

Numerical Second Law Analysis of a Refrigeration Phase-Change Storage*

Françoise STRUB and Jean-Pierre BEDECARRATS
Laboratoire de Thermodynamique et Energétique,
Université de Pau et des Pays de l'Adour, avenue de l'Université,
64000 Pau, France.
E-mail: francoise.strub@univ-pau.fr

Abstract

Second law analysis techniques, based on the calculation of entropy generation, are applied to the design and operation of an industrial refrigeration phase-change storage. The studied process consists in the use of encapsulated phase-change materials (PCM). A detailed analysis of the system permitted the development of a simulation program. So, the calculation of entropy generation has been made in a numerical way. The results are only presented for the discharge mode corresponding to the melting of the PCM.

Key words: energy storage, refrigeration, entropy generation, phase change.

1. Introduction

Second law analysis techniques, based on the calculation of entropy generation (Bejan 1996), are applied to the design and operation of an industrial refrigeration phase-change storage.

This method has been applied when a complete experimental study of the behaviour of the system permitted the development of a simulation program (Dumas et al. 1994, Bédécarrats et al. 1996). Entropy generation has been calculated in a numerical way and influence of different parameters on the thermodynamic irreversibilities has been analysed. Results are presented only for removal process.

2. Description of the Studied Phase-Change Storage System and Short Review of Previous Works

The presented refrigeration storage process, described at length in a previous paper (Bédécarrats et al. 1996), is a system using encapsulated phase-change materials (PCM). It consists in using plastic spherical containers, whose diameter is 77 mm, filled with PCM. These capsules called nodules are immersed in a chilled fluid in a tank. The PCM inside the containers releases its latent heat and freezes. To discharge the cooling from storage, warm fluid carrying energy from the load flows through the tank, melting the encapsulated PCM.

Experimental investigation has been made on a test plant which is a tank with a reduced size (1 m³ containing 2500 nodules) placed in a refrigeration loop. The test plant permits a detailed study of the behaviour of the tank during the entire cycle.

A numerical simulation that considers the aspects of both the surrounding heat transfer fluid and the phase change material packed inside the nodules has been developed in the cases of the charge and the discharge processes.

In this article, we only present results concerning the removal process. The model, presented in several previous papers (Dumas et al. 1994, Bédécarrats et al. 1996), permits to estimate the melting of each nodule, the temperatures in the PCM and in the heat transfer fluid during the discharge process. The tank is divided into several meshes containing N nodules. As the heat transfer fluid temperature is considered uniform in each layer, all the nodules of each layer simultaneously pass through the phase change at the melting temperature T_M . Applying the laws of conservation of mass and energy to each layer yields:

$$\rho c_p V \frac{dT}{dt} = \dot{m} c_p (T_i - T_e) + N \phi_i \quad (1)$$

where $T = (T_i + T_e)/2$ is the average temperature of the heat transfer fluid of the layer (T_i and T_e

* This paper was presented at the ECOS'98 Conference in Nancy, June 8-10, 1998.

are respectively the inlet and the outlet temperatures of the layer) and ϕ_i is the flux exchanged by a nodule.

The quasistationary approximation (Alexiades and Solomon 1993) is applied to the determination of ϕ_i during melting.

Consider a nodule of inner radius r_i . Uniform heating of its surface will result in a spherically symmetric melt-front, $r = r_L(t)$ the inner radius of liquid PCM, propagating inwards from $r = r_i$ with solid at T_M in $0 \leq r < r_L(t)$ and liquid in $r_L(t) < r \leq r_i$. Assuming constant thermal properties in each phase, the steady-state solution of the heat equation in the liquid has the form :

$$\theta(r, t) = T_M + [T(t) - T_M] \times \frac{1 - \frac{r_L(t)}{r}}{\left(\frac{k_{eff}}{k_p} - 1\right) \frac{r_L(t)}{r_i} + \left(\frac{k_{eff}}{hr_e} - \frac{k_{eff}}{k_p}\right) \frac{r_L(t)}{r_e} + 1} \quad (2)$$

The interface conditions here have the standard form :

$$\rho_L L_F \frac{dr_L(t)}{dt} = k_{eff} \left[\frac{\partial \theta(r, t)}{\partial r} \right]_{r=r_L(t)} = \frac{\phi_i}{4\pi r_L^2(t)} \quad (3)$$

The determination of ϕ_i before the melting and after the melting is finished is done considering the uniform PCM temperature.

The simulation results have been compared with a good agreement with experimental observations (Dumas et al. 1994, Bédécarrats et al. 1996).

3. Second Law Analysis

The tank being well insulated, the first law efficiency is, of course, close to the value $\eta_1 = 1$, only demonstrating that the energy actually removed from the tank during the process period is the same as the energy that has been stored in the tank.

It is obvious that the first law point of view is not sufficient enough to account for the actual performance of the storage process. For example, whatever the removal process may operate, the energy stored will be entirely removed whereas the outlet temperature can be very different from the melting temperature T_M of the PCM. Naturally, for a refrigeration process, it would be better if the outlet temperature (i.e. useful temperature) were close to T_M .

Following the pioneering study of Bejan (1978) about the optimisation of a sensible heat storage, the second law techniques have been applied to our case with the help of simulation.

3.1 Entropy generation

The tank which has been well insulated, filled with nodules whose diameter is given, can be considered as an open system as sketched on the *Figure 1*. During the discharge mode, the fluid enters the system, flows through the nodules and exits colder.

In terms of instantaneous rate of entropy \dot{S}_{gen} , the second law of thermodynamics states :

$$\dot{S}_{gen} = \frac{\partial S}{\partial t} - \dot{m}(s_{in} - s_{out}) \geq 0 \quad (4)$$

- \dot{m} is the mass flow rate through the tank.
- $\frac{\partial S}{\partial t}$ is the instantaneous variation of the entropy of the system defined by the dashed boundary of the figure and called control volume.
- s_{in} and s_{out} are the specific entropy of the fluid at the inlet and the outlet of the tank.

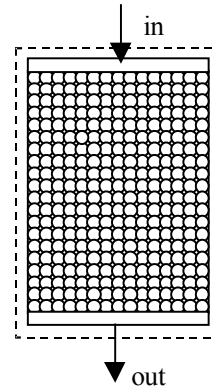


Figure 1.

Because of the low practical speed of the fluid in the tank, the frictional pressure drop on the fluid side, which is one of the irreversibility sources is neglected in the present analysis.

During each step of time Δt it is possible to calculate the variation of the entropy since model allows us to know the temperatures everywhere in the tank at each time and the different states of the nodules (melted, partly melted).

The entropy variation of the system (control volume) is the sum of the entropy variation of all the parts in each layer j :

$$\frac{\partial S}{\partial t} = \sum_j \left[\frac{\partial S_{fluid,j}}{\partial t} + \frac{\partial S_{PCM,j}}{\partial t} \right] \quad (5)$$

with for the heat transfer fluid :

$$\frac{\partial S_{fluid,j}}{\partial t} = \rho c_p V_j \frac{\partial}{\partial t} (\ln T_j)$$

It is important to note that the volume of the flowing fluid takes up about 40% of the volume of the tank.

For the PCM :
before and after the melting :

$$\frac{\partial S_{PCM,j}}{\partial t} = N M_{PCM} c_{PCM} \frac{\partial}{\partial t} (\ln \theta_j)$$

during the melting process :

$$\frac{\partial S_{PCM,j}}{\partial t} = N \frac{4}{3} \pi \frac{\rho_L L_F}{T_M} \frac{\partial}{\partial t} (r_{L,j}^3(t))$$

In this model, we neglect the entropy variation of the plastic capsules.

The rate of entropy transfer can be calculated simply :

$$\dot{m}(s_{in} - s_{out}) = \dot{m} c_p \ln \left(\frac{T_{in}}{T_{out}} \right) \text{ where } T_{in} \text{ and } T_{out}$$

are respectively the inlet and the outlet temperatures of the control volume.

The importance of the time-step size used in the numerical modelling is recognised. The accuracy of the results depends on the value of Δt . Δt must be chosen rigorously not to calculate mixing entropy of the fluid introduced by the fact that T is considered the average temperature of the layer. Δt is the step of time which permits the fluid to throw a layer.

3.2 Entropy generation number

In a real process, for a given volume of tank, flow rate and inlet temperature can vary. Then performances vary i.e. the value of the outlet temperature and duration of the complete process vary.

To compare different operations of the discharge system in order to optimise it, the entropy generation number N_S has been calculated as the ratio of the destroyed exergy during the time interval 0-t to the removed exergy from the system during the same interval.

$$N_S = \frac{T_0 \int_0^t \dot{S}_{gen} dt}{\left| \int_0^t \dot{E}x_{cv} dt \right|} \quad (6)$$

$\dot{E}x_{cv}$ is the time rate of change of exergy of the control volume defined as :

$$\dot{E}x_{cv} = \dot{E}x_{in} - \dot{E}x_{out} - T_0 \dot{S}_{gen} \quad (7)$$

where

$$\dot{E}x_{in} - \dot{E}x_{out} = \dot{m} c_p (T_{in} - T_{out}) - T_0 \dot{m} c_p \ln \left(\frac{T_{in}}{T_{out}} \right)$$

is the rate of exergy transfer and $T_0 \dot{S}_{gen}$ is the time rate of exergy destruction due to irreversibilities within the control volume.

In order to compare different situations, the dimensionless number N_S is represented versus the percentage of change of control volume exergy X defined like this :

$$X = \frac{\int_0^t \dot{E}x_{cv} dt}{\int_0^{t_R} \dot{E}x_{cv} dt}$$

where t_R is the complete duration of the removal evaluated when the outlet temperature is the same as the inlet temperature.

3.3 Results from the discharge mode

The results concern the removal process where T_{in} remains at a constant value during the process.

Extending the investigation at any size of the tank, flow rate and inlet temperature, the variation of N_S versus X is represented for different values of τ and ΔT defined as :

$\tau = \frac{\rho V_{cv}}{\dot{m}}$ representing the time required for the fluid to flow through the tank from the inlet to the outlet. It is independent of the shape of the tank. V_{cv} is the volume of fluid in the tank. In practical cases it takes about twenty minutes for the fluid to flow across the tank.

$\Delta T = T_{in} - T_M$ representing the difference between the inlet and the melting temperature. For the calculations the values of $k_{eff} = 1.1 \text{ W/mK}$, $h = 130 \text{ W/m}^2\text{K}$, $T_M = 273 \text{ K}$ and $T_0 = 293 \text{ K}$ were taken.

On *Figure 2* and *Figure 3*, we can see that the fraction of destroyed exergy varies from about 7 % to 30 %, for a complete removal, according to the conditions of flow rate and inlet temperature.

As expected, N_S increases as ΔT increases and τ decreases. Moreover, the influence of ΔT is more important than the one of τ (i.e. the flow rate).

At the very beginning, before the warm fluid reaches the nodules, corresponding to the values $X < 0.025$, the exergy removed is zero. Then up to about $X = 0.15$, N_S suddenly increases up to a more or less considerable value according to the condition of inlet temperature. During this period, the stored fluid has given way to the entering fluid and T_{out} has begun to increase quickly.

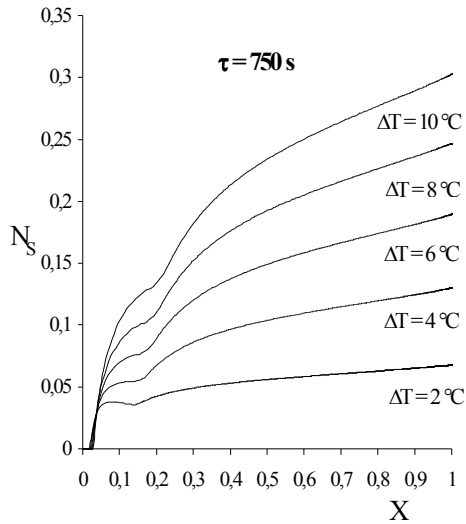


Figure 2. Fraction of destroyed exergy versus X for different ΔT .

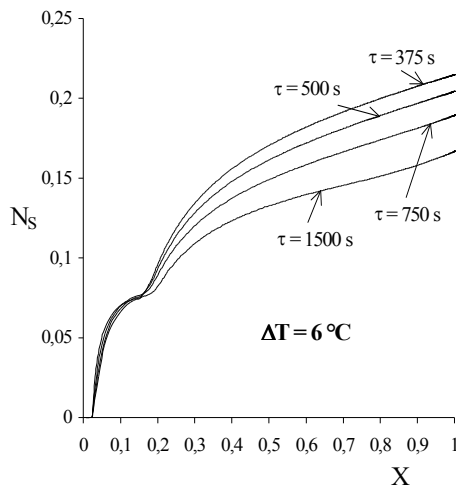


Figure 3. Fraction of destroyed exergy versus X for different τ .

At this period the temperature difference between fluid and nodules is the most important and causes irreversibilities due to heat exchange. Next N_S increases monotonically during the phase when the nodules are melting. T_{out} increases very slowly during this period.

The three dimensional view offered by Figures 4 and 5 shows the entropy generation number surface $N_S(\tau, \Delta T)$ for two values of X : $X = 1$ and $X = 0.1$.

On Figure 4, we see variation of N_S versus ΔT and τ for a complete removal ($X = 1$). The curves formed by the entropy generation number surface with the plane for which N_S is constant

are represented. We can observe that it is especially ΔT that has an influence on the variation of N_S .

Some previous studies (Bédécarrats et al. 1996) have shown the influence of the flow rate and the inlet temperature on the refrigeration process. According to the load, we can choose the appropriate couple flow rate, inlet temperature.

So, practically, to minimize the irreversibility, it is better to choose a small ΔT (an inlet temperature near T_M) and a value of τ (i.e. flow rate) which permits to obtain the expected removed thermal power.

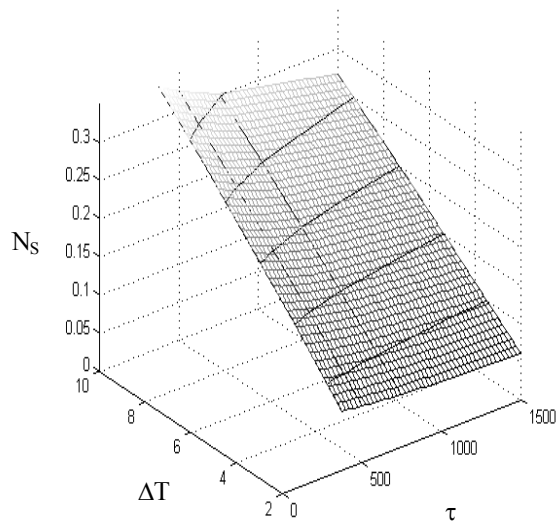


Figure 4. Entropy generation number versus ΔT and τ for $X = 1$.

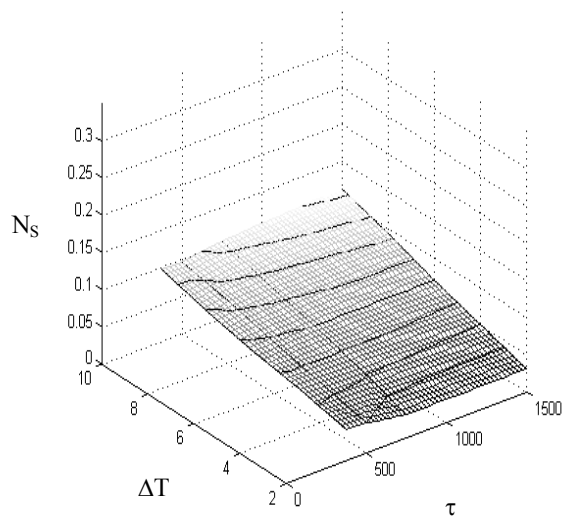


Figure 5. Entropy generation number versus ΔT and τ for $X = 0.1$.

On *Figure 5* the same representation for $X = 0.1$ shows a surface differently inclined. On the whole, the variation of τ has no influence on the variation of N_S . It corresponds to the first moment when the warm fluid is flowing through the tank suddenly bringing about a great temperature difference between the fluid and the nodules.

A particular application of the calculation of N_S has been made to evaluate the destroyed exergy rate in the following case : according to the kind of the encapsulated PCM, the melting temperature can vary and thus thermal energy can be removed at the suitable temperature. In order to maintain the tank outlet temperature the lowest possible for a refrigeration application it is possible to put in the tank several kinds of nodules.

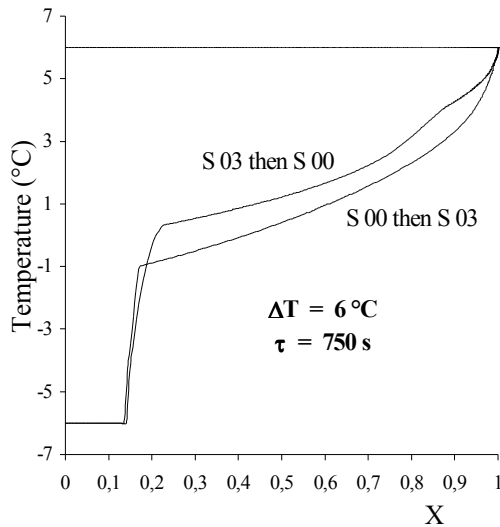


Figure 6. Inlet and outlet temperatures of the tank ($T_{in} = 6^\circ\text{C}$) versus X .

In the examples presented on *Figures 6* and *7* two kinds of nodules have been mixed equally in the tank: one kind S 00 whose melting temperature is 0°C and another one S 03 whose melting temperature is -3°C . *Figures 6* and *7* represent the variation of the outlet temperature and the entropy generation number N_S in the two following cases : on the one hand nodules S 00 take up the inlet half part of the tank while nodules S 03 take up the outlet part and on the other hand nodules change place.

We can observe that the first configuration (S 00 then S 03) brings less irreversibility (lower values of N_S on *Figure 7*). The practical consequence is obvious by considering outlet temperature variation on *Figure 6* : for this arrangement energy is removed at a lower temperature. Whatever the place of the nodules, first law point of view indicates that the tank has the same capacity

to remove energy. Second law analysis points out that the same quantity of energy is removed but its refrigeration value is different because entropy generation is different.

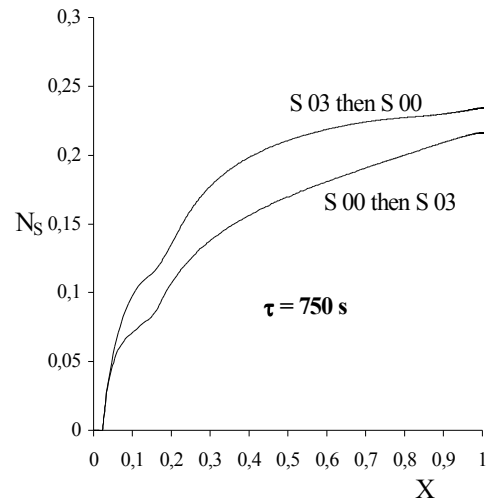


Figure 7. Fraction of destroyed exergy versus X .

4. Conclusion

In this study we consider the numerical analysis of the second law of thermodynamics applied to a refrigeration phase-change storage in the case of the removal process. The entropy generation number N_S permits to evaluate the effects of the duration of the process, the difference of temperature and the flow rate, on the fraction of destroyed exergy. The important volume of flowing fluid in the tank which represents a not inconsiderable part of the control volume explains the particular variation of N_S versus X .

Nomenclature

c_{PCM}	specific heat of PCM
c_p	constant pressure specific heat of the fluid
$\dot{E}_{X_{cv}}$	rate of exergy change of the control volume
h	heat transfer coefficient nodule - fluid
k_{eff}	effective thermal conductivity of liquid PCM
k_p	thermal conductivity of nodule envelope
L_F	latent heat of melting of PCM
M_{PCM}	mass of PCM in a nodule
\dot{m}	mass flow rate of heat transfer fluid
N	number of nodules in each layer
N_S	entropy generation number
$r_{L(t)}$	internal radius of liquid PCM
$r_{e,i}$	outer and inner radius of nodule
s	specific entropy of the fluid
S	entropy

\dot{S}_{gen}	entropy generation rate
t_R	duration of an entire removal process
T_M	melting temperature of PCM
T	heat transfer fluid temperature
V	volume of the heat transfer fluid in a mesh
X	percentage of removed exergy

Greek symbols

Δt	step of time for the calculation
ΔT	difference between the inlet and the melting temperatures
ϕ_i	heat flux exchanged by a nodule
$\theta(r,t)$	temperature of PCM.
ρ	density of heat transfer fluid
ρ_L	density of liquid PCM
τ	time required for the fluid to flow through the tank

Subscripts

Fluid	heat transfer fluid
in	inlet
j	refers to the number of a layer
out	outlet
PCM	Phase Change Material

References

Alexiades, V. and Solomon, A. D., 1993, *Mathematical Modelling of Melting and Freezing Processes*, ed., Hemisphere Publishing Corporation, pp.126-152.

Bédécarrats, J. P., Strub, F., Falcon B. and Dumas J. P., 1996, "Phase-change thermal energy storage using spherical capsules : performance of a test plant", *Int. J. Refrigeration*, Vol. 19, No.3, pp.187-196.

Bejan, A., 1978, "Two thermodynamic optima in the design of sensible heat units for energy storage", *J. Heat Transfer*, Vol. 100, pp.708-712.

Bejan, A., 1996, *Entropy Generation Minimization*, CRC Press, Boca Raton New York.

Dumas, J. P., Bédécarrats, J. P., Strub, F. and Falcon B., 1994, "Modelization of a tank filled with spherical nodules containing a phase change material", *Proceedings, 10th Int. Heat Transfer Conf.*, Brighton, Vol. 7, pp.239-244.