# Exergy – Aided Cost Optimization Using Evolutionary Algorithms[*]

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**Abstract**

In this paper an algorithm is presented, which combines a powerful optimizing tool, Evolutionary Algorithms, with an exergo-economic analysis. The latter supports the optimizer by evaluating certain parameters as the exergetic efficiency and the relative cost difference. A simple cogeneration system has been chosen to illustrate this algorithm.

*Key words: exergy analysis, exergo-economics, evolutionary algorithms, cogeneration-plant*

## 1. Introduction

The economic optimization of thermal energy systems and chemical processes has often proven to be very complicated due to the great amount of subsystems and their interrelation. Conventional optimization algorithms do not accomplish the requirements, as they either do not converge or persist in local optima. With developments in computer technology Evolutionary Algorithms have become more popular in recent years. In the majority of cases they converge and they have the ability to overcome local optima (Baeck, 1996). Nevertheless, the required computing time is still very high due to the simulation procedures of complex processes. Therefore, it is desirable to reduce the number of the simulation steps. To attain this goal a new method has been developed by combining Evolutionary Algorithms with an exergo-economic analysis. Instead of using pure stochastic mutation to choose the set of decision variables for the next generation, the information attained by an exergo-economic analysis is used to affect the mutation operator.

To illustrate the application of the method the CGAM problem has been chosen, which is well-known from literature (Bejan et al., 1996; Valero et al., 1994). The design of the cogeneration system representing the CGAM problem is shown in *Figure 1*. Driven by natural gas, which is taken as methane, the cogeneration system delivers a net power output of 30 MW and 14 kg/s of saturated water vapor at 20 bars. The compressor pressure ratio $\Pi_C$, the isentropic efficiencies of the compressor $\eta_{sC}$ and of the turbine $\eta_{sT}$ as well as the temperature of the preheated air $T_{air}$ and the turbine inlet temperature $T_{gas}$ are regarded as the system's decision variables, which will be optimized.

After a brief view over Evolution Strategies in section 2 the exergy-aided optimization technique is presented in section 3. Finally the results for the cogeneration system are discussed in section 4.

## 2. Evolution Strategies

Evolutionary Algorithms are optimization techniques based on the biological evolution. Generally, Evolutionary Algorithms are divided in Evolution Strategies (ES) (Rechenberg, 1994; Schwefel, 1995) and Genetic Algorithms (GA) (Holland, 1992). Traditionally, GA work on bit

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strings, while ES use real-valued decision variables. The latter has been used for the process under consideration since using real-valued parameters allows for greater flexibility in designing the mutation operator, which is crucial to the combination of ES and exergy analysis.

Corresponding to nature, the main mechanisms of ES being used here are *mutation* and *selection*. Fulfilling the physical and technical constraints, any set of n decision variables $\{x_1,...,x_n\}$ can be chosen as the parent of the first generation. This parent generates one or more offspring when imposed to the mutation operator, which is obtained by multiplying normal distributed random numbers and step sizes for each $x_i$. Fulfilling all constraints, the offspring's fitness is determined by evaluating the objective function. The offspring with the better fitness is selected to be the parent of the next generation. In some variants of the ES the parent could be selected again to be the new parent of the next generation. The kind of selection is identified by using a general notation for the ES (Schwefel, 1995): $(\mu+\lambda)$-ES or $(\mu,\lambda)$-ES. The first one is called "plus strategy", because the $\mu$ parents are included in the selection and the second one is called "comma strategy", because only the $\lambda$ offspring are compared. In this paper only one parent per generation is used, i.e. a $(1,\lambda)$-ES is applied.

The mutation operator is determined by normal distributed random numbers *z* and step sizes $\delta$ underlying a self-adapting step size control, which itself uses the mechanism of mutation (Rechenberg, 1994). The use of a step size control is of considerable importance as this is the most significant item that distinguishes the ES from the Monte-Carlo method. Offspring are generated with both enlarged and reduced step sizes. The step size of the best offspring will be the parent's step size of the next generation. Therefore, that offspring whose step size is best adapted to the objective function will be selected and finally, hopefully, lead to the global optimum. Using a $(1,\lambda)$-ES with self-adapting step size control, the offspring are determined as follows

$$x_i^O = x_i^P + \delta_i^O \cdot z_i$$
$$\delta_i^O = \delta_i^P \cdot \xi \qquad i = 1...n, \qquad (1)$$

where $\xi$ adjusts the offspring's step size by enlarging or decreasing the parent's step size randomly.

## 3. Exergy – Aided Cost Optimization

The objective function F of the optimization problem under consideration, which will be minimized is the total annual costs

$$F = \dot{C}_F + \kappa \cdot \sum_k I_k \qquad (2)$$

The fuel costs $\dot{C}_F$ are determined by the market price of natural gas, while the investment costs $I_k$ for each system component k are quantified by cost functions given in (Bejan et al., 1996). Supposed that those cost functions are in full agreement with market prices, they enable the calculation of investment costs as a function of thermodynamic data obtained by the thermodynamic model solving the mass and energy balances. The economic analysis, which is based on (Bejan et al., 1996), is taken into account by the capital factor $\kappa$ considering both the annuity factor and the operating and maintenance expenses, which are assumed to be calculated by a factor applied to investment costs.
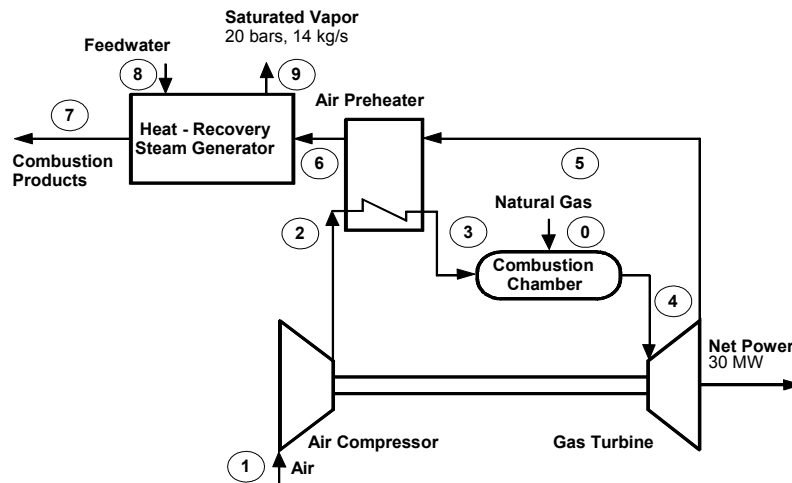


*Figure 1. Design of the cogeneration system.*

### 3.1 Exergo-economic analysis

The optimization algorithm should be supported by information gained from an exergo-economic analysis. Therefore, the thermodynamic model for the single components is enhanced by exergy and cost balances. The basic concept of this methodology is the evaluation of average cost per exergy unit $c_i$ associated with the i'th stream. In fact, the cost flow can be traced throughout the whole process with the cost per exergy unit. It is possible to detect for each component, whether the cost of the component's product increases mostly due to capital investment or due to exergy loss. This analysis yields to certain parameters for each component, which are used to guide the Evolutionary Algorithms.

Using suitable exergy balances, the exergetic efficiencies are determined for each component. Generally, the exergetic efficiency of the system component k is defined as

$$\zeta_k = \frac{\dot{E}_{P,k}}{\dot{E}_{F,k}} \tag{3}$$

where $\dot{E}_{P,k}$ represents the exergy rate of product and $\dot{E}_{F,k}$ the exergy rate of fuel. Depending on the service rendered by each component, product and fuel have to be defined carefully. Furthermore, the average cost per exergy unit of fuel $c_{F,k}$ and product $c_{P,k}$ for each component are necessary. They result from the cost balances and the definition of fuel and product (Bejan et al., 1996). A parameter of the exergo-economic analysis introducing these cost contributions is the relative cost difference $r_k$, which describes the ratio of cost increase to the cost flow of fuel

$$r_k = \frac{c_{P,k} - c_{F,k}}{c_{F,k}} \tag{4}$$

These parameters are evaluated in order to provide the optimizer with information. Basically, the components are ranked in descending order of the amount of the total costs. Special attention should be turned to those components which have also high values of the relative cost difference. Following this list of components, the exergetic efficiency is compared to its value at a reference point for each component and at each step of the ES. It is then decided in which direction the decision variables shall be changed in order to approach the exergetic efficiency at the reference point. An increase or decrease of the exergetic efficiency can be realized by mutating the decision variables in a certain direction. It is desirable to find a rational way to the evaluation of the reference points for the single components. They should not depend on heuristic rules, as this limits the universality of this method. Here, an isolated optimization of each system component has been used, whereby some process data calculated previously remain constant.

### 3.2 Reference points

Considering each component isolated from the remaining process, not only the complexity of the objective function but also the number of decision variables is reduced. In principle, there is still more than one decision variable left. Therefore, the optimization problem should be reduced to one variable, the exergetic efficiency, which is a function of the decision variables. The exergetic efficiency has been chosen, because it is related to both fuel and capital costs. *Figure 2* illustrates the most common case, that the fuel costs of each component increase with decreasing exergetic efficiency, while the component's capital costs increase rapidly with increasing exergetic efficiency. Therefore, an optimum of the total costs of the component under consideration exists and can be found easily. The corresponding optimum exergetic efficiency $\zeta_k^{\text{opt}}$ of the component k is taken as reference point for guiding the optimization of the whole process. In fact, those reference points are not identical with the final optimum exergetic efficiencies. The isolated optimization is used to calculate reference points analytically, which are close to the optimum. Especially those components with the greatest impact on total costs are assumed to yield reasonable reference points.

Considering a small interval around the actual set of variables, the component's cost function, which in fact depends on the decision variables, can be formulated depending on the exergetic efficiency only by applying a general equation introduced by Tsatsaronis (Bejan et al., 1996), which has been slightly modified here

$$I_k = B_k \cdot \left(\frac{\zeta_k}{1 - \zeta_k}\right)^{n_k} \tag{5}$$

Certainly, this approximation is only valid for small changes in the decision variables. Therefore, only one of the decision variables is slightly changed in each case in order to determine the constants $B_k$ and $n_k$ by evaluating both the exergetic efficiency as well as the capital costs with the thermodynamic and the economic model for the component under consideration. This is done for both a positive and a negative change of the current value of each variable leading to a lower and an upper value for the optimum exergetic efficiency. If the actual exergetic efficiency of the component k lies in between this interval, no decision can be made about the optimum direction of the variable. The total costs are calculated corresponding to Equation {2}, whereby the fuel costs depend on the average cost per exergy unit of fuel and the exergy flow rate of the product, which both are assumed to remain constant, and on the exergetic efficiency:
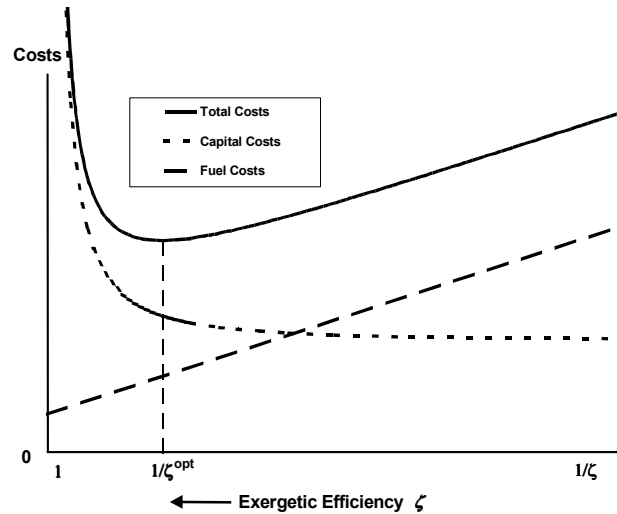
*Figure 2. Cost as a function of the exergetic efficiency.*

$$\dot{C}_{F,k} = c_{F,k} \cdot \frac{\dot{E}_{P,k}}{\zeta_k} \qquad (6)$$

This optimization problem can be solved analytically and leads to the optimum exergetic efficiency of the component k (Bejan et al., 1996)

$$\frac{\zeta_k^{opt}}{1 - \zeta_k^{opt}} = \left( \frac{c_{F,k} \cdot \dot{E}_{P,k}}{\kappa \cdot B_k \cdot n_k} \right)^{\frac{1}{1+n_k}} \qquad (7)$$

In each generation the reference points are recalculated in order to improve their reliability. But according to the assumptions and simplifications, the guiding information can only lead to a parameter set which is close to the optimum. From this point on the information obtained by an isolated optimization will mislead the optimizer. Therefore, a conventional optimization technique can be used in order to determine the optimum exactly as can be seen in the example in section 4.

### 3.3 Key design variables

Comparing the current exergetic efficiencies with the reference points yields the information on increasing or decreasing the exergetic efficiencies. But, in fact, information on changing the decision variables is needed to guide the optimizer. Therefore, the key design variables are defined. The key design variables are the decision variables which are most promising for the optimization of one component. In principle, each of the decision variables can act as the key design variable for each of the components. To decide which variables affect the component's cost most significantly, the cost functions as well as the exergy loss must be examined considering

changes in the decision variables. In addition it has to be proven that the capital costs increase with increasing exergetic efficiency and the fuel costs of a component increase with decreasing exergetic efficiency. Otherwise the optimization of a single component is impossible as *Figure 2* confirms. Thus, the key design variables are determined for each component of the system. In principle, one decision variable can act as the key design variable for more than one component. To avoid contradictory information, the components are ranked in a priority list (see section 3.4).

The compressor's key design variable is the isentropic efficiency as it affects the compressor's costs significantly. The influence of the pressure ratio on the compressor's costs is not negligible, too. But this parameter cannot be chosen as a key design variable for the compressor, as the fuel costs for the compressor increase with increasing exergetic efficiency in the process as a whole. Therefore, the reference point gained from an isolated optimization would mislead the optimizer. Regarding the gas turbine the isentropic efficiency is the most promising variable and not the turbine inlet temperature. The pressure ratio is ruled out, because the capital costs increase with decreasing exergetic efficiency. The combustion chamber (CC) depends on the inlet temperature of the air and the outlet temperature of the hot gas. The two heat-exchangers, the air preheater as well as the heat recovery steam generator, depend on all five decision variables, but in both cases the temperatures $T_{air} = T_3$ and $T_{gas} = T_4$ are most important. Therefore, they are selected both.

### 3.4 Algorithm for guiding the optimization

The following methodology is proposed for the automated algorithm using the exergo-economic analysis to guide the Evolution Strategy.

- Rank the components in descending order of total costs with special attention to those components with high values for the relative cost difference $r_k$.
- The system components heading this list have priority, i.e. the component ranked higher decides on the direction. If the heat recovery steam generator, for example, leads the ranking, a decision is made on $T_{air}$ and $T_{gas}$. Therefore, the combustion chamber and the air preheater need not to be taken into account.
- Determine the upper and the lower optimum exergetic efficiencies following the methodology described in section 3.2.
- Decide on the direction the key design variable shall be changed to. This information is passed on to the mutation operator of the ES. If the current exergetic efficiency is lower than the lower optimum exergetic efficiency, increase $\zeta_k$ by changing the key design variable. If the current exergetic efficiency is higher than the upper optimum exergetic efficiency, decrease $\zeta_k$ by changing the key design variable. Otherwise the mutation operator is free to choose the value of the key design variable for the offspring.

The information on the direction the key design variable shall be changed affects the normal distributed random numbers z of the mutation operator (Eq. {2}). Depending on the information only positive or negative z-values are allowed. This limits the search space and therefore reduces computing time.

### 4. Results

A workable design for the cogeneration system under consideration has to be developed first. Here, the same values of the decision variables as in (Bejan et al., 1996) have been chosen:

$\Pi_C = 10.0,$ $\qquad \eta_{sC} = \eta_{sT} = 0.86,$
$T_{air} = 850$ K, $\qquad T_{gas} = 1520$ K.

Evaluating the objective function, the total annual costs amount 28.06 million \$/a. To minimize the total costs a (1,50)-ES has been applied. To find the optimum of this simple problem the conventional ES requires about 20 generations as shown in *Figure 3*. The cost-optimal case calculated by the ES reduces the total annual costs to 25.5 million \$/a. The corresponding values of the decision variables are:

$\Pi_C = 6.417,$ $\qquad \eta_{sC} = 0.821,$
$\eta_{sT} = 0.856,$ $\qquad T_{air} = 919.10$ K,
$\qquad\qquad\qquad T_{gas} = 1469.89$ K.

The dotted line in *Figure 3* represents the progression of the total annual costs versus the number of generations required for optimization aided by an exergy analysis as described above. Compared with the continuous line, which represents the conventional ES, the new methodology obviously leads faster to the optimum until the sixth generation is reached. As optimization proceeds, the value of the objective function deteriorates. Apparently, this is caused by the underlying assumptions made for the isolated optimization of the system components. Close to the optimum these assumptions mislead the optimizer, as the simplified evaluation of optimum exergetic efficiencies can only guide to a point, which is adjacent to the optimum. Otherwise the reference points would be identical with the real optimum exergetic efficiencies. As a consequence, the exergy analysis has to be switched off, if the offspring get worse. Hereupon, a conventional optimization technique can be used starting from the last best result in order to determine the optimum exactly.

TABLE I. VALUES OF THE DECISION VARIABLES FOR THE EXERGY-AIDED OPTIMIZATION.

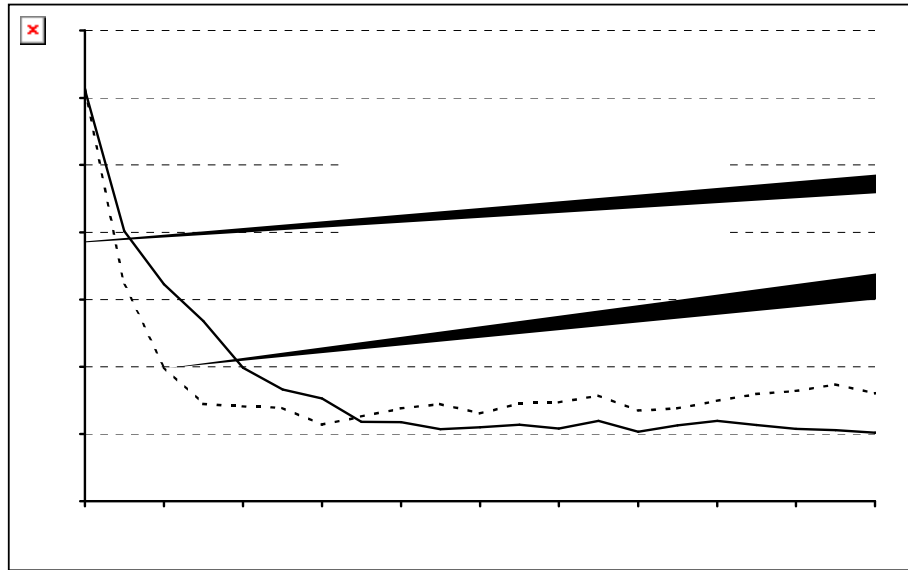| Gen. | $\Pi_C$ | $\eta_{sC}$ | $\eta_{sT}$ | $T_{air}$ / K | $T_{gas}$ / K | Total Annual Costs in Million \$/a |
|------|---------|-------------|-------------|---------------|---------------|-----------------------------------|
| 0 | 10.000 | 0.860 | 0.860 | 850.000 | 1520.000 | 28.060 |
| 1 | 9.292 | 0.843 | 0.848 | 850.658 | 1497.202 | 26.606 |
| 2 | 8.599 | 0.834 | 0.853 | 860.726 | 1468.520 | 26.000 |
| 3 | 7.639 | 0.831 | 0.854 | 880.144 | 1464.224 | 25.723 |
| 4 | 7.620 | 0.829 | 0.857 | 877.145 | 1455.076 | 25.707 |
| 5 | 7.216 | 0.825 | 0.862 | 875.067 | 1465.254 | 25.695 |
| 6 | 7.195 | 0.816 | 0.865 | 895.937 | 1462.432 | 25.569 |

*Figure 3. Evolution of the total annual cost during the optimization*

TABLE I summarizes the values of the decision variables of the best offspring for the first six generations. A closer inspection of the temperatures, which are guided by the exergy analysis, shows, that their direction changes during the optimization process. The temperature $T_{gas}$, for example, is led to decrease until a value is attained nearby the optimum value. Henceforth the direction obtained by the exergy analysis alternates and, consequently, the value of $T_{gas}$ remains in a small interval around the optimum. The same can be observed, too, for the following generations which are not reported in TABLE I. The isentropic efficiency of the compressor is changed as well in a proposed direction leading to the correct value in the first six generations. If a value occurs which is lower than the optimum value, however, the variable is still forced to decrease. Obviously, the simplified assumptions mislead the optimizer nearby the optimum in this case. The misled isentropic efficiency of the compressor is mainly responsible for the increase of the total annual costs as indicated by the dotted line in *Figure 3*.

## 5. Conclusions

The methodology presented in this paper uses the information of an exergo-economic analysis to find the global economic optimum of an energy process without any interaction between engineer and optimization algorithm during the optimization. Some preparations have to be made for the algorithm described in section 3.4, whereby special care should be applied to the choice of the key design variables. A remark should be made to the fact, that no prediction can be made to optimize the pressure ratio. The total costs are affected significantly by the tempera-

ture $T_{gas}$ and the compressor's isentropic efficiency, but the impact of the pressure ratio is not negligible as well. The pressure ratio is mutated by the ES itself, whereas the mutation of the other decision variables is aided by the exergo-economic analysis. It has further to be emphasized that a prescription should not be made for each decision variable, as this could restrict the search space too much and therefore may limit the capability of the ES. This applies especially to complex processes. A notable reduction in computing time has been achieved for this particular case of a cogeneration system. The optimizer is led to a point which is adjacent to the real optimum in the early steps. This does not necessarily imply general validity. Further investigations of other thermal systems, which are more complex than the CGAM problem, are required to analyze the capabilities of this methodology.

## Nomenclature

| | |
|---|---|
| $\dot{C}$ | cost rate ($/a) |
| c | cost per unit of exergy ($/MJ) |
| $\dot{E}$ | exergy flow rate (MJ/a) or (MW) |
| F | objective function ($/a) |
| I | investment cost ($) |
| p | pressure (MPa) |
| r | relative cost difference |
| T | temperature (K) |
| x | decision variable |
| B, n | constants of cost function |
| z | random number |

Greek symbols

| | |
|---|---|
| δ | step size |
| $\eta_s$ | isentropic efficiency |

κ        capital factor (1/a)
λ        number of offspring
μ        number of parents
ζ        exergetic efficiency
$\Pi = p_2/p_1$  pressure ratio

Subscripts

C        compressor
F        fuel
P        product
T        turbine
k        component k

Superscripts

O        offspring
opt      optimum
P        parents

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