Int.J. Applied Thermodynamics, ISSN 1301-9724
Vol.2 (No.4), pp.159-164, December-1999

Control System of Substance and Energy Balances of Combined Heat-and-Power Plants Applying the Least Squares Adjustment Method

Andrzej ZIĘBIK, Marcin SZEGA, Henryk RUSINOWSK
Institute of Thermal Technology
Technical University of Silesia
ul. Konarskiego 22, 44-101 Gliwice, POLAND
Tel. +(48 32) 237 16 61, Fax +(48 32) 237 28 72
E-mail: ziebik@itc.ise.polsl.gliwice.pl

Abstract

The set of substance and energy balance equations of water and steam collectors, together with the balance of boilers and turbines (including the regeneration system) is the base of the control system of the exploitation of a combined heat-and power plant. The initial values of this system are the results of measurements in the course of exploitation. Due to inevitable errors in the measurements the substance balances display discrepancies between the balance of steam and the balance of the feed water, and the calculated technical indices are uncertain. In order to increase the reliability of the results of the technical analysis of exploitation and to co-ordinate the balance equations we may use the least squares adjustment method. It may be applied on the condition that we have a sufficient surplus of measuring data. The number of balance equations must be higher than the number of unknown values. This makes it possible to apply the criterion of least squares warranting a maximum of the reliability function in an n dimensional. space of errors (n - number of measuring data). This method has been applied in the balance systems of the combined heat-and-power generating plant.

Keywords: control system, substances and energy balances, combined heat and power plant, least squares adjustment method

1. Introduction

In combined heat-and-power plants, similarly as in power stations, commonly systems of monitoring and automatic acquisition of measuring data are used. Next these data are processed by a computer using adequate mathematical models in order to determine the indices expressing the level of exploitation of the energy machines and equipment.

The bases of the mathematical model of controlling the exploitation indices of the heat-and-power station are a set of balance equations of substances and energy of the feed water and steam collectors together with the balances of the boilers and turbines (including the regeneration systems). The input data are the result of exploitation measurements. The mathematical model of controlling the exploitation indices suggested in this paper differs from conventionally applied models by the fact that it contains the condition of obtaining a surplus of measurement data. This surplus of information makes it possible to check the pre-set accuracy of measurements. In this way we get a set of balance equations containing more equations than the number of unknown values. Only then the least squares adjustment method may be applied. Due to measurement errors equations without unknown values do not prove true (that means the left-hand side of the equation differs from the right-hand side). Only the application of the least squares adjustment method leads to an agreement of the balance equations and consequently to a higher credibility of the calculated technical and economical indices. The least squares adjustment method is the basis of the procedure of co-ordination of substance and energy balances (Brandt, 1970 and Szargut, 1968).

In the control system of balances using the least squares adjustment method corrections of the measured values are determined. If the correction does not exceed the maximum error, the result of exploitation measurements may be considered to be correct and the obtained corrected values to be credible basis for further analysis. The control signals the situation when the correction exceeds the maximum error. Such a case must be eliminated by improving the measurements conditions.

2. Co-ordination of Substance and Energy Balances

The operation of every thermal installation, as well as a distribution network of energy carriers (steam, water), is checked by means of measurements. Basing on the results of these measurements the substance and energy balances are set up. Most of them are used to determine those quantities which have not been measured (e.g. the flux of escaping combustion gases and the flux of combustion air in the case of boilers). The results of measurements and balance calculations are then used to determine the energy efficiency, energy losses and technical indices characterizing the thermal processes. This is the basis for the evaluation of the technical state of the thermal installation and the correctness of its exploitation.

In thermal installations most often the number of unmeasured (unknown) quantities is smaller than the number of balance equations. Therefore, traditionally some part of the balance equations is usually not utilized. Substituting in this redundant equations the results of measurements and calculations of unknown quantities, the result is a discrepancy between the right-hand and the left-hand side of the balance equations due to the inevitable errors of the measured quantities and errors in calculating the unknown values. Redundant balance equations may, however, be used to calculate corrections of measured quantities and preliminarily calculated unknown values. Such a procedure has been called the co-ordination of substance and energy balances (Szargut, 1968 and 1984, Szargut et al.,1996). The application of the procedure of coordinating substance and energy balances yields the following advantages:

- explicit and most reliable values of measurements and unmeasured quantities,
- it becomes possible to control whether the assumptions concerning the accuracy of measurements have been adhered to.

The co-ordination algorithm firstly constitutes a set of equations for the substance and energy balance, as well as equations of the sum of shares. These equations are generally called conditional equations. In order to co-ordinate them, the number of conditional equations must be larger than the number of unknown values. Previous to the co-ordination the set of conditional equations ought to be properly prepared (Szargut et al., 1968 and 1984). Firstly, from the set of equations the equation which contains an unknown value that does not occur in the remaining equations must be eliminated. If then from the balance of energy the losses of heat to the environment are determined, the energy balance cannot be included in the process of co-ordination. Next the mutual independence of the conditional equations is to be checked. For the sake of convenience normally non-linear conditional equations are linearised by expanding them into Taylor's series. Then the set of conditional equations takes the form:

$$\bigwedge_{1...r} : \sum_{i=1}^{n} a_{ki}\,\nu_i + \sum_{l=1}^{u} b_{kl}\,y_l = w_k \qquad (1)$$

where:

a_{ki} -partial derivative of k-th conditional equation in relation to the i-th measured value,
v_i -correction of the i-th measured value,
b_{kl} -partial derivative of k-th conditional equation in relation to the I-th unknown value,
y_l -corrections of the initially determined 1-th unknown value,
w_k -discrepancy of the k-th conditional equation,
r -number of conditional equations,
n -number of measured values,
u -number of unknown values.

In the set of Eqs. 1 the number of unknown values is larger than the number of equations. This makes it possible to condition the determination of corrections of the measured values. The most credible values of the corrections v; result from the maximum of the reliability function formulated in an n-dimensional space of errors. This means that the weighted sum of squares of the corrections ought to be at its minimum:

$$\sum_{i=1}^{n} m_i^{-2}\,\nu_i^2 \Rightarrow \min \qquad (2)$$

where m_i; denotes the average absolute error of the i-th measured value.

Relation (2) is a conditional extremum. The equations of condition are expressed by the set of Eq.(1). This is solved by means of Lagrange's method. The determined correction helps to control the accuracy of measurements:

$$|\nu_i| \le 3|m_i| \qquad (3)$$

If the inequality is true for all the i = 1, 2, ..., n , this means that the assumed accuracy of measurements is satisfied.

3. The Balance Procedure of the Control System of Thermal Balances in a Combined Heat-and-Power Generating Plant Making Use of the Least-Squares Adjustment Method

In the thermal system of a combined heat-and power generating plant generally the following balance subsystems are to be distinguished:

- collectors of feed water,
- steam boilers and possibly water boilers,
- turbines with a regenerating system,
- collectors of technological and heating steam,
- district heating exchangers.

The subsystem of feed water, as well as collectors of technological and heating steam, may be considered to be a set of a priori linear conditional equations containing no unknown values. The redundance of measurement information favours the exactitude of co-ordination calculations. In such a case it is convenient to make use of the matrix convention:

$$\mathbf{A}\dot{\mathbf{G}} = \mathbf{0} \tag{4}$$

$$\dot{\mathbf{G}} = \mathbf{L} + \mathbf{V} \tag{5}$$

$$\mathbf{V}^{T}\mathbf{M}\mathbf{V} \Rightarrow \min \tag{6}$$

where:

A -matrix consisting of the elements 0, +1, -1,
G -vector of the flux of media (water, steam),
L -vector of the measurement results of fluxes,
V -vector of corrections,
M -diagonal matrix of the absolute average errors of measurements.

By means of solving the conditional extremum (5) at the condition (4) it is easy to determine the vector of corrections **V**. Matrix **A** is particularly easy to be set up if we have a graphical diagram of the network. We can then utilize the theory of graphs. Matrix **A** is identical to the incidence matrix.

The general algorithm of co-ordination is considerably simplified in the case of only one conditional equation without any unknown value. Lagrange's method leads then to a solution of the following unconditional extremum:

$$\sum_{i=1}^{n} m^{-2}{}_{i}\upsilon^{2}{}_{i} + \lambda\left(\sum_{i=1}^{n} a_{i}\upsilon_{i} - w\right) \Rightarrow \min \tag{7}$$

where λ denotes Lagrange's multiplier.

In the case of steam and water boilers the algorithm of coordination is used in a general form (Eqs. 1 and 2). The set of conditional equations is then mainly a non-linear balance equation of the elements C, S, H, O and N. This procedure has been described in detail in (Szargut and Rusinowski, 1996). In this form it is introduced into the complete algorithm of the control of balances.

The balance control of the thermal system of a turbine, together with the regenerating system, is analyzed in detail in this paper on the example of a combined heat-and-power generating plant.

4. Example of the Control System of the Balance of an Extraction Turbine

Figure 1 shows a diagram of an extraction turbine together with a regeneration system and district heating exchangers. The unknown values calculated from balance equations subjected to co-ordination are the fluxes of steam passed into the regenerating exchangers and to district heating exchangers, as well as the outlet flux from the turbine to the the condenser. The set of balance equations concerning turbine, regeneration system and district heating exchanger look as follows:

$$\dot{G}_{4} - A_{1}\dot{G}_{16} = 0 \tag{8}$$

$$\dot{G}_{5r} + A_{2}\dot{G}_{4} - A_{3}\dot{G}_{16} = 0 \tag{9}$$

$$\dot{G}_{7} - A_{4}\dot{G}_{14} = 0 \tag{10}$$

$$\dot{G}_{8r} + A_{5}\dot{G}_{7} - A_{6}(\dot{G}_{14} - \dot{G}_{8h}) = 0 \tag{11}$$

$$\dot{G}_{8c} - A_{7}\dot{G}_{w} = 0 \tag{12}$$

$$\dot{G}_{3} - \dot{G}_{4} - \dot{G}_{5} - \dot{G}_{6} - \dot{G}_{7} - \dot{G}_{8} - \dot{G}_{9} - \delta\dot{G}_{p} = 0 \tag{13}$$

$$\dot{G}_{3}h_{3} - \dot{G}_{4}h_{4} - \dot{G}_{5}h_{5} - \dot{G}_{6}h_{6} - \dot{G}_{7}h_{7} + - \dot{G}_{8}h_{8} - \dot{G}_{9}h_{9} - \delta\dot{G}_{p}h_{p} - \frac{N_{el}}{\eta_{me}} = 0 \tag{14}$$

$$\dot{G}_{3} = \dot{G}_{1} - \delta\dot{G}_{1} - \delta\dot{G}_{e} \tag{15}$$

$$\dot{G}_{5} = \dot{G}_{5r} + \dot{G}_{5c} \tag{16}$$

$$\dot{G}_{8} = \dot{G}_{8r} + \dot{G}_{8h} \tag{17}$$

where:

$\dot{G}_{1}$,…. , $\dot{G}_{9}$ fluxes of steam
$\dot{G}_{14}$,…., $\dot{G}_{18}$ fluxes of feed water,
h_3,…..,h_9 specific enthalpy of steam,
δG_p, h_p flux and enthalpy of steam from the packing
N_{el} power rating of the turbo generator

η_{me}	electromechanical efficiency of the turbo generator
$\delta\dot{G}_1$, $\delta\dot{G}_e$	fluxes of steam leakages and flux to ejector
$\dot{G}_{5r}$, $\dot{G}_{5c}$	fluxes of steam (1.5 Mpa) to the regenerating preheater and to the collector
$\dot{G}_{8r}$, $\dot{G}_{8h}$	fluxes of steam (0.25 Mpa) to the regenerating preheater and to the district heating exchanger
$\dot{G}_w$	flux of district heating water

The auxiliary values A1,….A-, depending on the thermal parameters of steam and water are expressed by the relations

$$A_1 = \frac{(1+\zeta_r)(h_{19}-h_{17})}{(h_4-h_{20})}$$

$$A_2 = \frac{h_{20}-h_{18}}{h_5-h_{18}}$$

$$A_3 = \frac{(1+\zeta_r)(h_{17}-h_{16})}{h_5 h_{18}}$$

$$A_4 = \frac{(1+\zeta_r)(h_{14}-h_{13})}{h_7-h_{15}}$$

$$A_5 = \frac{(h_{15}-h_{12})}{h_8-h_{12}}$$

$$A_6 = \frac{(1+\zeta_r)(h_{11}-h_{10})}{h_8-h_{12}}$$

$$A_7 = \frac{(1+\zeta_h)(h_{w2}-h_{w1})}{h_8-h_{21}}$$

where:

ζ_r, ζ_h	relative losses of heat from regenerating
$h_{10}\ldots h_{21}$	specific entalphy of feed water and condensate

TABLE I contains the results of measurements, as well as calculations of corrections of the measured data and initially calculated unknown values. There are also the values of average absolute errors. The last column presents the signalization of erroneous measurements. The lack of an asterisk denotes that the correction does not exceed, the average error, whereas the asterisk informs that the average error has been exceeded. The presence of two asterisks signalizes that the error has been doubled. Three asterisks denote that the error is gross. Such a measurement cannot be accepted.

The results of control calculations of the substance and energy balances (TABLE I) indicate that the exploitation measurements are correct. In no case may the correction of the measured value exceed the threefold value of the average absolute error. The assumed accuracy of measurements has been satisfied. The corrected results of measurements and preliminary calculated unknown values may be used in of the technical indices of exploitation.

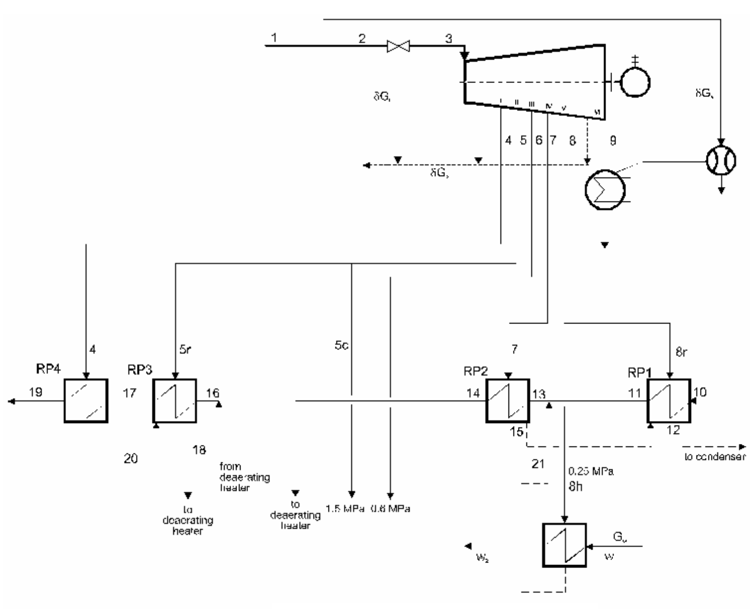

Figure 1. Balance scheme of turbine with the regeneration system.

TABLE I. RESULTS OF MEASUREMENT AND CALCULATIONS OF CORRECTIONS.

Measured quantities	Symbol	Unit	Results before coordination	Results after coordination	Absolute error	Correction	Control of accuracy
High pressure steam	G1	t/h	164.0	166.7	1.64	2.748	*
Extr. steam II to collector 1.5 Mpa	G5c	t/h	1.0	1.0	0.	0	
Extr. steam III to collector 0.6 Mpa	G6	t/h	0.0	0.0	0	0	
Feed water after RP2	G14	t/h	141.0	140.9	1.41	-0.1136	
Feed water after RP4	G19	t/h	168.0	167.8	1.68	-0.1924	
District heating water	Gw	t/h	1143.0	1103.7	57.15	-39.3025	
Production of electric energy	Nel	MW	40.0	40.0	0.04	-0.0059	
Entalphy of inlet steam	h3	kJ/kg	3348.7	3376.2	13.0672	27.5273	**
Entalphy of extraction steam I	h4	kJ/kg	3175.9	3175.8	5.3962	-0.1039	
Entalphy of extraction steam II	h5	kJ/kg	2956.1	2955.6	7.1722	-0.4356	
Entalphy of extraction steam III	h6	kJ/kg	2829.3	2829.3	9.356	0	
Entalphy of extraction steam IV	h7	kJ/kg	2733.8	2732.1	12.5663	-1.6494	
Entalphy of extraction steam V	h8	kJ/kg	2588.3	2526.8	33.262	-61.5061	*
Entalphy of outlet steam	h9	kJ/kg	2322.0	2234.2	36.3115	-87.7445	**
Entalphy of feed water	h10	kJ/kg	175.8	176.0	4.1868	0.1767	
Entalphy of feed water	h11	kJ/kg	360.2	360.0	4.1879	-0.1767	
Entalphy of feed water	h12	kJ/kg	364.3	364.8	4.1879	0.4365	
Entalphy of feed water	h13	kJ/kg	563.3	562.9	4.2037	-0.4398	
Entalphy of feed water	h14	kJ/kg	662.4	663.2	4.2188	0.869	
Entalphy of feed water	h15	kJ/kg	802.5	802.7	4.2458	0.2039	
Entalphy of feed water	h16	kJ/kg	879.2	878.1	4.2678	-1.0353	
Entalphy of condensate	h17	kJ/kg	368.6	368.6	4.1889	-0.0394	
Entalphy of condensate	h18	kJ/kg	503.6	503.6	4.1967	-0.0127	
Entalphy of condensate	h19	kJ/kg	784.8	784.7	4.242	-0.0848	
Entalphy of condensate	h20	kJ/kg	874.7	874.7	4.267	-0.0069	
Entalphy of condensate	h21	kJ/kg	368.6	368.5	4.1889	-0.1087	
Entalphy of district heating water	hw1	kJ/kg	255.3	257.5	4.185	2.2139	
Entalphy of district heating water	hw2	kJ/kg	364.1	361.9	4.185	-2.2139	
Steam to valves	dG1	t/h	1.5	1.5	0.0015	0	
Steam to ejectors	dGe	t/h	0.5	0.5	0.0005	0	
Steam from packing	dGp	t/h	2.5	2.5	0.0025	0	
Entalphy of steam from packing	hp	kJ/kg	3000.0	2999.0	20	-0.9951	

Not Measured Quantities	Symbol	Unit	Results before coordination	Results after coordination	Correction	Absolute error	
Extr. Steam I to regen.preh.RP4	G7	t/h	5.6	5.5	-0.101	0.4427	
Extr. Steam IV to regen.preh.RP2	G7	t/h	12.6	12.5	-0.0563	0.4025	
Extr. Steam V to distr. heating exh.	G8h	t/h	56.6	53.9	-2.6837	4.2202	
Extr. steam II to regen. preh. RP3	G5r	t/h	10.6	10.6	-0.0582	0.4865	
Extr. steam V to regen. preh. RP1	G8r	t/h	6.2	6.6	0.3761	0.4223	
Outlet Steam	G9	t/h	66.9	72.1	5.2771	4.1298	

5. Conclusions

The control systems of the exploitation of combined heat and-power generating plants, as well as power stations are based on thermal measurements and on substance and energy balances. Due to inevitable errors of measurements the calculated indices of exploitation are not quite reliable.

In order to improve the credibility of control systems the introduction of the least squares adjustment method into the algorithm has been suggested in order to correct the measured values and next the calculated unknown values. This method can be applied if the number of balance equations exceeds the number of unknown values. The procedure of coordinating the balance is based on the criterion of the maximum of the a reliability function in an n-dimensional space of errors.

The control system of substance and energy balances with the least squares adjustment method includes the signalization of accuracy of measurements. If the maximum error is exceeded, this is signalized by three asterisks. Only the results of measurements which satisfy the criterion of the assumed accuracy of measurements may be used in further analyses.

If the maximum error is exceeded either the measuring method or the measuring device or the way of operating the device be checked. After the gross error has been eliminated, the measurements must be repeated.

Thanks to the least squares adjustment method the measurements which have already been taken may be used again. Because if we have an adequate redundance of balance equations, exceeding the number of unknown values, we may assume the measured quantity with a gross error as an unknown value, repeating the coordination procedure.

Acknowledgement

This paper was supported by the Polish Government within the frame of *Polish-German Institute without Walls*. The authors would like express their gratitude to the State Committee for Scientific Research.

References

Brandt S., 1970, *Statistical and Computational Methods in Data Analysis*, North Holland Publishing Company, Amsterdam, London.

Szargut J., Kolenda Z., 1968*, Theory of Co-ordination of Material and Energy Balances in Metallurgical Chemical Processes.* Archives of Metallurgy Vol. 12, No.2.

Szargut J. at al., 1984, *Co-ordination Calculus in Thermal Engineering (in Polish)*. Polish Academy of Sciences, Ossolineum Press, Wroclaw.

Szargut J., Kolenda Z., Styrylska T.,1996, *Justification of Measurements Results in Thermal Technology*. Proceedings of ECOS'96, Stockholm.

Szargut J., Rusinowski H., 1996, *Justification of Measurement Results of Combustion Processes*. Proceeding of ECOS'96, Stockholm.