Int.J. Applied Thermodynamics,
Vol.3, (No.4), pp.171-180, December-2000

ISSN 1301-9724

On Applied Thermodynamics in Atmospheric Modeling*

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Abstract

Recently some atmospheric researchers have turned to thermodynamics to avoid the complexity of conventional atmospheric models. This approach has resulted in a search for an entropy production extremum principle that governs circulation. Research has focused on the maximum dissipation theorems of Paltridge and Zeigler and the minimum entropy production principle of Prigogine. Stephens and O'Brien have calculated the entropy production rate of Earth based on satellite energy measurements and have concluded that its value supports the maximum dissipation conjecture. In this paper, we present evidence that Prigogine's minimum entropy production principle is not applicable to atmospheric circulation. The calculation of the extremum simply shows that the entropy production rate is minimum with respect to any force when the corresponding flux has ceased. The force-flux equations completely govern the response of the system, under certain constraints, to applied external forces or radiative sources and sinks. Secondly, it is shown that for a number of reasons the conclusion of Stephens and O'Brien that the calculated entropy production rate supports the maximum dissipation conjecture is not justified. Thirdly, an improved radiative model of the planet is presented that provides insight into the thermodynamic behaviour of the Earth system. For example, the results from this model show, surprisingly, that the Earth's mean temperature has a tendency to be independent of planetary albedo (or independent of the fraction of sunlight absorbed) while being dependent on phenomena like the greenhouse effect.

Key words: irreversible thermodynamics, atmospheric modeling, entropy production principles, radiative planetary model

1. Introduction

Predicting the likely global effects caused by rising greenhouse gas levels in the atmosphere is a primary motivating factor for atmospheric modeling. It is expected that rising carbon dioxide levels will not only cause global warming, and consequently rising sea levels, but may also cause sudden changes in climate due to the climate's inherent meta-stability. The interaction of many contributing phenomena, however, makes modeling of the atmosphere a difficult task. For an overview of this subject see, for example, Peixoto and Oort (1992).

Conventional general circulation models incorporate the principles of energy, momentum (linear and angular), and mass conservation. To avoid the complexity encountered with these models some researchers have tried to identify a

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* This paper was presented at the ECOS2000 Conference in Enschede, July 5-7, 2000

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thermodynamic extremum principle that can be used to predict steady states without modeling the large number of processes that occur in the atmosphere (O'Brien and Stephens, 1995). Research has focused on the minimum entropy production principle of Prigogine (1967) and the maximum dissipation principles of Paltridge (1981) and Zeigler (1983).

Stephens and O'Brien (1993) state that the fundamental question regarding the entropy production rate of Earth is whether or not it is at a maximum or minimum. Stephens and O'Brien's perspective is that the observed entropy production rate of Earth supports the maximum dissipation viewpoint. They conclude that "the observed rate of entropy production is close to the upper bound for a planet with the albedo of Earth ... it is not possible to prove that the planet is in a state of maximum dissipation, but we have accumulated evidence that suggests that the conjecture may be true" (1995).

Also, researchers have used simple blackbody (BB) type planetary models to theoretically estimate planetary entropy production rates. The analysis of simple radiative models of the Earth system provides insight into its thermodynamic behavior even though it is complex. This is because radiative processes play a major role in the thermodynamics of the Earth system. From a thermodynamic perspective thermal radiation (TR) exchange, i.e., incoming sunlight and outgoing TR, is the only significant form of energy transfer between the Earth and the universe. Further, processes such as absorption and emission dominate planetary entropy production, and the non-uniform absorption of solar radiation (SR) on the Earth causes circulation of the atmosphere and oceans.

In the research reported in this paper the objectives are:

1) to determine the significance of Prigogine's minimum entropy production principle as a governing principle in atmospheric modeling,
2) to examine Stephens and O'Brien's conclusion that the observed entropy production rate of Earth supports the maximum dissipation viewpoint, and
3) to present an improved, yet simple, graybody planetary model to provide insight into the thermodynamic behavior of the Earth system.

The analysis of the topics in this paper utilize recent results by the authors on improving the understanding and simplifying the calculation of the entropy flux of radiative heat transfer (Wright et. al 2001). The most pertinent results from this related work include (1) the finding that, on a per unit of energy basis, the entropy flux carried by non-blackbody radiation (NBR), such as graybody radiation (GR), can be much greater than that of blackbody radiation (BR) with the same material emission temperature, (2) the entropy flux of NBR can be accurately approximated as that of BR with the same energy flux, but different material emission temperature, over a large range of spectral emissivities (the error is least significant at high emissivities), and (3) accurate approximations presented for the entropy flux of GR with low emissivities are utilized that are convenient because they avoid laborious numerical calculations.

2. Prigogine's Minimum Entropy Production Principle

The maximum entropy production principles of Paltridge (1981) and Ziegler (1983) appear to be in direct opposition to Prigogine's minimum entropy production principle. It was our purpose in the present work to determine whether Prigogine's principle is relevant in atmospheric modeling. In Prigogine's analysis, restrictions are applied to the character of the force-flux relationships that are quite restrictive and well known in the field of thermodynamics:

1) The force-flux relationships are linearly coupled.
2) The phenomenological coefficients are constants (L_{ij} = constant)[1].
3) Onsager's Reciprocity[2] applies ($L_{ij} = L_{ji}$). In Prigogine's example Onsager's reciprocity means that $L_{21} = L_{12}$.
4) The boundary conditions are time-independent.

Some researchers have considered whether Prigogine's four restrictions are sufficient for differentiating between systems where the minimum entropy production principle applies and where it does not. For example, Pelkowski (1997) considers Prigogine's principle for continuous systems applied to a system consisting of a radiating graybody layer between two blackbody plates near equilibrium. The present analysis questions the general significance of Prigogine's result as a governing principle for any system. This is done by analyzing Prigogine's approach and conclusions for the example system used in his analysis.

Prigogine's approach was to extend the second law of thermodynamics to non-equilibrium processes in order to develop an entropy

[1] Note that in Paltridge's approach the specification of constant phenomenological coefficients is not necessary as it is in Prigogine's and Ziegler's approaches.

[2] Note that this does not cause any additional loss of generality with regard to entropy production because the antisymmetric part of L_{ij}, if it were to exist, would not contribute to the entropy production rate $L_{ij}X_iX_j$ because X_iX_j is symmetric.

production rate expression (see Prigogine 1967, pp. 18-21 for closed systems and pp. 28-29 for open systems). From the entropy production expression generalized forces and fluxes are identified. Then Prigogine states that a stationary (steady) state "may be characterized by an extremum principle which states that in the stationary state, the entropy production has its minimum value compatible with some auxiliary conditions to be specified in each case."

To determine the significance of this observation, consider the illustrative example that Prigogine presents in his analysis. The system is composed of two vessels, I and II, containing the same gas, which communicate by means of a small hole, capillary, membrane, or porous wall. An external thermodynamic force, a temperature difference, is applied and maintained across the two-vessel system such that the net heat flow is perpendicular to the membrane. When this external driving force is applied and maintained (stationary) the system will eventually reach steady state (stationary state) where the fluxes, internal forces, entropy production rate, and all other quantities are no longer functions of time.

Now, the specific entropy production rate $\dot{\pi}$ for the two-flux system that Prigogine considers is

$$\dot{\pi}=\sum_{i=1}^{N} J_i X_i = J_m X_m + J_{th} X_{th} \quad (1)$$

where X is a thermodynamic force, J is a flux, and the subscripts 'm' and 'th' denote respectively 'mass' and 'thermal'. When the force-flux relationships are linear,

$$\begin{aligned} J_{th} &= L_{11} X_{th} + L_{12} X_m, \\ J_m &= L_{12} X_{th} + L_{22} X_m. \end{aligned} \quad (2)$$

The entropy production rate is given by

$$\dot{\pi}=J_{th} X_{th} + J_m X_m = L_{11} X_{th}^2 + 2 L_{12} X_{th} X_m + L_{22} X_m^2 \quad (3)$$

The entropy production rate (3) has an extremum (minimum) with respect to the force X_m when the flux of mass flow J_m is zero. In this case (2) gives $X_m=-X_{th}(L_{12}/L_{22})$ and the entropy production rate (3) becomes

$$\dot{\pi}=J_{th} X_{th} = \left(L_{11} - \frac{L_{12}^2}{L_{22}} \right) X_{th}^2 \le L_{11} X_{th}^2 \quad (4)$$

Note that coupling in the force-flux relationships leads to a lower value of $\dot{\pi}$ at the stationary state than if coupling were not present ($L_{12} = L_{21} = 0$). Coupling causes the system to increase its 'thermal resistance' to the applied external thermal force X_{th} by re-distribution of mass; the hot side becomes less dense while the density of the cold side increases.

In the particular example under consideration the combined system is closed to mass flow so at the stationary state the mass flux J_m must be zero. Prigogine concludes that the stationary condition is characterized by the condition that the entropy production is a minimum for a given value of the applied thermal force X_{th}. Prigogine's conclusion implies that, given the constraints on the system, the stationary state can be determined from all possible states as the state with the lowest entropy production rate.

However, for the same system under different constraints a 'parallel' extremum is obtained, $J_{th} = 0$, for the entropy production rate with respect to the other force X_{th}. By extending this exercise we see that it would lead to an absolute minimum for $\dot{\pi}$, namely zero, when all the fluxes are zero - the equilibrium condition. Prigogine's analysis regarding the variation of the entropy production rate seems to only show that the entropy production rate is a minimum with respect to a certain force when the corresponding flux has ceased.

The only reason the entropy production rate decreases with respect to one of the forces as the stationary state is approached is because this particular system is closed to the corresponding flux. To speak of Prigogine's observation of the variation of the entropy production rate as representing a governing principle appears to be misleading. Prigogine's analysis requires a knowledge of the force-flux relationships, and his choice of which force to find an extremum of $\dot{\pi}$ with respect to was based on knowledge of the constraints on the system. However, knowledge of the constraints on the system and the force flux relationships completely determines the response or behaviour of the system. For the particular system considered by Prigogine, knowledge of the constraint to mass flow at the steady state can be used directly in (2) and (3) to arrive at (4).

3 Maximum Dissipation Conjectures

Stephens and O'Brien (1993) estimated the total entropy production rate of Earth based on satellite TR energy measurements. These measurements were carried out in the Earth Radiation Budget Experiment (ERBE) by satellites orbiting at the top of the atmosphere (TOA). Barkstrom et al. (1986) presents an overview of the ERBE experiment. Stephens and O'Brien found that the entropy production rate of Earth is approximately constant at 6.8×10^{14} W/K with an annual cycle variation of 1 to 2%.

Stephens and O'Brien (1993) suggest that the value of the calculated total entropy production rate of Earth $\dot{\Pi}_{Earth}$ supports the maximum dissipation viewpoint for atmospheric circulation[3]. They state "we present an analysis that suggests the Earth is near its maximum entropy production *for a given observed albedo* of the planet. This arises from the fact that the long-wave entropy flux to space is very closely approximated by the *equivalent blackbody entropy flux*." (1993). Note that the emitted entropy flow rate dominates the entropy production rate calculation because the entropy of SR is comparatively low.

Stephens and O'Brien's conclusion is based on the fact that the entropy of the TR emitted from Earth can be accurately approximated by the entropy of *BR with the same energy flux*, which they term the *equivalent* BR entropy flux. BR is associated with maximum entropy. It appears that Stephens and O'Brien reason that the emitted entropy is near that of the 'equivalent' BR entropy, and BR is associated with maximum entropy, therefore $\dot{\Pi}_{Earth}$ is near its maximum in support of the maximum dissipation viewpoint. However, from thermodynamic arguments we feel that this conclusion is not justified for two reasons.

First, we question the validity of the assumption that it can be determined whether the planet is in a state of maximum dissipation based on considering only planetary states with the same albedo (*a*) as the present value. The entropy production rate is strongly dependent on the albedo. For the graybody model discussed in section 4, the maximum entropy production rate occurs when the albedo is zero because the energy flow rate is maximized[4].

At the outset one might expect that the maximum entropy production rate for the Earth would occur for an albedo of zero (BB character) because the energy flow rate is at a maximum. However, due to the actual radiative character of material that is present on Earth and feedback effects such as the ice-feedback effect, plant-feedback effect, and the cloud-feedback effect the state of maximum entropy production rate cannot be easily determined. For example, if the planet gets too hot plants will die and ice will melt in the northern regions thereby affecting the albedo of the planet. Thus, to find an extremal state, factors like planetary albedo must be allowed to vary.

Secondly, the emitted entropy is closely approximated as that of BR with the same energy over a large range of emissivities and consequently over a large range of possible states of the system. For $\varepsilon_{LW} > 0.5$ the emitted entropy flux is within 1% of the BR entropy with the same energy flux, and within 5% for $\varepsilon > 0.2$ (Wright et al., 2001). This large range of emissivity corresponds to a large range in mean planetary temperature *T* given by $T = \varepsilon^{-1/4} T_{BR}$ for GR where T_{BR} is the emission temperature of BR with the same energy. For $\varepsilon_{LW} > 0.5$ the corresponding mean planetary temperature range is $T_{BR} < T < 1.189(T_{BR})$ and for $\varepsilon_{LW} > 0.2$ it is $T_{BR} < T < 1.495\ (T_{BR})$. For Earth with the albedo equal to 0.30 the BB emission temperature is T_{BR} = 254 K (-19°C) so the temperature ranges are 254 K < T < 302 K and 254 K < T < 380 K for $\varepsilon_{LW} > 0.5$ and $\varepsilon_{LW} > 0.2$ respectively.

In conclusion, the entropy flux of the emitted longwave radiation from Earth is closely approximated as that of BR with the same energy flux. However, it does not seem justified to conclude that the Earth is in a state of maximum dissipation based on the accuracy of this approximation. Further, the maximum entropy production rate of the Earth system cannot be easily determined due to the complexity of the system including feedback effects such as the plant, ice and cloud feedback effects. Thus, the evidence presented here suggests that the calculated value of the entropy production rate based on satellite energy measurements does not necessarily support the maximum dissipation conjecture.

4 Graybody Radiative Planetary Model

Researchers have analyzed simple blackbody type radiative models to investigate the thermodynamic behaviour of the Earth system and to estimate planetary temperature and entropy production rates (Aoki, 1982; Stephens and O'Brien,1993; Weiss, 1996). It is more accurate to model the Earth system as a graybody because absorption of sunlight and emission of TR are substantially less than that of a blackbody. In this paper we present a simple graybody planetary model that more closely models the radiative behaviour of the Earth system and results in more accurate expressions for planetary entropy production rates and mean temperatures.

In this analysis the Earth system is represented by an isothermal, solid sphere with uniform properties. The analysis is at steady state and the 'long wave' emissivity of the sphere is taken as one minus the 'short wave' albedo of

[3] This conclusion relies on the assumption that, if the *planet* is in a state of maximum dissipation, then the process of *atmospheric circulation* is in a state of maximum dissipation. This assumption is made because circulation and radiative behaviour of the planet are strongly coupled. Note that the fraction of Earth's entropy production rate actually due to fluid motion (circulation) is small compared to radiative processes (such as absorption and emission), as described further in section 4.2.

[4] For the GR model the maximum entropy production rate of 788 TW/K occurs with a = 0 and $\varepsilon_{LW} = 1$. Note that for a fixed albedo, such as a = 0, the entropy production rate is dependent on longwave emissivity and the maximum occurs with $\varepsilon_{LW} = 1$ because BR has the highest entropy flux for a given energy flux. For $\varepsilon_{LW} < 1$ the mean planetary temperature is higher and the entropy production rate is lower.

Earth ($\varepsilon_{LW} = 1 - a$), where the albedo a is an overall measure of the reflectivity of the planet. In actuality this relation only holds approximately; for Earth the long wave emissivity ε_{LW} is 0.61 while the short wave albedo a is 0.30. The purpose for using this approximation is discussed in section 4.2.

The effective planetary temperature is determined by specifying that the energy inflow (absorbed) and outflow (emitted) of the planet are equal. Otherwise, the planet would be heating up if there were a continual net influx of energy or cooling down if there were a net outflux. The energy flow absorbed by the planet is by definition

$$\dot{E}_{Abs} = (1-a)\dot{E}_{Inc}, \tag{5}$$

and the GR energy flow emitted by the planet is

$$\dot{E}_{Emi} = 4\pi R^2 \varepsilon_{LW} \sigma T_P^4, \tag{6}$$

where T_P is the effective temperature of the planet. To determine this temperature we first need to calculate the incident energy flow rate in (5). The incident energy flow rate on a planet with cross-sectional area πR^2 is

$$\dot{E}_{Inc} = \pi R^2 H_{Inc} = \pi R^2 \Omega K_{Inc}. \tag{7}$$

The influx of sunlight to the Earth is contained in a small solid angle as shown in *Figure 1*. Sunlight is uniformly incident on the Earth so random positions are chosen to illustrate the influx of sunlight within a small solid angle.

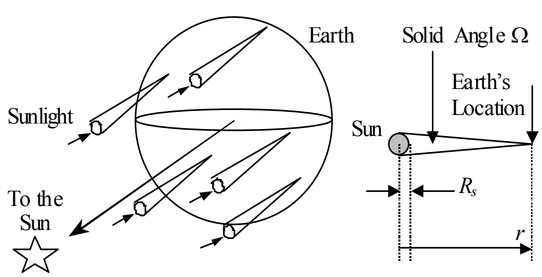

Figure 1. Illustration of SR incident on Earth and the SR solid angle.

The solid angle of sunlight incident on a planet is equal to 4π times the ratio of the cross-sectional area of the Sun to the surface area of the sphere with radius equal to the mean orbital radius of the planet, i.e.,

$$\Omega \approx 4\pi\left(\frac{\pi R_S^2}{4\pi r^2}\right) = \pi\left(\frac{R_S}{r}\right)^2 \tag{8}$$

where R_S is the radius of the Sun, and r is the distance from the center of the Sun to the planet and is approximated as constant in this analysis. The next step is to determine the energy radiance K of solar radiation contained in the solid angle Ω. From conservation of energy, the flow rate of energy at any distance from the Sun is equal to the energy flow rate at the surface of the Sun:

$$4\pi R_S^2 H_{Sf} = 4\pi r^2 H \tag{9}$$

where the energy irradiances H and H_{Sf} are the energy flux per unit area (irradiance) at distance r from the Sun and at the surface of the Sun, respectively. By approximating emission from the Sun as blackbody radiation (BR) we have

$$H_{Sf} = \pi K_{Sf} = \sigma T_S^4, \tag{10}$$

where T_S is the effective temperature of the Sun based on the BR approximation, and K_{Sf} is the energy radiance at the surface of the Sun. Thus, using (9) and (10), the energy irradiance H at distance r from the Sun is given by

$$H = \left(\frac{R_S}{r}\right)^2 \sigma T_S^4 \tag{11}$$

The energy irradiance H is equal to the energy radiance K, times the solid angle given by (8) so we have

$$K = \frac{H}{\Omega} = \frac{\sigma}{\pi} T_S^4 \tag{12}$$

Note that the energy radiance K at any distance from the Sun is the same as at the surface of the Sun K_{Sf}. The entropy radiance L, and the energy and entropy spectrums, are also invariant with the distance from the Sun. However, the energy irradiance H, equal to $K\Omega$, decreases as r increases because the solid angle of the sunlight given by (8) decreases.

After substituting for the incident energy irradiance from (11) the incident energy flow rate on Earth given by (7) becomes

$$\dot{E}_{Inc} = \pi R^2 H_{Inc} = \pi R^2 \sigma T_S^4 \left(\frac{R_S}{r}\right)^2 \tag{13}$$

By equating (5) and (6), and using (13), the effective planetary temperature can be expressed as

$$T_P = \left(\frac{1-a}{\varepsilon_{LW}}\right)^{\frac{1}{4}} \left(\frac{R_S}{2r}\right)^{\frac{1}{2}} T_S = \left(\frac{R_S}{2r}\right)^{\frac{1}{2}} T_S \tag{14}$$

by taking $\varepsilon_{LW} = 1 - a$; the reason for this approximation is discussed at the end of section 4.2. Surprisingly, the graybody planetary temperature only depends on the mean planetary orbital radius and not the planetary albedo. A blackbody planet absorbs all incident SR but does not have a higher temperature than a graybody planet because the emitted energy is correspondingly higher as well.

The entropy production rate of the planet is simply the difference between the entropy of incoming and outgoing TR. The incoming TR is incident SR and the outgoing TR is a combination of reflected SR and TR emitted from the planet. For simplicity the entropy flux of the reflected and emitted radiation is calculated separately so the entropy production rate is given by

$$\dot{\Pi}=\dot{S}_{Emi}+\dot{S}_{Ref}-\dot{S}_{Inc}. \qquad (15)$$

The inaccuracies in Aoki's (1982) and Weiss' (1996) analyses are summarized as follows:

1) The entropy production rate is incorrectly calculated as the difference between the absorbed and emitted entropy flow rates. This approach is incorrect because the absorbed entropy flow rate is not equal to the difference between the reflected and incident entropy flow rates.
2) The absorbed entropy flow rate is incorrectly calculated as a linear fraction (1 - a) of the incident SR entropy. This is incorrect because the entropy of GR with emissivity of (1 - a) is not equal to (1 - a) times the entropy of BR with the same material emission temperature.
3) The entropy production rate due to diffuse reflection of incident SR over a large solid angle is neglected (as noted in Aoki's paper).
4) The emitted and reflected entropy flow rates are calculated independently without specifying that this is an approximation.

In the current analysis we first calculate the incident entropy flow rate followed by the reflected and emitted contributions to the entropy production rate. The incident entropy flow rates on a planet with cross-sectional area πR^2 is

$$\dot{S}_{Inc}=\pi R^2\Omega L_{Inc}=\pi R^2\frac{4}{3}\sigma T_S^3\left(\frac{R_S}{r}\right)^2 \qquad (16)$$

The reflected *energy* flow rate from the planet is simply the albedo times the incident energy flow rate in (13). However, the reflected *entropy* is not as easily calculated because it is not linearly related to the energy. In addition, the reflected SR can be approximated as GR but it is diluted by diffuse reflection so its emissivity is less than the albedo of the planet. This is because the incoming SR is contained in a small solid angle whereas it is reflected into a large solid angle. In this analysis we approximate the reflected solid angle as 2π, a hemisphere of directions, and approximate the reflected SR as isotropic. That is, we approximate the reflected SR as uniformly distributed over the solid angle of 2π. For isotropic radiation the irradiance is equal to π times the radiance so for the reflected TR we have $H_{Ref}=\pi K_{Ref}$.

The incident SR is distributed over a small angle perpendicular to the cross-section of the planet, as depicted in *Figure 2*, so $H_{Inc}=\Omega K_{Inc}$. A fraction of the incident SR is reflected so by definition of the albedo we have

$$H_{Ref}=aH_{Inc}=a\Omega K_{Inc} \qquad (17)$$

and thus (8) for the reflected energy radiance becomes

$$K_{Ref}=\frac{H_{Ref}}{\pi}=\frac{a\Omega}{\pi}K_{Inc}=a\left(\frac{R_S}{r}\right)^2 K_{Inc} \qquad (18)$$

The incident SR is approximated as BR with emission temperature T_S so the reflected SR is GR with emission temperature T_S and an emissivity

$$\varepsilon_{Ref}=a\left(\frac{R_S}{r}\right)^2 \qquad (19)$$

The entropy-to-energy ratio of GR is higher than that of BR with the same material emission temperature Wright et al. (2001), i.e.

$$\frac{L_{GR}}{K_{GR}}=I(\varepsilon)\frac{L_{BR}}{K_{BR}}>\frac{L_{BR}}{K_{BR}} \qquad (20)$$

For the entropy of the emitted and reflected entropy we use the approximations (Wright et al., 2001)

$$I_1(\varepsilon)=1-\frac{45}{4\pi^4}(2.292-0.150\varepsilon)\ln\varepsilon \qquad (21)$$

which is within 0.03% for $\varepsilon > 0.2$ and

$$I_2(\varepsilon)=1-\frac{45}{4\pi^4}(2.336-0.260\varepsilon)\ln\varepsilon \qquad (22)$$

which is within 0.33% for $0.005 < \varepsilon < 0.2$.

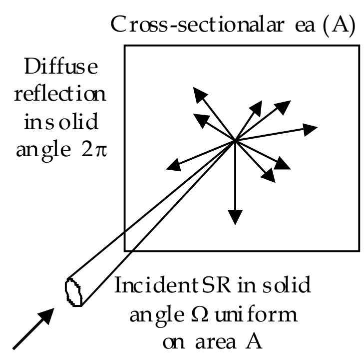

Figure 2. An illustration of diffusely reflected SR.

For the reflected TR we use $I_2(\varepsilon)$ to obtain:

$$\frac{L_{Ref}}{K_{Ref}}=I_2(\varepsilon_{Ref})\frac{L_{Inc}}{K_{Inc}}>\frac{L_{Inc}}{K_{Inc}} \qquad (23)$$

The emissivity given by (19) is outside the range indicated for an approximation within 0.33% using (22). However, when applied to the reflected SR for planets of our solar system, (22) is within 2% of the numeric value. Note that the reflected entropy flow is a relatively small term in the calculation of the total entropy production rate for the Earth.

The reflected entropy irradiance is then

$$J_{Ref} = \pi L_{Ref} = \pi I_2(\varepsilon_{Ref}) \frac{K_{Ref}}{K_{Inc}} L_{Inc} = a I_2(\varepsilon_{Ref}) J_{Inc} \quad (24)$$

and the reflected entropy flow rate is

$$\dot{S}_{Ref} = \pi R^2 J_{Ref} = a I_2\left(\frac{a R_S^2}{r^2}\right) \dot{S}_{Inc} \quad (25)$$

Now the entropy flow of GR emitted from the planet is

$$\dot{S}_{Emi} = 4\pi R^2 (1-a) I(1-a) \frac{4}{3} \sigma T_P^3 \quad (26)$$

and using (14) this becomes

$$\dot{S}_{Emi} = 4\pi R^2 (1-a) I(1-a) \left[\frac{R_S}{2r}\right]^{\frac{3}{2}} \frac{4}{3} \sigma T_S^3 \quad (27)$$

The entropy production rate of the planet is the difference between the entropy flow rates of the incoming and outgoing TR. Thus, using (16), (25) and (27) the planetary entropy production rate may be approximated as

$$\dot{\Pi} = \pi R^2 \left[\frac{R_S}{r}\right]^{\frac{3}{2}} \left\{ \sqrt{2}(1-a) I_1(1-a) - \left[1 - a I_2\left(\frac{a R_S^2}{r^2}\right)\right]\left(\frac{R_S}{r}\right)^{\frac{1}{2}} \right\} \frac{4}{3} \sigma T_S^3 \quad (28)$$

By substituting data for the Earth ($a = 0.30$, $R = 6.37 \times 10^6$ m and $r = 1.486 \times 10^{11}$ m) and the Sun ($R_S = 6.923 \times 10^8$ m and $T_S = 5760$ K) the planetary entropy production rate of the Earth is estimated as

$$\dot{\Pi}_{Earth} = 6.44 \times 10^{14} \text{ W/K} = 644 \text{ TW/K} \quad (29)$$

4.1 Discussion of planetary model results

The planetary entropy production rate for the Earth calculated using the graybody model of 644 TW/K is within 5.3% of Stephens and O'Brien's (1993) calculation based on satellite energy measurements of 680 TW/K. This is in reasonable agreement considering the uncertainty in Stephens and O'Brien's calculation. Aoki's 1982 result for the planetary entropy production rate may be expressed as

$$\dot{\Pi} = \pi R^2 \left[\frac{R_S}{r}\right]^{\frac{3}{2}} \left\{ \sqrt{2}(1-a)^{3/4} - [1-a]\left(\frac{R_S}{r}\right)^{\frac{1}{2}} \right\} \frac{4}{3} \sigma T_S^3 \quad (30)$$

The planetary entropy production rate for Earth using Aoki's approach resulting in (30) is 606 TW/K. Weiss (1996) incorrectly calculates the entropy of the GR emitted by the planet and consequently his result is of minor significance because the emitted entropy flow rate dominates the entropy production rate calculation. Weiss calculates the entropy of the emitted GR as equal to the emissivity times the entropy flux of BR. For an emissivity of 0.7 used for the Earth Weiss' calculation results in a 9% error in the entropy flux calculation (for other planets with lower emissivities the error is much greater; for example, the error is 61% for an emissivity of 0.1).

To explain the difference between the graybody approach and Aoki's approach it is convenient to examine the results for all the planets in our solar system. In TABLE I planetary temperatures and entropy production rates are presented for Aoki's approach and the graybody model.

Aoki's planetary temperatures are substantially different from those of the graybody model. In Aoki's model the entropy of emitted NBR from the planets is approximated as that of BR with the same energy, and the effective temperature is taken as that of a blackbody planet emitting the same energy flow rate. However, for the same energy flow rate a blackbody must be at a lower emission temperature. Thus, in Aoki's blackbody model all planetary temperatures are underestimated. Note that Weiss' (1996) calculation of planetary temperature is the same as Aoki's (1982).

TABLE I. MEAN PLANETARY TEMPERATURE

Planet	Albedo, a	Planetary Temperature, T_P (K)		
		Measured	Present GB Model	Aoki's Approach
Mercury	0.058	440	445	439
Venus	0.77	733*	326	226
Earth	0.30	288	278	254
Mars	0.20	220	224	212
Jupiter	0.42	124	121	106
Saturn	0.76	95	90	63
Uranus	0.93	59	63	33
Neptune	0.84	59	51	32
Pluto	0.14	50	44	42

* The estimates for the temperature of Venus are inaccurate largely due to the greenhouse effect.

Aoki's planetary entropy production rates are relatively close to those of the graybody model. There are at least three sources of inaccuracy in Aoki's calculation but they have a canceling effect on one another. First, Aoki calculates the emitted entropy as that of BR. The inaccuracy introduced by this simplification is usually within about 1% but it becomes more significant when the albedo of the planet is above 0.5, as is the case for the planets Venus, Saturn, Uranus and Neptune. Second, Aoki neglects the entropy production due to diffuse reflection. The inaccuracy introduced by this omission becomes more significant the higher the amount of SR that is reflected, that is the higher the albedo as with the first source of error. Third, Aoki inappropriately calculates the entropy of absorbed SR. The entropy of sunlight is relatively low compared to entropy of TR emitted from a planet except for planets that are very close to the Sun. The first source of error mentioned causes an overestimation of the entropy production rate while the second and third sources cause the opposite effect.

TABLE II. PLANETARY ENTROPY PRODUCTION RATES

Planet	Albedo, a	Entropy Production Rate (TW/K)	
		Present GB Model	Aoki's Approach
Mercury	0.058	448	Mercury
Venus	0.77	519	Venus
Earth	0.30	644	Earth
Mars	0.20	104	Mars
Jupiter	0.42	5960	Jupiter
Saturn	0.76	926	Saturn
Uranus	0.93	26.7	Uranus
Neptune	0.84	20.4	Neptune
Pluto	0.14	0.164	Pluto

4.2 Relevance of the graybody radiative model to the thermodynamics of the Earth

Simple radiative models of the Earth system are insightful because radiative processes play a major role in the thermodynamics of the Earth system. The present graybody model provides more accurate expressions for planetary entropy production rate and mean temperature than the BB type models. The estimated entropy production rate for Earth emphasizes that planetary entropy production is dominated by radiative processes. A small part of the total entropy production rate of Earth is due to circulation. The high entropy production rate due to absorption/emission results because SR is emitted from a high-temperature source near 6000 K while the absorbing material on Earth is near 300K.

The present model is also useful for investigating the sensitivity of the thermodynamic behaviour of the Earth system to variations in its radiative parameters, as is discussed in section 3. For example, this model illustrates that, surprisingly, the mean planetary temperature has a tendency to be independent of the planet's albedo (or the fraction of sunlight absorbed) – modified slightly because the thermal emissivity (ε_{LW}) of the Earth is not strictly equal to 1 minus the shortwave albedo (a). For the Earth this difference (ε_{LW} is 0.61 while 1 - a = 0.70) is mainly due to the 'greenhouse effect', the difference between longwave and shortwave absorption and emission behavior in the atmosphere.

The greenhouse effect is relatively mild on Earth as can be seen by the closeness of the mean planetary temperature predicted by the model (278 K) compared to the actual value near 288 K. However, we are not indicating that the fact that $\varepsilon_{LW} \neq 1 - a$ should be ignored but that insight can be gained into the behavior of the Earth system by considering the case with $\varepsilon_{LW} = 1 - a$. Therefore, analysis of the model with $\varepsilon_{LW} = 1 - a$ reveals that the Earth's mean temperature has a tendency of being independent of planetary albedo (or the fraction of sunlight absorbed) while it is dependent on phenomena such as the greenhouse effect. This observation would be somewhat obscured by considering only the case with $\varepsilon_{LW} \neq 1 - a$.

Taking into account the fact that $\varepsilon_{LW} \neq 1 - a$ results in improved estimates of mean planetary temperature as in (14) and improved entropy production rates. In this case the total planetary entropy production rate is estimated as

$$\dot{\Pi} = \pi R^2 \left[\frac{R_S}{r}\right]^{\frac{3}{2}} \left\{ \sqrt{2}(1-a)^{3/4}\varepsilon_{LW}^{1/4} I_1(\varepsilon_{LW}) - \left[1 - aI_2\left(\frac{aR_S^2}{r^2}\right)\right]\left(\frac{R_S}{r}\right)^{\frac{1}{2}} \right\} \frac{4}{3}\sigma T_S^3 \quad (31)$$

By substituting data for the Earth (a = 0.30, ε_{LW} = 0.61, R = 6.37 x 10^6 m and r = 1.486 x 10^{11} m) and the Sun (R_S = 6.923 x 10^8 m and T_S = 5760 K), the planetary entropy production rate of the Earth is estimated as

$$\dot{\Pi}_{Earth} = 6.41 \times 10^{14} \text{ W/K} = 641 \text{ TW/K} \quad (32)$$

compared to 644 TW/K with ε_{LW} = 1 - a = 0.70 in (31) or (28). The sensitivity of the planetary entropy production rate can be demonstrated by considering hypothetical values of ε_{LW}. For ε_{LW} = 1.0 the planetary entropy production rate is 647 TW/K, it is 638 TW/K for ε_{LW} = 0.50, and it is 615 TW/K for ε_{LW} = 0.20. Therefore, although the planetary entropy production rate is strongly dependent on planetary albedo it is only weakly dependent on longwave emissivity, particularly when $\varepsilon_{LW} > 0.50$.

5. Conclusions

Prigogine's minimum entropy production principle does not appear to be applicable as a governing principle for atmospheric circulation. The force-flux relationships completely govern the response of a system, under certain constraints to applied external forces or radiative sources and sinks. It appears that finding the extremum only illustrates that the entropy production rate is a minimum with respect to any force when the corresponding flux has ceased.

The findings presented in this paper suggest that the calculated entropy production rate of Earth does not necessarily support the maximum dissipation conjecture. First, the question of whether the planet is in a state of maximum dissipation cannot be determined by only considering planetary states with the same albedo. Secondly, the approximation used by Stephens and O'Brien is accurate for a large range of thermal emissivities and thus over a large range of possible states of the system.

The graybody model presented in this paper provides more accurate expressions for planetary entropy production rate and mean temperature than previous blackbody type models. For the Earth the estimated entropy production rate of 644 TW/K is within 5% of Stephens and O'Brien's value calculated from satellite energy measurements, 680 TW/K. The estimated mean planetary temperatures are very close to the accepted experimental values for planets in our solar system. For Earth the estimated value is 278 K compared to the accepted value of 288 K.

The high value of the estimated entropy production rate for the radiative model emphasizes that planetary entropy production is dominated by radiative processes whereas a small portion is due to circulation. The entropy production rate due to radiative processes is high primarily because SR has a high emission temperature near 6000 K while material on Earth absorbing the SR is near 300 K.

The present model also shows that, although the planet's entropy production rate is strongly dependent on planetary albedo (or the fraction of sunlight absorbed), it is only weakly dependent on changes in longwave emissivity, particularly if ε_{LW} remains above 0.50. Finally, the radiative model illustrates that, surprisingly, the mean planetary temperature has a tendency of being independent of fraction of sunlight absorbed (or planetary albedo) although its dependent on phenomena like the greenhouse effect.

Acknowledgements

This work was partially supported by the Natural Sciences and Engineering Research Council (NSERC) of Canada.

Nomenclature

TABLE III. TR ENERGY AND ENTROPY QUANTITIES

	Energy		Entropy	
	Symbol	Units	Symbol	Units
Internal	U	J	S	J/K
Specific*	u	J/m^3	s	J/m^3K
Flow Rate	$\dot{E}$	W	$\dot{S}$	W/K
Irradiance (Flux)	H	W/m^2	J	W/Km^2
Radiance	K	W/m^2sr	L	W/Km^2sr

* The specific energy and entropy for TR are per unit volume rather than per unit mass as they are for material related quantities.

a	Planetary albedo
A	Surface area (m^2)
c	Speed of light = $(2.9979)10^8$ m/s
I_1, I_2	Entropy functions in (21) and (22)
J_i	Generalized thermodynamic flux (i.e. J_m, J_{th})
L_{ij}	Phenomenological coefficients (i.e. L_{11}, L_{12}, L_{21}, L_{22})
r	Mean radius of planetary orbit (m)
R	Radius of the planet (m)
T	Material emission temperature (K)
X_i	Generalized thermodynamic force (i.e. X_M, X_{Th})
ε	Emissivity
π	Physical constant, 3.14159…
$\dot{\pi}$	Entropy production rate per unit surface area (W/Km^2)
$\dot{\Pi}$	Entropy production rate (W/K)
σ	Stefan-Boltzmann constant = $(5.67)10^{-8}$ W/m^2K^4
Ω	Solid angle (sr)

Subscripts

Abs	Absorbed
BR	Blackbody radiation
Emi	Emitted
Inc	Incident
LW	Long-wave
P	Planet
Ref	Reflected
S	Sun
Sf	Surface of the Sun

Abbreviations

BB	Blackbody
BR	Blackbody radiation

ERBE	NASA Earth Radiation Budget Experiment
GR	Graybody radiation
SR	Solar radiation
TOA	Top of the atmosphere
TR	Thermal Radiation

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