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Generalized class of estimators for population median using auxiliary information

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Abstract

This article suggests a generalized class of estimators for population median of the study variable in simple random sampling using information on an auxiliary variable. Asymptotic expressions of bias and mean square error of the proposed class of estimators have been obtained. Asymptotic optimum estimator has been investigated along with its approximate mean square error. It has been shown that proposed generalized class of estimators are more efficient than estimators considered by [26] , [5],[6], [22], [1] , [19] and other estimators . In addition theoretical findings are supported by an empirical study based on two populations to show the superiority of the constructed estimators over others.

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1. Introduction

In the sampling literature, Statisticians are often interested in dealing with variables that have highly skewed distributions such as consumptions and incomes. In such situations median is considered the more appropriate measure of location than mean. It has been well recognised that use of auxiliary information results in efficient estimators of population parameters. Initially, estimation of median without auxiliary variable analyzed, after that some authors including [6], [9], [24] and [7] used the auxiliary information in median estimation. [6], proposed the problem of estimating the population median M_y of study variable Y using the auxiliary variable X for the unites in the sample and its median M_x for the whole population. Some other important references in this context

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are [3], [11], [8], [15], [2], [4], [21, 20],[25] and [19].
Let Yi and Xi (i =1,2,....N) be the values of the population unites for the study variable Y and auxiliary variable X respectively .Further suppose that y_i and x_i (i=1,2.....n) be the values of the unites including in the sample say, s_n of size n drawn by simple random sampling without replacement scheme. [6] suggested a ratio estimator for population median M_y of the study variable Y, assuming population median of auxiliary variable X, M_x is known, given as

$$\hat{M}_r = \hat{M}_y \frac{M_x}{\hat{M}_x} \tag{1.1}$$

where $\hat{M}_y$ (due to [5]) and are the sample estimators of M_y and M_x respectively. Suppose that $Y_{(1)}$,$Y_{(2)}$,.....$Y_{(n)}$ are the y values of sample unites in ascending order. Further, suppose t be an integer satisfying and p=t/n be the proportion of y values in the sample that are less than or equal to the median value M_y, an unknown population parameter. If $Q_y(t)$ denote the t-quantile of Y then $\hat{M}_y = Q_y(0.5)$. [6] defined a matrix of proportion (p_{ij}) is

	$Y \leq M_y$	$Y \leq M_y$	Total
$X \leq M_x$	p_{11}	p_{21}	$p_{.1}$
$X > M_x$	p_{12}	p_{21}	$p_{.2}$
Total	p_1	p_2	1

Following [14] and [10], the product estimator for population median M_y is defined as

$$\hat{M}_p = \hat{M}_y \frac{\hat{M}_x}{M_x} \tag{1.2}$$

The usual difference estimator for population median M_y is given by

$$\hat{M}_d = \hat{M}_y + d(M_x - \hat{M}_x) \tag{1.3}$$

Where d is a constant to be determined such that the mean square error of $\hat{M}_d$ is minimum.
[21] proposed the following modified product and ratio estimator for population median M_y , respectively, as

$$\hat{M}_1 = \hat{M}_y \frac{a - \hat{M}_x}{a + M_x} \tag{1.4}$$

and

$$\hat{M}_2 = \hat{M}_y \frac{a + M_x}{a - \hat{M}_x} \tag{1.5}$$

Where a is suitably chosen scalar.
[26] type estimator for median estimation is

$$\hat{M}_3 = \hat{M}_y \frac{M_x}{\hat{M}_x} \tag{1.6}$$

[12, 13] and [30]-type estimator is given by

$$\hat{M}_4 = \hat{M}_y \left[\frac{M_x}{M_x + \beta(\hat{M}_x - M_x)} \right] \tag{1.7}$$

[16]-type esatimator is given by

$$\hat{M}_5 = \hat{M}_y \left[2 - \left(\frac{M_x}{\hat{M}_x} \right)^{\nu} \right] \tag{1.8}$$

[29]- type esatimator is given by

$$\begin{cases} \hat{M}_6 = w\hat{M}_y + (1-w)\hat{M}_y \dfrac{\hat{M}_x}{M_x} \\ \hat{M}_7 = w\hat{M}_y + (1-w)\hat{M}_y \dfrac{M_x}{\hat{M}_x} \end{cases} \tag{1.9}$$

Where w is suitably chosen scalar .

All the estimator considered from (1.1) to (1.9) and conventional estimator $\hat{M}_y$ are members of the [27] and [28]-type class of estimators

$$G = \left\{ \hat{M}_y^{(G)} : \hat{M}_y^{(G)} = G\left(\hat{M}_y, \frac{\hat{M}_x}{M_x} \right) \right\} \tag{1.10}$$

Where the function G assumes a value in a bounded closed convex subset $Q \subset R_2$, which contains the point (M_y ,1) and is such that G$(M_y, 1) = 1$

Using first order Taylor's series expansion about the point$(\hat{M}_y, 1)$, we have

$$(\hat{M}_y^{(G)} = G(M_y, 1) + (\hat{M}_y - M_y)G_{10}(M_y, 1) + O(n^{-1})) \tag{1.11}$$

Where $U = \dfrac{\hat{M}_x}{M_x}$

and $G_{01}(M_y, 1) = \dfrac{\partial G(.)}{\partial U}\Big|_{(M_y,1)}$

Using condition we have

$$\hat{M}_y^{(G)} = M_y + (\hat{M}_y - M_y) + (U-1)G_{01}(M_y, 1) + O(n^{-1})$$

$$or$$

$$(\hat{M}_y^{(G)} - M_y) = (\hat{M}_y - M_y) + (U-1)G_{01}(M_y, 1) + O(n^{-1}) \tag{1.12}$$

Squaring and taking expectations both sides of (1.12),we get the MSE of $\hat{M}_y^{(G)}$ to the first order of approximation as

$$MSE(\hat{M}_y^{(G)}) = \left[V(\hat{M}_y) + \frac{V(\hat{M}_y)}{{M_x}^2} G_{01}^2(\hat{M}_y, 1) + 2\frac{Cov(\hat{M}_y, \hat{M}_x)}{M_x} G_{01}^2(\hat{M}_y, 1) \right] \tag{1.13}$$

Here as $N \to \infty$,$n \to \infty$ then n/N→ f and we assumed that as $N \to \infty$ the distribution of (X, Y) approaches a continues distribution with marginal densities $f_x(x)$ and $f_y(y)$ of X and Y respectively. Super population model framework is necessary for treating the values of X and Y in a realization of N independent observation from a continuous distribution. It is also assumed that $f_x(M_x)$ and $f_y(M_y)$ are positive. Under these conditions, sample median $\hat{M}_y$ is consistent and asymptotically normal (due to [5]) with mean M_y and variance

$$V(\hat{M}_y) = \gamma M_y^2 C_y^2 \tag{1.14}$$

$$and \;\; V(\hat{M}_x) = \gamma M_x^2 C_x^2 \tag{1.15}$$

$$Cov(\hat{M}_y, \hat{M}_x) = \gamma \rho_c M_y M_x C_y C_x \tag{1.16}$$

Where $\gamma = (1-f)/4n, f = n/N, C_y = [M_y f_y(M_y)]^{-1}, C_x = [M_x f_x(M_x)]^{-1}$ and $\rho_c = (4p_{11} - 1))$ with $p_{11} = P(M_x, M_y)$goes from -1 to $+1$ as p_{11}increase from 0 to 0.5.

Substituting these values we get the MSE of $\hat{M}_y^{(G)}$ to the first degree of approximation as

(1.17) $$MSE(\hat{M}_y^{(G)}) = \gamma\left[M_y^2C_y^2 + C_x^2\{G_{01}(M_y,1)\}^2 + 2\rho_c C_x C_y M_y G_{01}(M_y,1)\right]$$

The MSE is minimum when

(1.18) $$G_{01} = (M_y,1) = -k_c M_y$$

Where $k_c = \rho_c\left(\frac{C_y}{C_x}\right)$

Thus the minimum MSE of $\hat{M}_y^{(G)}$ is given by

(1.19) $$MSE_{min}(\hat{M}_y^{(G)}) = \gamma C_y^2 M_y^2(1-\rho_c^2) = MSE_{min}(\hat{M}_d)$$

Which equal to the minimum MSE of the estimator $\hat{M}_d$ defined at (1.3).

It is to be mentioned that minimum MSEs of the estimators $\hat{M}_r$,$\hat{M}_p$ and $\hat{M}_i (i = 1,2,...,7)$ are equal to MSE expression given in equation (1.19). It is obvious from (1.19) that the estimators of the form $\hat{M}_y^{(G)}$ are asymptotically no more efficient than the difference estimator at its optimum value or the regression type estimator given as

(1.20) $$\hat{M}_{lr} = \hat{M}_y + \hat{d}(M_x - \hat{M}_x)$$

where $\hat{d} = \frac{\hat{f}_x\hat{M}_x}{\hat{f}_y\hat{M}_y}(4\hat{p}_{11} - 1)$

[19] Suggested following Classes of estimator

(1.21) $$\hat{M}_d^{(1)} = d_1\hat{M}_y + (1-d_1)(M_x - \hat{M}_x)$$

(1.22) $$\hat{M}_d^{(2)} = d_1\hat{M}_y + d_2(M_x - \hat{M}_x)$$

(1.23) $$\hat{M}_d^{(3)} = d_1\hat{M}_y + d_2\hat{M}_x + (1-d_1-d_2)M_x$$

(1.24) $$\hat{M}_d^{(4)} = \left[d_1\hat{M}_y + d_2(M_x - \hat{M}_x)\right]\left(\frac{(\phi M_x + \delta)}{(\phi\hat{M}_x + \delta)}\right)^{\beta}$$

where d_1 and d_2 are suitable constants to be determined such that MSEs of the estimators considered in (1.21) to (1.24) are minimum, ϕ and δ are either real numbers or the functions of the known parameters of auxiliary variable X.

Biases and minimum MSEs of the estimators considered in (1.21) to (1.24) are given as

(1.25) $$B\left(\hat{M}_d^{(1)}\right) = (d_1 - 1)M_y$$

(1.26) $$B\left(\hat{M}_d^{(2)}\right) = (d_1 - 1)M_y$$

(1.27) $$B\left(\hat{M}_d^{(3)}\right) = (d_1 - 1)(1-R)M_y$$

(1.28) $$B\left(\hat{M}_d^{(4)}\right) = M_y\left[d_1^2\{1 + \gamma\delta C_x^2(\delta - k_c)\} + d_2 R\gamma\delta C_x^2 - 1\right]$$

(1.29) $$MSE_{min}\left(\hat{M}_d^{(1)}\right) = M_y^2\left[1 + R^2\gamma C_x^2 - \frac{\{1 + R\gamma C_x^2(R+k_c)\}^2}{\{1+\gamma(C_y^2 + RC_x^2(R+2k_c))\}}\right]$$

(1.30) $$MSE_{min}\left(\hat{M}_d^{(2)}\right) = \frac{M_y^2\gamma C_y^2\left(1-\rho_c^2\right)}{\left[1+\gamma C_y^2\left(1-\rho_c^2\right)\right]}$$

(1.31) $$MSE_{min}\left(\hat{M}_d^{(3)}\right) = \frac{M_y^2\gamma C_y^2\left(1-\rho_c^2\right)(1-R)^2}{\left[(1-R)^2 + \gamma C_y^2\left(1-\rho_c^2\right)\right]}$$

(1.32) $$MSE_{min}\left(\hat{M}_d^{(4)}\right) = \frac{\left(1-\delta^2\gamma C_x^2\right)M_y^2\gamma C_y^2\left(1-\rho_c^2\right)}{\left[\left(1-\delta^2\gamma C_x^2\right) + \gamma C_y^2\left(1-\rho_c^2\right)\right]}$$

2. The Suggested Generalised Class of Estimators

We propose a generalized family of estimators for population median of the study variable Y, as

$$t_m = \left\{ w_1 \hat{M}_y \left(\frac{M_x}{\hat{M}_x}\right)^{\alpha} exp\left(\frac{\eta(M_x - \hat{M}_x)}{\eta(M_x + \hat{M}_x) + 2\lambda}\right)\right\} + w_2 \hat{M}_x + (1 - w_1 - w_2) M_x \tag{2.1}$$

where w_1 and w_2 are suitable constants to be determined such that MSE of t_m is minimum, η and λ are either real numbers or the functions of the known parameters of auxiliary variables such as coefficient of variation C_x, skewness $\beta_{1(x)}$, kurtosis $\beta_{2(x)}$ and correlation coefficientρ_c (see [17]).
It is to be mentioned that
(i) For (w_1, w_2) =(1,0), the class of estimator t_m reduces to the class of estimator as

$$t_{mp} = \left\{ \hat{M}_y \left(\frac{M_x}{\hat{M}_x}\right)^{\alpha} exp\left(\frac{\eta(M_x - \hat{M}_x)}{\eta(M_x + \hat{M}_x) + 2\lambda}\right)\right\} \tag{2.2}$$

(ii) For $(w_1, w_2) = (w_1, 0)$, the class of estimator t_m reduces to the class of estimator as

$$t_{mq} = \left\{ w_1 \hat{M}_y \left(\frac{M_x}{\hat{M}_x}\right)^{\alpha} exp\left(\frac{\eta(M_x - \hat{M}_x)}{\eta(M_x + \hat{M}_x) + 2\lambda}\right)\right\} \tag{2.3}$$

A set of new estimators generated from (2.1) using suitable values of w_1, w_2, α, η and λ are listed in Table 2.1.

Table 2.1:Set of estimators generated from the class of estimators t_m

Subset of proposed estimator	w_1	w_2	α	η	λ
$t_{m1} = \hat{M}_y$ [5]	1	0	0	0	1
$t_{m2} = \hat{M}_y \left(\frac{M_x}{\hat{M}_x}\right) = \hat{M}_r$ [6]	1	0	1	0	1
$t_{m3} = \hat{M}_y \left(\frac{M_x}{\hat{M}_x}\right)^{\alpha} = \hat{M}_3$ [26]	1	0	α	0	1
$t_{m4} = \hat{M}_y \left(\frac{\hat{M}_x}{M_x}\right) = M_p$ [10]	1	0	-1	0	1
$t_{m5} = w_1 \hat{M}_y \left(\frac{M_x}{\hat{M}_x}\right)$ [1]	1	0	1	0	1
$t_{m6} = w_1 \hat{M}_y \left(\frac{\hat{M}_x}{M_x}\right)$	w_1	0	-1	0	1
$t_{m7} = w_1 \hat{M}_y$ [1]	w_1	0	0	0	1
*$t_{m8} = w_1 \hat{M}_y + w_2 \hat{M}_x + (1 - w_1 - w_2) M_x = \hat{M}_d^3$	w_1	w_2	0	0	1

*Estimator proposed by [19] given in equation (1.23). Another set of estimators generated from class of estimator t_{mq} given in (2.3) using suitable values of η and λ are summarized in table 2.2

Table 2.2:Set of estimators generated from the estimator t_{mq}

Subset of proposed estimator	α	η	λ
$t_{mq}^{(1)} = \left\{ w_1 \hat{M}_y \left(\frac{M_x}{\hat{M}_x}\right) exp\left(\frac{(M_x - \hat{M}_x)}{(M_x + \hat{M}_x) + 2}\right)\right\}$	1	1	1
$t_{mq}^{(2)} = \left\{ w_1 \hat{M}_y \left(\frac{M_x}{\hat{M}_x}\right) exp\left(\frac{(M_x - \hat{M}_x)}{(M_x + \hat{M}_x) + 2\rho_c}\right)\right\}$	1	1	ρ_c
$t_{mq}^{(3)} = \left\{ w_1 \hat{M}_y \left(\frac{M_x}{\hat{M}_x}\right) exp\left(\frac{(M_x - \hat{M}_x)}{(M_x + \hat{M}_x) + 2M_x}\right)\right\}$	1	1	M_x
$t_{mq}^{(4)} = \left\{ w_1 \hat{M}_y \left(\frac{M_x}{\hat{M}_x}\right) exp\left(\frac{(M_x - \hat{M}_x)}{(M_x + \hat{M}_x)}\right)\right\}$	1	1	0
$t_{mq}^{(5)} = \left\{ w_1 \hat{M}_y \left(\frac{\hat{M}_x}{M_x}\right) exp\left(\frac{(M_x - \hat{M}_x)}{(M_x + \hat{M}_x)}\right)\right\}$	-1	1	0
$t_{mq}^{(6)} = \left\{ w_1 \hat{M}_y \left(\frac{M_x}{\hat{M}_x}\right) exp\left(\frac{M_x(M_x - \hat{M}_x)}{M_x(M_x + \hat{M}_x) + 2\rho_c}\right)\right\}$	1	M_x	ρ_c
$t_{mq}^{(7)} = \left\{ w_1 \hat{M}_y exp\left(\frac{M_x(M_x - \hat{M}_x)}{M_x(M_x + \hat{M}_x) + 2\rho_c}\right)\right\}$	0	M_x	ρ_c
$t_{mq}^{(8)} = \left\{ w_1 \hat{M}_y \left(\frac{M_x}{\hat{M}_x}\right) exp\left(\frac{\rho_c(M_x - \hat{M}_x)}{\rho_c(M_x + \hat{M}_x) + 2M_x}\right)\right\}$	1	ρ_c	M_x
$t_{mq}^{(9)} = \left\{ w_1 \hat{M}_y \left(\frac{\hat{M}_x}{M_x}\right) exp\left(\frac{\rho_c(M_x - \hat{M}_x)}{\rho_c(M_x + \hat{M}_x) + 2M_x}\right)\right\}$	-1	ρ_c	M_x

Expressing (2.1) in terms of e's, we have

(2.4) $t_m = w_1 M_y(1+e_0)(1+e_1)^{-\alpha} exp\{-ke_1(1+ke_1)^{-1}\} + w_2 M_x(1+e_1) + (1-w_1-w_2)M_x$

where $k = \dfrac{\eta M_x}{2(\eta M_x + \lambda)}$

Up to the first order of approximation we have,

(2.5) $$(t_m - M_y) = \left[(w_1 - 1)b + w_2 M_y\{e_0 - ae_1 + de_1^2 - ae_0e_1\} + w_2 M_x e_1\right]$$

where $a = (\alpha + k)$, $b = (M_y - M_x)$ and $d = \left\{\frac{3}{2}k^2 + \alpha k + \frac{\alpha(\alpha+1)}{2}\right\}$

Squaring both sides of equation (2.5) and neglecting terms of e's having power greater than two, we have

(2.6)
$$(t_m - \bar{Y})^2 = \left[(1-2w_1)b^2 + w_1^2\{b^2 + m_y^2(e_0^2 + a^2e_1^2 - 2ae_0e_1)\} + w_2^2 M_x^2 e_1^2 + 2w_1w_2 M_y M_x(e_0e_1 - ae_1^2)\right]$$

Taking expectations both sides, we get the MSE of the estimator t_m to the first order of approximation as

(2.7) $$MSE(t_m) = \left[(1-2w_1)b^2 + w_1^2 A + w_2^2 B + 2w_1w_2 C\right]$$

where,
$A = b^2 + M_y^2\gamma(C_y^2 + a^2C_x^2 - 2a\rho_c C_y C_x)$
$B = M_x^2\gamma C_x^2$
$C = M_y M_x \gamma(\rho_c C_y - aC_x)C_x$

The optimum values of w_1 and w_2 are obtained by minimizing (2.7) and is given by

$$w_1^* = \frac{b^2 B}{(AB - C^2)} \quad and \quad w_2^* = \frac{-b^2 C}{(AB - C^2)} \tag{2.8}$$

Substituting the optimal values of w_1 and w_2 in equation (2.7) we obtain the minimum MSE of the estimator t_m as

$$MSE_{min}(t_m) = b^2 \left[1 - \frac{b^2 B}{(AB - C^2)}\right] \tag{2.9}$$

Putting the values of A, B, C and b and simplifying, we get the minimum MSE of estimator t_m as

$$MSE_{min}(t_m) = \left[\frac{M_y^2(1-R)^2\gamma C_y^2(1-\rho_c^2)}{(1-R)^2\gamma C_y^2(1-\rho_c^2)}\right] \tag{2.10}$$

where $R = \frac{M_x}{M_y}$

MSE expression given in (2.10) is same as the minimum MSE of Estimator $\hat{M}_d^3$ given in (1.31)

Similarly, the minimum MSE of the class of estimators t_{mq} is given by

$$MSE_{min}(t_{mq}) = M_y^2 \left[\frac{(\gamma C_y^2 + a^2\gamma C_x^2 - 2a\gamma\rho_c C_y C_x)}{(1 + \gamma C_y^2 + a^2\gamma C_x^2 - 2a\gamma\rho_c C_y C_x)}\right] \tag{2.11}$$

3. Efficiency Comparisons

From equations (1.19) and (2.10) we have

(3.1)

$$\left\{MSE_{min}\left(\hat{M}_y^{(G)}\right) = MSE_{min}\left(\hat{M}_d\right)\right\} - MSE_{min}\left(\hat{t}_m\right) = \frac{(1-R)^2 MSE_{min}\left(\hat{M}_d\right)}{(1-R)^2 + \dfrac{MSE_{min}\left(\hat{M}_d\right)}{M_y^2}} > 0$$

From equations (1.19) and (2.11) we have

$$\left\{MSE_{min}\left(\hat{M}_y^{(G)}\right) = MSE_{min}\left(\hat{M}_d\right)\right\} - MSE_{min}\left(t_m\right) > 0$$

$$\gamma C_y^2 M_y^2(1-\rho_c^2) - M_y^2\left[\frac{(\gamma C_y^2 + a^2\gamma C_x^2 - 2a\gamma\rho_c C_y C_x)}{(1 + 1 + \gamma C_y^2 + a^2\gamma C_x^2 - 2a\gamma\rho_c C_y C_x)}\right] > 0$$

$$\gamma C_y^2 M_y^2(1-\rho_c^2)(1+1+\gamma C_y^2 + a^2\gamma C_x^2 - 2a\gamma\rho_c C_y C_x) > \gamma C_y^2 + a^2\gamma C_x^2 - 2a\gamma\rho_c C_y C_x) \tag{3.2}$$

From equations (1.30) and (2.10)

$$MSE_{min}\left(t_m\right) - MSE_{min}\left(\hat{M}_d\right)$$

(3.3)

$$= \frac{M_y^2 R(R-2) MSE_{min}\left(\hat{M}_d\right)}{M_y^2 + MSE_{min}\left(\hat{M}_d\right)\left\{M_y^2(1-R)^2 + MSE_{min}\left(\hat{M}_d\right)\right\}} < 0, \qquad When \;\; 0 < R < 2$$

Since, $MSE_{min}\left(\hat{M}_d^{(2)}\right) - MSE_{min}\left(\hat{M}_d^{(4)}\right) > 0$

$$\frac{\delta^2\gamma C_x^2 M_y^2\left\{MSE_{min}\left(\hat{M}_d\right)\right\}^2}{M_y^2 + MSE_{min}\left(\hat{M}_d\right)\left\{M_y^2(1-\delta^2\gamma C_x^2) + MSE_{min}\left(\hat{M}_d\right)\right\}} > 0 \tag{3.4}$$

and from (3.3) we have, $MSE_{min}\left(t_m\right) - MSE_{min}\left(\hat{M}_d^{(2)}\right) < 0$

$$(3.5)\qquad Therefore, MSE_{min}\left(t_m\right) - MSE_{min}\left(\hat{M}_d^{(4)}\right) < 0, When \;\; 0 < R < 2$$

It follows from (3.1), (3.2),(3.3), (3.4) and (3.5) that the proposed class of estimators t_m is better than the Conventional difference estimator $\hat{M}_d$, the class of estimators ${M_y}^{(G)}$ and estimator belonging to the class of estimators ${M_y}^{(G)}$ i.e. usual unbiased estimator $\hat{M}_y$,due to [5], usual ratio-type estimator $\hat{M}_y$ due to [6], product estimator $\hat{M}_p$ and $\hat{M}_i$ (i=3,4...7) at their optimum conditions. Further it is shown that the proposed class of estimators t_m is better than the estimators $M_d^{(2)}, M_d^{(4)}$ and $M_d^{(1)}$ considered by [19].

Remark 3.1: Estimator Based on optimum values
Putting the optimum values of w_1^* and w_2^* in the equation (2.1) we get the optimum estimator as:

$$(3.6)\qquad t'_m = \left\{ w_1^* \hat{M}_y \left(\frac{M_x}{\hat{M}_x}\right)^{\alpha} exp\left(\frac{\eta(M_x - \hat{M}_x)}{\eta(M_x + \hat{M}_x) + 2\lambda}\right)\right\} + w_2^* \hat{M}_x + (1 - w_1^* - w_2^*) M_x$$

If the experimenter is not able to specify the value precisely, then it may be desirable to estimate the optimum values from the samples, therefore the values of w_1^* and w_2^* are given as: $w_1^* = \dfrac{\hat{b}^2 \hat{B}}{(\hat{A}\hat{B} - \hat{C}^2)}$ $\; and \;$ $w_2^* = \dfrac{\hat{b}^2 \hat{C}}{(\hat{A}\hat{B} - \hat{C}^2)}$
where, $\quad A = \hat{b}^2 + {\hat{M}_y}^2 \gamma ({\hat{C}_y}^2 + \hat{a}^2 {\hat{C}_x}^2 - 2a\hat{\rho}_c \hat{C}_y \hat{C}_x)$
$B = {\hat{M}_x}^2 \gamma {\hat{C}_x}^2, \hat{\rho}_c = 4(4\hat{p_{11}} - 1)$
$C = \hat{M}_y \hat{M}_x \gamma (\hat{\rho}_c \hat{C}_y - a\hat{C}_x)\hat{C}_x, \hat{C}_x = \left\{\hat{M}_x \hat{f}_x \left(\hat{M}_x\right)\right\}^{-1}, \hat{C}_y = \left\{\hat{M}_y \hat{f}_y \left(\hat{M}_y\right)\right\}^{-1}$
$\hat{a} = (\alpha + \hat{k})$, $\hat{b} = (\hat{M}_y - \hat{M}_x)$ and $\hat{k} = \dfrac{\eta \hat{M}_x}{2(\eta \hat{M}_x + \lambda)}$
Here, we have assumed that the population median of auxiliary variable x is known, therefore $\hat{M}_x$ can also be remain as M_x .
Expressing (3.6) in terms of e's, we have
$t'_m = w_1^* M_y (1+e_0)(1+e_1)^{-\alpha} exp\{-\hat{k}e_1(1+\hat{k}e_1)^{-1}\} + w_2^* M_x(1+e_1) + (1 - w_1^* - w_2^*)M_x$
Proceeding as above, we get the minimum MSE of the estimator t'_m given as:

$$(3.7)\qquad MSE_{min}(t'_m) = \left[\frac{{\hat{M}_y}^2 (1-\hat{R})^2 \hat{\gamma} {\hat{C}_y}^2 (1 - {\hat{\rho}_c}^2)}{(1-\hat{R})^2 \hat{\gamma} {\hat{C}_y}^2 (1 - {\hat{\rho}_c}^2)}\right]$$

Remark 3.2. It may be noted here that the minimum MSEs of the estimators considered in (2.10) and (2.11) are usable only if we know the exact values of C_x, C_y, R, k_c and ρ_c . If these values are unknown then we can estimate them from samples as $\hat{C}_y = \left\{\hat{M}_y \hat{f}_y \left(\hat{M}_y\right)\right\}^{-1}$,$\hat{C}_x = \left\{\hat{M}_x \hat{f}_x \left(\hat{M}_x\right)\right\}^{-1}$, $\hat{R} = \hat{M}_x / \hat{M}_y$, $\hat{k_c} = \hat{\rho}_c \left(\hat{C}_y / \hat{C}_x\right)$ and $\hat{\rho}_c = 4(4\hat{p}_{11} - 1)$ with $\hat{p}_{11}$ being the sample values analogues of p_{11} ([18]; [24]).

4. Empirical study

Data Statistics: To illustrate the efficiency of proposed generalized class of estimators in the application, we consider the following two population data sets.
Population I. (Source [23])
y : The number of fish caught by marine recreational fisherman in 1995.
x : The number of fish caught by marine recreational fisherman in 1964.
The values of the required parameters are :
N=69, n=17, $M_y = 2068$, $M_x = 2011$, $f_y(M_y) = 0.00014$,$f_x(M_x) = 0.00014$, $\rho_c = 0.1505$,

R=0.97244

Population II. (Source [23])

y : The number of fish caught by marine recreational fisherman in 1995.

x : The number of fish caught by marine recreational fisherman in 1993.

The values of the required parameters are:

N=69, n=17, $M_y = 2068$, $M_x = 2307$, $f_y(M_y) = 0.00014$,$f_x(M_x) = 0.00013$, $\rho_c = 0.3166$, R=1.11557

Table 3.1: Variances / MSEs/minimum MSEs of different Estimators

Estimators	Population I	Population II
$V\left(\hat{M}_y\right)$	565443.57	565443.57
$MSE\left(\hat{M}_r\right)$	988372.76	536149.50
$MSE_{min}\left(\hat{M}_d\right)$	552636.13	508766.02
$MSE_{min}\left(\hat{M}_d^{(G)}\right)$	552636.13	508766.02
$MSE_{min}\left(\hat{M}_i\right)$	552636.13	508766.02
$MSE_{min}\left(\hat{M}_d^1\right)$	485969.06	495484.97
$MSE_{min}\left(\hat{M}_d^2\right)$	489395.24	454675.78
$MSE_{min}\left(\hat{M}_d^3\right)$	3229.34	51355.17
$MSE_{min}\left(\hat{M}_d^3\right)$	480458.97	454616.15
$MSE_{min}\left(t_m\right)$	3229.34	51355.17
$MSE_{min}\left(t_{mq}^1\right)$	3267.42	58727.72
$MSE_{min}\left(t_{mq}^2\right)$	3267.43	58729.63
$MSE_{min}\left(t_{mq}^3\right)$	3254.89	55919.25
$MSE_{min}\left(t_{mq}^4\right)$	3267.43	58730.48
$MSE_{min}\left(t_{mq}^5\right)$	3238.55	55037.68
$MSE_{min}\left(t_{mq}^6\right)$	3267.43	58730.48
$MSE_{min}\left(t_{mq}^7\right)$	3232.56	51514.08
$MSE_{min}\left(t_{mq}^8\right)$	3247.25	54709.03
$MSE_{min}\left(t_{mq}^9\right)$	3253.88	59211.32

(for i=1,2,....,7)

Analysing table 3.1 we conclude that the estimators based on auxiliary information are more efficient than the one which does not use the auxiliary information as $\hat{M}_y$ .The members of the class of estimators t_{mq} ,obtained from generalized class of estimators t_m , are almost equally efficient but more than the usual unbiased estimator $\hat{M}_y$ (due to [5]), usual ratio estimator $\hat{M}_r$ (due to [6]), difference type estimator $\hat{M}_d$, the class of estimators $\hat{M}_y^{(G)}$, the estimators $\hat{M}_i$ (i=1,2,...7) and the estimators $\hat{M}_d^{(1)}$, $\hat{M}_d^{(2)}$ and $\hat{M}_d^{(3)}$ (due to [19]). Among the proposed estimators t_m and t_{mq}^j (j=1,2,...9) the performance of the estimator t_m ,which is equal efficient to the estimator $\hat{M}_d^{(3)}$ (due to [19]) , is best in the sense of having the least MSE followed by the estimator $\hat{M}_{mq}^{(7)}$ which utilize the information on population median M_x and ρ_c

5. Conclusion

In this article we have suggested a generalized class of estimators for the population median of study variable y when information is available on an auxiliary variable in simple random sampling without replacement (SRSWOR). In addition, some known estimators of population median such as usual unbiased estimator for population median $\hat{M}_y$ due to [5], estimators due to [6], [26], [10] , [1] and [19] are found to be members of the proposed generalized class of estimators. Some new members are also generated from the proposed generalized class of estimators. We have determined the biases and mean square errors of the proposed class of estimators up to the first order of approximation. The proposed generalized class of estimators are advantageous in the sense that the properties of the estimators, which are members of the proposed class of estimators, can be easily obtained from the properties of the proposed generalized class. Thus the study unifies properties of several estimators for population median. In theoretical and empirical efficiency comparisons, it has been shown that the proposed generalized class of estimators are more efficient than the estimators considered here and equally efficient to the estimator $\hat{M}_d^{(3)}$

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