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Second Law Analysis of the Turbulent Flat Plate Boundary Layer*

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Abstract

Until now the second law analysis of turbulent flow relied only on the irreversibilities performed by the mean velocity and mean temperature gradients. Using the Reynolds decomposition of the volumetric entropy generation rate expression we found that the dissipation rates of both, turbulent kinetic energy and fluctuating temperature variance, also represent the irreversibilities of the flow. Applying the above results, the second law analysis of the turbulent boundary layer shows that the maximum values of the "mean motion irreversibilities" (generated by the mean velocity and mean temperature gradient) are located at the wall, while the maximum values of the "turbulent irreversibilities" (performed by the dissipation rate of turbulent kinetic energy and fluctuating temperature variance) are located in the buffer sublayer. As a consequence, for a given location on the plate, the integral values of the "mean motion irreversibilities" are approximately constant and the "turbulent irreversibilities" grow up with the boundary layer thickness.

Key words: Entropy generation, turbulent flow, flat plate boundary layer

1. Introduction

Most flows occurring in engineering applications are turbulent. Taking into account the well known liaison between the thermodynamic irreversibilities and the lost exergy, the second law analysis accuracy of such a flow become very important for a good technical performance prediction of engineering devices.

Following Sciubba (1994), the second law analysis can be performed at a bulk, microscopic and macroscopic level. In the bulk level, the whole system or macroscopic subsystems are considered and the entropy generation rate is computed using the thermodynamic procedures. At this level, the second law analysis can be easy applied, but, in the case of complex flows, it does not offer a reasonable accuracy. In the microscopic level the system is divided into an infinite number of subsystems of point size for which the calculus of the volumetric entropy generation rate is performed. Using the microscopic level, the entire flow field solution needs to be known, but at the same time, the entire flow field irreversibilities are revealed. This method was first formulated by Bejan (1982) in the case of laminar flows. It was also used for the turbulent flows (Bejan, 1982; Sciubba, 1994; Natalini and Sciubba, 1994), but only the viscous and thermal irreversibilities due to the mean velocity and mean temperature gradients have been considered.

The turbulence, itself, generates specific mechanisms of irreversibilities because the kinetic energy of fluctuating velocity and the thermal exergy of fluctuating temperature are dissipated through the viscosity and the thermal diffusivity of the fluid. In order to identify these irreversibilities and their magnitude, the expression of the volumetric entropy generation rate will be first, averaged in time and next, particularized for the case of the turbulent incompressible flow over a flat plate.

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2. Entropy Generation Rate in Turbulent Incompressible Flows

In the incompressible turbulent flow hypothesis the instantaneous volumetric rate of entropy generation is expressed by:

$$\dot{S}_{gen}^{(\Omega)} = \left(\dot{S}_{gen}^{(\Omega)}\right)_V + \left(\dot{S}_{gen}^{(\Omega)}\right)_Q \tag{1}$$

where:

$$\left(\dot{S}_{gen}^{(\Omega)}\right)_V = 2\frac{\mu}{T}\left(S_{ij}S_{ij}\right) \tag{2}$$

$$\left(\dot{S}_{gen}^{(\Omega)}\right)_Q = \frac{\lambda}{T^2}\left(\frac{\partial T}{\partial x_j}\frac{\partial T}{\partial x_j}\right) \tag{3}$$

represent the viscous part and the thermal part of irreversibilities and S_{ij} denote the strain rate tensor. Decomposing the flow properties into the mean and the fluctuating part, the Reynolds averaged expression of eq. (1) can be written as:

$$\begin{aligned}\overline{\dot{S}_{gen}^{(\Omega)}} &= 2\frac{\mu}{\overline{T}}\overline{\left(1+T'/\overline{T}\right)^{-1}\left(\overline{S}_{ij}+S'_{ij}\right)\left(\overline{S}_{ij}+S'_{ij}\right)}+\\ &+\frac{\lambda}{\overline{T}^2}\overline{\left(1+T'/\overline{T}\right)^{-2}\frac{\partial\left(\overline{T}+T'\right)}{\partial x_j}\frac{\partial\left(\overline{T}+T'\right)}{\partial x_j}}\end{aligned} \tag{4}$$

Because of the fluid incompressibility the fluctuating temperature are much smaller than the mean temperature. Then, using the serial decomposition of $(1+T'/\overline{T})^{-1}$ and $(1+T'/\overline{T})^{-2}$ the expression (4) becomes:

$$\begin{aligned}\overline{\dot{S}_{gen}^{(\Omega)}} &= \left(\overline{\dot{S}_{gen}^{(\Omega)}}\right)_{VM} + \left(\overline{\dot{S}_{gen}^{(\Omega)}}\right)_{QM} +\\ &+\left(\overline{\dot{S}_{gen}^{(\Omega)}}\right)_{VT} + \left(\overline{\dot{S}_{gen}^{(\Omega)}}\right)_{QT} + \overline{\Delta'_V} + \overline{\Delta'_Q}\end{aligned} \tag{5}$$

The first two terms of the above expression are:

$$\left(\overline{\dot{S}_{gen}^{(\Omega)}}\right)_{VM} = 2\frac{\mu}{\overline{T}}\overline{S}_{ij}\overline{S}_{ij} \tag{6}$$

$$\left(\overline{\dot{S}_{gen}^{(\Omega)}}\right)_{QM} = \frac{\lambda}{\overline{T}^2}\frac{\partial\overline{T}}{\partial x_j}\frac{\partial\overline{T}}{\partial x_j} \tag{7}$$

and model the viscous and thermal irreversibilities generated in the mean motion field by the gradients of average velocity and average temperature. They are the homologues of the terms modeling the laminar flow irreversibilities because they are generated by the same mechanisms.

The following two terms, containing the correlations of the fluctuating velocity and fluctuating temperature gradients model the proper irreversibilities of the flow turbulence. Thus, the term:

$$\left(\overline{\dot{S}_{gen}^{(\Omega)}}\right)_{VT} = \frac{\rho}{\overline{T}}\left(2\nu\overline{S'_{ij}S'_{ij}}\right) = \frac{\rho\varepsilon}{\overline{T}} \tag{8}$$

corresponds to the irreversibilities generated by the viscous dissipation ε of the kinetic energy K, and the term:

$$\left(\overline{\dot{S}_{gen}^{(\Omega)}}\right)_{QT} = \frac{\lambda}{\overline{T}^2}\overline{\frac{\partial T'}{\partial x_j}\frac{\partial T'}{\partial x_j}} = \frac{\rho c_p}{\overline{T}^2}\varepsilon_\theta \tag{9}$$

characterizes the irreversibilities due to the thermal dissipation ε_θ of the fluctuating temperature variance K_θ. As in the previous case, both, the viscosity and the thermal diffusivity of the fluid are involved in the act of mechanism's dissipation. Of course, in the eq. (8) and (9) the last equalities result from the definitions of ε and ε_θ (Tennekes and Lumley, 1972; Mohammadi and Pironneau, 1994).

The last two terms, $\overline{\Delta'_V}$ and $\overline{\Delta'_Q}$ contain all of the correlations build from the serial decomposition of $(1+T'/\overline{T})^{-1}$ and $(1+T'/\overline{T})^{-2}$. Generally they can be neglected because they are formed with the upper powers of $(T'/\overline{T})$.

As in the laminar case, the expressions (5)-(9) show that the only sources of the turbulent flow irreversibilities are the viscous and the thermal dissipations. Unlike the laminar flows the structure of turbulent dissipations are more complicated because they act not only on the mean exergy but also on the turbulent fluctuating exergy. In order to emphasize the relative importance of the fluctuating exergy dissipation mechanisms, we define two irreversibility distribution ratios:

$$\Phi_V^{(\Omega)} = \left(\overline{\dot{S}_{gen}}\right)_{VT}\Big/\left(\overline{\dot{S}_{gen}}\right)_{VM} = \frac{\rho\varepsilon}{2\mu\overline{S}_{ij}\overline{S}_{ij}} \tag{10}$$

$$\begin{aligned}\Phi_Q^{(\Omega)} &= \left(\overline{\dot{S}_{gen}}\right)_{QT}\Big/\left(\overline{\dot{S}_{gen}}\right)_{QM} =\\ &= \frac{\varepsilon_\theta}{\alpha\left(\partial\overline{T}/\partial x_j\right)\left(\partial\overline{T}/\partial x_j\right)}\end{aligned} \tag{11}$$

Using the above definitions, and neglecting the corelations $\overline{\Delta'_V}$ and $\overline{\Delta'_Q}$ the expression for mean volumetric entropy generation rate becomes:

$$\begin{aligned}\overline{\dot{S}_{gen}^{(\Omega)}} &= \left(\overline{\dot{S}_{gen}^{(\Omega)}}\right)_{VM}\left(1+\Phi_V^{(\Omega)}\right)+\\ &+\left(\overline{\dot{S}_{gen}^{(\Omega)}}\right)_{QM}\left(1+\Phi_Q^{(\Omega)}\right)\end{aligned} \tag{12}$$

The relations (5)-(12) show that the turbulence act not only by growing the classical viscous and thermal ireversibilities, (noted with the subscripts "VM" and "QM") but also by generating new kinds of viscous and thermal irreversibilities, (noted with subscripts "VT" and "QT") due to the dissipation of turbulent kinetic energy and fluctuating temperature variance.

3. Entropy Generation Rate Calculation for Turbulent Boundary Layer Flow

Most flows occurring in the field of power generation are turbulent and often very complicated. In such a case, the calculation of the entropy generation rate is performed only by numerical methods. The flow in boundary layer represents one of the few exceptions for which the distribution of all the volumetric entropy generation rate components can be obtained analytically.

In terms of the wall coordinates the common dimensionless quantities are:

$$u^+ = \frac{u}{u_\tau};\ T^+ = \frac{T_w - T}{T_\tau};$$

$$K^+ = \frac{K}{u_\tau^2};\ K_\theta^+ = \frac{1}{2}\frac{\overline{T'^2}}{T_\tau^2};$$

$$\varepsilon^+ = \frac{\nu\varepsilon}{u_\tau^4};\ \varepsilon_\theta^+ = \frac{\nu\varepsilon_\theta}{T_\tau^2 u_\tau^2}$$

where u_τ and T_τ are the friction velocity and the friction temperature, and:

$$\left(\dot{S}_{gen}^{(\Omega)}\right)^+ = \dot{S}_{gen}^{(\Omega)}\frac{\nu T_w}{u_\tau^2 \tau_w} \qquad (13)$$

With the classical simplifications used for boundary layer flows the expressions (6) and (7) become:

$$\left(\dot{S}_{gen}^{(\Omega)}\right)_{VM}^+ = \frac{\nu_V^+\left(\partial u^+/\partial y^+\right)^2}{1 - bT^+}; \qquad (14)$$

$$\left(\dot{S}_{gen}^{(\Omega)}\right)_{QM}^+ = \frac{a\alpha_V^+\left(\partial T^+/\partial y^+\right)^2}{\left(1 - bT^+\right)^2} \qquad (15)$$

where a and b are two coefficients containing in their definition the initial flow properties U_∞ and T_∞, the wall temperature T_w, the difference $\Delta T = T_w - T_\infty$, the skin friction $C_{f,x}$ and the Stanton number St_x:

$$a = \frac{u_\tau^2}{c_p T_w}\left(\frac{\dot{q}_w}{u_\tau \tau_w}\right)^2 = \left(\frac{St_x}{\frac{1}{2}C_{f,x}}\right)^2 \frac{\Delta T}{T_w}\frac{c_p \Delta T}{U_\infty^2};$$

$$b = \frac{T_\tau}{T_w} = \frac{St_x}{\frac{1}{2}C_{f,x}}\frac{u_\tau}{U_\infty}\frac{\Delta T}{T_w} \qquad (16)$$

The skin friction and the Stanton number are related by the Colburn analogy ($St_x = 0.5C_{f,x}/Pr^{2/3}$) that remove the dependence of a and b from the x-direction. If the classical laws of the wall:

$$u^+ = \begin{cases} y^+ & y^+ < 11{,}63 \\ (1/\kappa)\ln y^+ + B & y^+ > 11{,}63 \end{cases}$$

$$T^+ = \begin{cases} Pr\, y^+ & y^+ < 11{,}63 \\ (Pr_T/\kappa)\ln y^+ + B_T & y^+ > 11{,}63 \end{cases} \qquad (17)$$

are used for the calculation of the two derivatives appearing in eq. (14) and (15), then the distributions of $(\dot{S}_{gen}^{(\Omega)})_{VM}$ and $(\dot{S}_{gen}^{(\Omega)})_{QM}$ are not continuos at the joining point $y^+=11.63$ and the derivative's approximations are not accurate in the buffer sublayer. Because of these $\partial u^+/\partial y^+$ and $\partial T^+/\partial y^+$ were computed using the relations:

$$\frac{u^+}{y^+} = \frac{1 + \sqrt{1 + 4\left(\kappa y^+ D\right)^2}}{2\left(\kappa y^+ D\right)^2}; \qquad (18)$$

$$\frac{T^+}{y^+} = \frac{Pr}{1 + \left(Pr/Pr_T\right)\left(\kappa y^+ D\right)^2\left(\partial u^+/\partial y^+\right)} \qquad (19)$$

resulting from the assumptions of the constant total shear stress and constant total heat flux. In the above equations $\kappa = 0.415$ is the von Karman constant and $D = 1 - \exp(-y^+/A^+)$ represents the van Driest correction for the Prandtl mixing length formula.

In the wall coordinates the eq. (8) and (9) can be written as:

$$\left(\dot{S}_{gen}^{(\Omega)}\right)_{VT}^+ = \frac{\varepsilon^+}{1 - bT^+}; \qquad (20)$$

$$\left(\dot{S}_{gen}^{(\Omega)}\right)_{QT}^+ = \frac{a\varepsilon_\theta^+}{\left(1 - bT^+\right)^2} \qquad (21)$$

If the turbulence equilibrium hypothesis is used then:

$$\varepsilon^+ \cong P_K^+ = \nu_T^+\left(\partial u^+/\partial y^+\right)^2; \qquad (22)$$

$$\varepsilon_\theta^+ \cong P_\theta^+ = \alpha_T^+\left(\partial T^+/\partial y^+\right)^2 \qquad (23)$$

where P_K^+ and P_θ^+ are the production terms of turbulent kinetic energy and fluctuating temperature variance. The dimensionless turbulent viscosity $\nu_T^+ = \nu_T/\nu$ and dimensionless thermal diffusivity $\alpha_T^+ = \alpha_T/\alpha$ can be evaluated with:

$$\nu_T^+ = \left(\kappa y^+ D\right)^2 \frac{\partial u^+}{\partial y^+} \tag{24}$$

$$\alpha_T^+ = \frac{\nu_T^+}{Pr_T} \tag{25}$$

The approximations (22) and (23) work very well in the logarithmic sublayer but leads to important errors in the vicinity of the wall. These errors are not relevant for the calculation of the volumetric entropy generation rate because at the wall :

$$(\dot{S}_{gen}^{(\Omega)})_{VM} >> (\dot{S}_{gen}^{(\Omega)})_{VT};\ (\dot{S}_{gen}^{(\Omega)})_{VM} >> (\dot{S}_{gen}^{(\Omega)})_{QT}$$

Anyway with the two-layer treatment of the K-ε equations (Patel and Chen, 1988) the hypothesis (22) can be avoided. In the low Reynolds number layer, ν_T^+ and ε^+ have the expressions:

$$\nu_T^+ = C_\mu L_\mu^+ \sqrt{K^+} \tag{26}$$

$$\varepsilon^+ = (K^+)^{3/2} / L_\varepsilon^+ \tag{27}$$

Using Eq. (18), (24), (26) and (27) the turbulent kinetic energy and the turbulent dissipation rate are the solution of the system:

$$1 + \sqrt{1 + 4\left(\kappa y^+ D\right)^2} = 2C_\mu^{1/4}\left\{1 - \exp\left[\ \left(K^+\right)^{1/2} y^+ / A_\mu\right]\right\}\left(K^+\right)^{1/2} \tag{28a}$$

$$\varepsilon^+ = \frac{\left(K^+\right)^{3/2}}{\kappa y^+ C_\mu^{-3/4}\left\{1 - \exp\left[-\left(K^+\right)^{1/2} y^+ / A_\varepsilon\right]\right\}} \tag{28b}$$

where A_μ=70, $A_\varepsilon=2\kappa C_\mu^{-3/4}$ and C_μ=0.09. In the present work the hypothesis (23) was maintained because for the K_θ-ε_θ equations the two layer approach does not exist.

4. Results and Discussions

Figure 1 shows the distributions of the volumetric entropy generation rate components for a=2 and b=0.00667. The great mean motion irreversibilities occur in the viscous and in the buffer sublayers and their maximal values are reached at the wall. The high turbulent irreversibilities are found in the same regions of the flow, but their distributions are quite different, because their maximal values are reached in the buffer sublayer. We note that, because of the approximation (23), which does not work well in the viscous sublayer, only $(\dot{S}_{gen}^{(\Omega)})_{VT}^+$ has a correct behavior in the wall vicinity, where "ε and ε_θ cannot vanish".

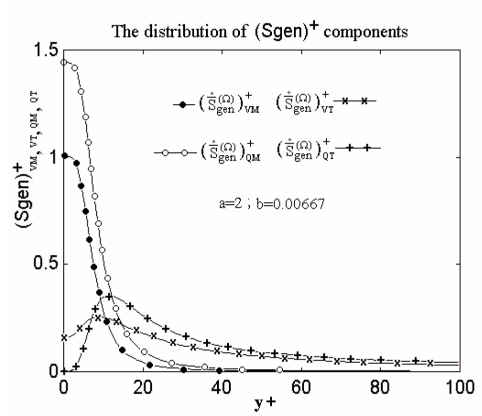

Figure 1. The variation of the volumetric entropy generation rate components

The distributions of the irreversibility ratios, computed with:

$$\Phi_V^{(\Omega)} = \frac{\varepsilon^+}{\left(\partial u^+/\partial y^+\right)^2}\ ;\ \Phi_Q^{(\Omega)} = \frac{\varepsilon_\theta^+}{\left(\partial T^+/\partial y^+\right)^2}$$

are presented in *Figure 2*. It can be seen that the turbulent irreversibility ratios increase monotonically with y^+, and the equalities $\Phi_V^{(\Omega)}$=1 and $\Phi_Q^{(\Omega)}$=1 are valid in the buffer sublayer. For this reason the rate of entropy generation by the mean velocity and mean temperature gradients prevail in the viscous sublayer while the rate of entropy generation due to the dissipations of turbulent kinetic energy and fluctuating temperature variance rule the logarithmic sublayer.

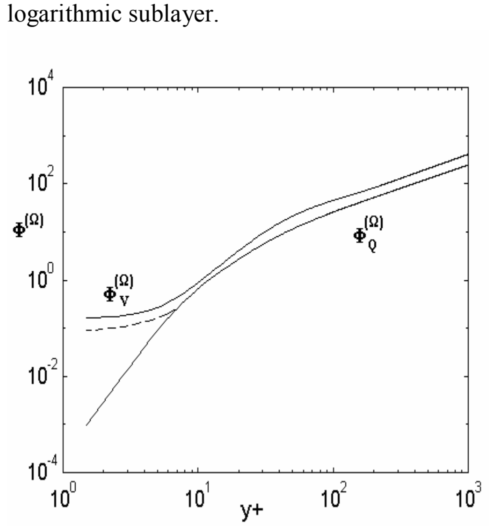

Figure 2. The variation of volumetric ireversibility ratios

Figure 3 presents the variation of the volumetric entropy generation rate along the coordinate normal to the wall. Obviously, the growth of parameter 'a' increases the flow irreversibilities but their profile remains unchanged. So, variation of the volumetric entropy generation rate follows the profile of the mean motion irreversibilities close to the wall and the variation of turbulent irreversibilities in the logarithmic sublayer.

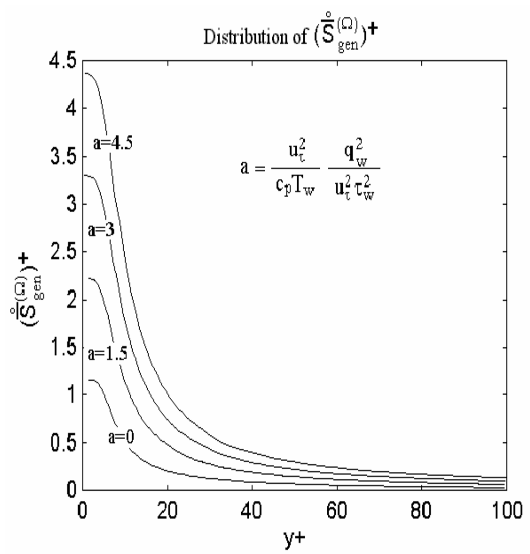

Figure 3. The variation of volumetric entropy generation rate

TABLE I shows the variations of the surface entropy generation rate components along y^+, obtained by numerical integration of:

$$(\dot{\bar{S}}{}^{(\Sigma)}_{gen})^+_{VM,VT,QM,QT} = \int_0^{y^+} (\dot{\bar{S}}{}^{(\Omega)}_{gen})^+_{VM,VT,QM,QT}\, dy^+$$

for a=2 and b=0.00667. It can be seen that the mean motion components of surface irreversibilities have an asymptotic behaviour on the wall normal coordinate so that their values remain practically unchanged over a certain value of y^+ (in our case $y^+>300$). Contrary, the values of surface turbulent irreversibilities grow continuos with the y^+ coordinate showing that the surface dissipations of the turbulent energy and the turbulent irreversibility ratios increases with the boundary layer thickness. It is interesting to note that, if the wall functions (17) had been used for determining the values of $\partial u^+/\partial y^+$ and $\partial T^+/\partial y^+$, the errors of the mean motion entropy generation calculation would have been greater than 20% for the surface viscous rate and about 11% for the surface thermal rate.

TABLE I. THE VARIATION OF SURFACE ENTOPY GENERATION RATE COMPONENTS

y^+	$(\dot{\bar{S}}{}^{(\Sigma)}_{gen})^+_{VM}$	$(\dot{\bar{S}}{}^{(\Sigma)}_{gen})^+_{QM}$	$(\dot{\bar{S}}{}^{(\Sigma)}_{gen})^+_{VT}$	$(\dot{\bar{S}}{}^{(\Sigma)}_{gen})^+_{QT}$
50	9.0445	14.5682	7.5573	10.2974
100	9.0955	14.7490	9.8201	13.8701
200	9.1285	14.8302	11.7647	17.4178
300	9.1395	14.8568	12.8513	19.5383
400	9.1441	14.8715	13.6249	21.0625
500	9.1484	14.8790	14.2373	22.2563
600	9.1496	14.8862	14.7216	23.2402
700	9.1518	14.8889	15.1417	24.0724
800	9.1529	14.8914	15.5056	24.8000
900	9.1540	14.8939	15.8273	25.4422
1000	9.1551	14.8963	16.1157	26.0184

5. Conclusion

The average volumetric irreversibilities of the incompressible turbulent flows are performed by the viscous and thermal dissipations that occur in the mean as well as in the fluctuating part of the motion. The expressions (6)-(9) show that in the both cases the mechanism of gradient interactions is involved in the dissipation process. Thus, the mean motion irreversibilities are born through the interactions of the average velocity or the average temperature gradients, while the turbulent irreversibilities are performed by the interactions of fluctuating velocity or fluctuating temperature gradients. In these conditions the irreversibility ratios (11) and (12) emphasize the relative importance of the volumetric turbulent irreversibilities.

Applying the above results, the second law analysis of the turbulent boundary layer shows that the “volumetric mean motion irreversibilities” (generated by the mean velocity and mean temperature gradient) prevail in the viscous sublayer and reach their maximal values at the wall, while the "volumetric turbulent irreversibilities" (performed by the dissipation rate of turbulent kinetic energy and fluctuating temperature variance) rule the logarithmic sublayer but have the maximal values located in the buffer sublayer. As a consequence, for a given location of the plate, the surface values of the mean motion irreversibilities are approximately constant while the turbulent irreversibilities grow up with the boundary layer thickness.

Nomenclature

a,b	Constants eq. 16
A_μ, A_ε	Constants of the two layer approach
c_p	Specific heat at constant pressure
C_{fx}	Local skin friction
C_μ	Constant of K-ε model
$D = 1 - \exp(-y^+/A^+)$	van Driest corection of Prandtl mixing length formula
$K = \frac{1}{2}\overline{u'_j u'_j}$	Turbulent kinetic energy
$K_\theta = \frac{1}{2}\overline{T'^2}$	Fluctuating temperature variance
S_{ij}	Strain rate tensor
St_x	Local Stanton number
$\dot{S}_{gen}$	Rate of entropy generation
T	Temperature
$T_\tau = \dot{q}_w / (u_\tau \rho c_p)$	Friction temperature
$u_\tau = (\tau_w/\rho)^{1/2}$	Friction velocity
$y^+ = y u_\tau / \nu$	Dimensionless wall coordinate
$\alpha = \lambda/(\rho c_p)$	Thermal diffusivity
$\varepsilon = 2\nu\overline{S'_{ij}S'_{ij}}$	Dissipation rate of K
$\varepsilon_\theta = \alpha\overline{\frac{\partial T'}{\partial x_j}\frac{\partial T'}{\partial x_j}}$	Dissipation rate of K_θ
κ	von Karman constant
Φ	Irrevresibility ratio
λ	Thermal conductivity
ν	Kinematic viscosity
μ	Viscosity
ρ	Density
$\tau_w = (\partial u/\partial y)_{y=0}$	Wall stress tensor

Superscript

$(\ )'$	Fluctuating part
$\overline{(\ )}$	Mean part
$(\ )^{(\Omega)}$	Volumetirc
$(\ )^+$	Wall coordinate

Subscript

$(\ )_V$	Viscous part
$(\ )_Q$	Thermal part
$(\ )_W$	Wall conditions
$(\ )_T$	Turbulent

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