right)-\lambda^2C_x^4\left(k-\tfrac{1}{2}\right)^2>0$$
$$\Rightarrow 1+\lambda C_y^2>\left(k-\frac{1}{2}\right)^2$$
$$\Rightarrow \text{Var}\left(\bar{y}\right)+\bar{Y}^2>\frac{1}{C_x^2}[\text{MSE}\left(\bar{y}_{\Re}\right)+\bar{Y}^2\lambda C_y^2(\rho^2-1)]$$

(5.11) $$\Rightarrow \text{Var}\left(\bar{y}\right)+\bar{Y}^2>\frac{1}{C_x^2}[\text{MSE}\left(\bar{y}_{\Re}\right)-\bar{Y}^2\lambda C_y^2(1-\rho^2)]$$

Clearly, from equation (5.11), $\text{MSE}\left(\bar{y}_{\Re}\right)$, the Mean Square Error of [1] is smaller than $\text{Var}\left(\bar{y}\right)$, the Variance of the simple random sample mean. Also, the second term in the bracket on the right hand side of equation (5.11) is the Variance of regression estimator, which is smaller than $V(\bar{y})$. Therefore, the expression on the left hand side of equation (5.11) is always greater than that of the right hand side. Hence, equation (5.11) holds. It follows therefore that [$\gamma_4\left(\gamma_1\gamma_4-\gamma_2^2\right)>0$] and the numerator of (5.10) is a square, which implies that (5.10) holds. Hence, the proposed estimator I is always more efficient than classical regression estimator.
Also,

$$V\left(\bar{y}_{\text{reg}}\right)-\text{MSE}\left(\bar{y}_{\text{pr2}}\right)>0$$
$$\Rightarrow \bar{Y}^2\left[\gamma_1-[(\gamma_4+\gamma_5)^2/\gamma_4]-1-(1-q_2)\right]>0$$
$$\Rightarrow \bar{Y}^2\left[\gamma_1-[(\gamma_4+\gamma_5)^2/\gamma_4]-2+q_2\right]>0$$

(5.12) $$\Rightarrow [\gamma_4\left(\gamma_1-1\right)-\gamma_5(\gamma_5+\gamma_4)]^2/\gamma_4\left(\gamma_1\gamma_4-\gamma_5^2\right)>0$$

Similarly, for (5.12) to be satisfied,

$$\gamma_4\left(\gamma_1\gamma_4-\gamma_5^2\right)>0$$
$$\Rightarrow \gamma_1\gamma_4^2-\gamma_5^2>0$$
$$\Rightarrow 1+\lambda C_y^2>(k-1)^2$$
$$\Rightarrow \bar{Y}^2+\text{Var}\left(\bar{y}\right)>\frac{1}{C_x^2}[\text{MSE}\left(\bar{y}_R\right)+\bar{Y}^2\lambda C_y^2(\rho^2-1)]$$

(5.13) $$\Rightarrow \bar{Y}^2+\text{Var}\left(\bar{y}\right)>\frac{1}{C_x^2}[\text{MSE}\left(\bar{y}_R\right)-\bar{Y}^2\lambda C_y^2(1-\rho^2)]$$

From (5.13), we observe that $\text{MSE}\left(\bar{y}_R\right)$, the Mean Square Error of the classical ratio estimator is always smaller than $\text{Var}\left(\bar{y}\right)$, the variance of simple random sample mean.

In addition, the second term in the bracket of the right hand side of (5.13) is the Variance of the classical regression estimator. Therefore the expression on the left hand side of equation (44) is greater than that of the right hand side. Hence, equation (44) holds and the numerator of (5.12) is positive, which implies that (43) always holds.

Remark II Since equations (5.10) and (5.12) are all greater than zero, then the proposed estimators are always more efficient than the regression estimator. Moreover, since the regression estimator is more efficient than the simple random sample mean, classical ratio estimator, estimators of [14], [9] and [10], and any other ratio estimators, it follows that the proposed estimators are more efficient than these estimators. The above remark is summarized in the following theorem.

5.1. Theorem. *If $\theta_1, \theta_2, \varphi_1\ \varphi_2$ attain or almost attain their optimal values in the proposed estimators, then the proposed estimators are always more efficient than the regression estimator.*

5.4. Gupta and Shabbir [5] estimator, $\bar{\mathbf{y}}_{\mathbf{GS}}$. The proposed estimators would be better than the Gupta and Shabbir's class of estimators if:

$$\mathrm{MSE}\left(\bar{y}_{\mathrm{GS}}\right) - \mathrm{MSE}\left(\bar{y}_{\mathrm{pr1}}\right) > 0$$

$$\Rightarrow \bar{Y}^2\left[(1-\nu_1) - (1-q_1)\right] > 0$$

$$\Rightarrow [q_1 - \nu_1] > 0 \qquad (5.14)$$

$$\mathrm{MSE}\left(\bar{y}_{\mathrm{GS}}\right) - \mathrm{MSE}\left(\bar{y}_{\mathrm{pr2}}\right) > 0$$

$$\Rightarrow [q_2 - \nu_1] > 0 \qquad (5.15)$$

5.5. Singh and Solanki [17] estimator, $\bar{\mathbf{y}}_{\mathbf{SS}}$. The proposed estimators would be more efficient than Singh and Solanki's class of estimators if:

$$\mathrm{MSE}\left(\bar{y}_{\mathrm{SS}}\right) - \mathrm{MSE}\left(\bar{y}_{\mathrm{pr1}}\right) > 0$$

$$\Rightarrow \bar{Y}^2\left[(1-\nu_2) - (1-q_1)\right] > 0$$

$$\Rightarrow [q_1 - \nu_2] > 0 \qquad (5.16)$$

and

$$\mathrm{MSE}\left(\bar{y}_{\mathrm{SS}}\right) - \mathrm{MSE}\left(\bar{y}_{\mathrm{pr2}}\right) > 0$$

$$\Rightarrow \bar{Y}^2\left[(1-\nu_2) - (1-q_2)\right] > 0$$

$$\Rightarrow [q_2 - \nu_2] > 0 \qquad (5.17)$$

6. Empirical Study

To investigate our theoretical results, as well as, test the optimality and efficiency performances of our proposed estimators over other existing ones considered in this study, we make use of data of the following populations.

Population I:

$N = 200, n = 50, \bar{Y} = 500, \bar{X} = 25, C_y = 15, C_x = 2, \rho = 0.90, \beta_2(x) = 50$

[Kadilar and Cingi [11]]

Population II:

$N = 106, n = 20, \bar{Y} = 2212.59, \bar{X} = 27421.70, C_y = 5.22, C_x = 2.10,$
$\rho = 0.86, \beta_2(x) = 34.57$

[Kadilar and Cingi,[9, 10]]

Population III:

$N = 104, n = 20, \bar{Y} = 625.37, \bar{X} = 13.93, C_y = 1.866, C_x = 1.653,$
$\rho = 0.865, \beta_2(x) = 17.516$

[Kadilar and Cingi [11]]

Population IV:

$N = 923, n = 180, \bar{Y} = 436.4345, \bar{X} = 11440.5, C_y = 1.7183, C_x = 1.8645,$
$\rho = 0.9543, \beta_2(x) = 18.7208$

[Koyuncu and Kadilar, [12]]

Table 4: Optimum values (θ_1^*, θ_2^*, MSEs and PREs of some Gupta and Shabbir[5] estimators and the proposed estimators.

Estimators	**Population I**				**Population II**				**Population III**				**Population IV**			
	θ_1^*	θ_2^*	MSE	PRE	θ_1^*	θ_2^*	MSE	PRE	θ_1^*	θ_2^*	MSE	PRE	θ_1^*	θ_2^*	MSE	PRE
$\bar{y}_{\mathrm{GS1}}$	0.60	76.61	95396.05	884.471	0.74	0.09	1043368.08	518.642	0.96	2.41	13321.23	412.828	0.999	-0.005	224.356	1121.019
$\bar{y}_{\mathrm{GS2}}$	0.59	76.48	95304.34	885.322	0.74	0.09	1043366.16	518.643	0.96	0.87	13316.65	412.970	0.999	-0.005	224.356	1121.019
$\bar{y}_{\mathrm{GS3}}$	059	76.49	95308.27	885.285	0.74	0.09	1043366.29	518.643	0.96	0.98	13316.98	412.960	0.999	-0.005	224.356	1121.019
$\bar{y}_{\mathrm{GS4}}$	0.60	76.71	95468.42	883.800	0.74	0.09	1043369.73	518.641	0.96	3.10	13323.21	412.766	0.999	-0.005	224.356	1121.019
$\bar{y}_{\mathrm{GS5}}$	0.60	76.59	95386.75	884.557	0.74	0.09	1043367.79	518.642	0.96	2.19	13320.59	412.848	0.999	-0.005	224.356	1121.019
$\bar{y}_{\mathrm{GS6}}$	0.59	76.48	95303.94	885.325	0.74	0.09	1043366.14	518.643	0.96	0.85	13316.58	412.972	0.999	-0.005	224.356	1121.019
$\bar{y}_{\mathrm{GS7}}$	0.59	76.48	95304.34	885.322	0.74	0.09	1043366.03	518.643	0.96	0.83	13316.52	412.974	0.999	-0.005	224.356	1121.019
$\bar{y}_{\mathrm{GS8}}$	0.60	76.73	95485.97	883.638	0.75	0.09	1049814.73	515.457	0.96	4.25	13326.36	412.669	0.999	-0.001	224.357	1121.014
$\bar{y}_{\mathrm{GS9}}$	0.59	76.48	95303.94	885.325	0.74	0.09	1043366.03	518.643	0.96	0.81	13316.47	412.975	0.999	-0.005	224.356	1121.019
$\bar{y}_{\mathrm{GS10}}$	0.59	76.47	95300.40	885.358	0.74	0.09	1043366.03	518.643	0.96	0.70	13316.13	412.986	0.999	-0.005	224.356	1121.019
$\bar{y}_{\mathrm{reg}}$	-	-	160312.5	526.316	-	-	1409113.09	384.025	-	-	13846.05	397.180	-	-	224.625	1119.677
$\bar{y}_{\mathrm{pr1}}$	0.58	83.08	72692.31	1160.7	0.71	0.13	713838.3	758.06	0.92	42.13	11051.73	497.60	0.99	6.23	212.519	1183.458
$\bar{y}_{\mathrm{pr2}}$	0.56	4.23	45870.36	1839.4*	0.65	1.73	240765.7	2247.6*	0.87	0.98	6819.164	806.5*	0.99	0.35	183.147	1373.254*
$\bar{y}$	-	-	843750	100.000	-	-	5411348.28	100.000	-	-	54993.75	100.000	-	-	2515.074	100.000

Table 5: Optimum values (φ_1^*, φ_2^*, MSEs and PREs of some Singh and Solanki [17] estimators and the proposed estimators

Estimator	**Population I**				**Population II**				**Population III**				**Population IV**			
	φ_1^*	φ_2^*	MSE	PRE	φ_1^*	φ_2^*	MSE	PRE	φ_1^*	φ_2^*	MSE	PRE	φ_1^*	φ_2^*	MSE	PRE
$\bar{y}_{\text{SS1}}$	0.53	3.98	45246.7	1864.791	0.50	1.57	202185.29	2676.43	0.94	0.13	12986.83	423.458	1.00	-0.12	224.527	1120.166
$\bar{y}_{\text{SS2}}$	0.53	3.98	44081.39	1914.073	0.50	1.57	202185.29	2676.82	0.94	0.09	13116.65	419.267	1.00	-0.12	224.527	1120.166
$\bar{y}_{\text{SS3}}$	0.53	3.98	44131.01	1911.921	0.50	1.57	202155.53	2676.80	0.94	0.10	13107.25	419.567	1.00	-0.12	224.527	1120.166
$\bar{y}_{\text{SS4}}$	0.53	3.98	46177.73	1827.179	0.50	1.57	202157.65	2676.09	0.94	0.15	12931.31	412.766	1.00	-0.12	224.527	1120.166
$\bar{y}_{\text{SS5}}$	0.53	3.98	45127.48	1869.703	0.50	1.57	202210.82	2676.49	0.94	0.12	13005.06	425.276	1.00	-0.12	224.527	1120.166
$\bar{y}_{\text{SS6}}$	0.53	3.98	44076.42	1914.289	0.50	1.57	202180.85	2676.83	0.94	0.09	13118.60	422.864	1.00	-0.12	224.527	1120.166
$\bar{y}_{\text{SS7}}$	0.53	3.98	44081.39	1914.073	0.50	1.57	202155.27	2676.82	0.94	0.09	13116.65	419.204	1.00	-0.12	224.527	1120.166
$\bar{y}_{\text{SS8}}$	0.53	3.98	46405.23	1818.222	0.54	1.52	202155.53	1817.35	0.94	0.17	12844.01	419.267	0.99	-0.03	224.510	1120.250
$\bar{y}_{\text{SS9}}$	0.53	3.98	44076.42	1914.289	0.50	1.57	297759.98	2676.85*	0.94	0.09	13121.61	428.167	1.00	-0.12	224.527	1120.166
$\bar{y}_{\text{SS10}}$	0.53	3.98	44031.68	1916.234*	0.50	1.57	202153.61	2676.85*	0.94	0.09	13131.21	419.108	1.00	-0.12	224.527	1120.166
$\bar{y}_{\text{reg}}$	-	-	160312.5	526.316	-	-	1409113.542	384.025	-	-	13846.05	397.180	-	-	224.625	1119.677
$\bar{y}_{\text{pr1}}$	0.58	83.08	72692.31	1160.7	0.71	0.13	713838.3	758.06	0.92	42.13	11051.73	497.60	0.99	6.23	212.519	1183.458
$\bar{y}_{\text{pr2}}$	0.56	4.23	45870.36	1839.4	0.65	1.73	240765.7	2247.6	0.87	0.98	6819.164	806.5*	0.99	0.35	183.147	1373.254*
$\bar{y}$	-	-	843750	100.000	-	-	5411348.28	100.000	-	-	54993.75	100.000	-	-	2515.074	100.000

***indicates the largest PRE**

7. Discussion

The ratio-type class of estimators considered in Tables (IV) and (V) was adapted from the work of [17], where he made corrections on the MSE of the general class of [5] estimators. It is observed from Table (IV) that the proposed estimator (I) fares better at optimum condition than the unbiased sample mean, regression estimator and [5] class of estimators in all the four populations. This is evident on the larger Percent Relative Efficiencies (PREs) and the smaller Mean Squared Errors of the proposed estimator (I) than those of sample mean, regression and estimators of [5]. On the other hand, the proposed estimator (II) becomes more efficient than the simple random sample mean, regression estimator, the class of estimators of [5] and proposed estimator (I) in the four populations. This is evident on the fact that the proposed estimator (II) has the largest PRE in the four populations considered in this study. This therefore, shows that the proposed estimators are more efficient than any other proposed estimators that have less efficiency than the regression estimator and estimators of [5]. Table (V) clearly shows that [17] and our proposed estimators fare better than the class of estimators of [5], regression estimator and simple random sample mean in all the populations considered in this study. A clear difference is also observed between the class of estimators of [17] and the proposed estimators. In populations (I) and (II), estimator of [17], $(\bar{y}_{\mathrm{SS10}})$ fares better than the proposed estimators. Also, [17] estimator $(\bar{y}_{\mathrm{SS9}})$ is equally efficient with $(\bar{y}_{\mathrm{SS10}})$ and more efficient than the proposed estimators. On the other hand, the proposed estimators (I) and (II) fares better than [17] estimators in populations (III) and (IV), but the proposed estimator (II) is most efficient in the populations (III) and (IV). This indicates that the proposed estimators using exponential estimator may fare in some populations better than [17] class of estimators, while [17] may be more efficient than the proposed estimators in some other populations. On the whole, the proposed estimators have shown significant efficiencies in the four populations considered in this study. It can also be deduced that the proposed estimators always fare better than the usual regression estimator and [5].

8. Conclusion

From the above result and discussion, It can be concluded that the two proposed estimators at optimal condition are each more efficient that the general regression estimator which have always been preferred because of its minimum MSE. The two proposed estimators are also more efficient than most of the exiting ratio estimators, thus providing better alternative estimators in practical situations.

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