Int.J. Applied Thermodynamics, **ISSN 1301-9724**
Vol.4, (No.2), pp.103-109, June-2001

Thermoeconomic Analysis at Optimal Performance of Non-isothermal Flat-Plate Solar Collectors

E. TORRES-REYES, B. A. IBARRA-SALAZAR
Instituto de Investigaciones Científicas, Universidad de Guanajuato
L. de Retana No. 5, C.P. 36000.Guanajuato, Guanajuato - México
Tel: 52 (4)7 327555, Fax: 52 (4)7 326252
E-mail: bibarra@quijote.ugto.mx

J. G. CERVANTES-DE G.
Departamento de Termoenergía y Mejoramiento Ambiental
Facultad de Ingeniería, Universidad Nacional Autónoma de México - México
Tel: 52 (5)6 228103
E-mail: jgonzalo@servidor.unam.mx

Abstract

A thermoeconomic analysis based on the Second Law is presented. It combines the annualized total cost (investment, installation and operating costs) with entropy generation during the collection and thermal use of solar energy. This thermoeconomic model is developed to determine the annualized total cost for air heating in a solar air heater by means of dimensionless parameters like the Entropy Generation Number and the Mass Flow Number. With these dimensionless groups, a set of relations between optimum operational variables, physical characteristics and properties of construction materials of the heater, are established. The function incorporates the cost and quality of the product, the cost by solar-to-thermal energy conversion, and also considers the thermal energy quality through the Second Law Efficiency.

Keywords: entropy generation minimization, thermoeconomic analysis, flat-plate solar collector

1. Introduction

Important investigations related with solar device analyses using methods derived from the Second Law were carried out by Bejan (1998). He determined the optimum operation temperature in a solar collector to minimum entropy generation.

Altfeld et al. (1988a,b) present an optimization about net flow of exergy in flat-plate collectors for heating air. They show in this work how the collection surface features, specially those of the extended surface, diminish airflowing; thus, net flow of exergy and thermal efficiency of the device increase.

Design methods of flat-plate solar collectors in other previous works, like the one published by Kwong and Chung (1991), based on a theoretical study, suggest determining the quality of the collected energy in a year, and comparing it with the global exergy given by the solar collector. The study includes a simple relation, like the annual energy collected times the Carnot efficiency, which can be directly related to the exergy content of the working fluid. This example is an optimization case of comparing certain types of collectors according to their global exergetic efficiency. However, it does not mean that the design of each one of the collectors is the thermodynamically best for the operating conditions.

It is required, in solar collector design, to establish a criterion to determine the best collector for a specific application. It can be also validated by a thermoeconomic analysis.

As a result of optimizing economic systems by means of mathematical models, an extension to energy systems has emerged and is now under continuous expansion under the subject of thermoeconomics (Sieniutycz and Salamon, 1990, Bejan et al., 1998). By considering exergy balances and related concepts, many complex energy systems can be thoroughly evaluated. The various exergy currents present in an industrial process can be associated to monetary costs, getting, therefore, a more meaningful concept of energy. A unifying and simple method to calculate costs and related applications of thermoeconomics has lately been proposed by Valero and Lozano (1990). Exergy costs are calculated by means of First and Second Laws of Thermodynamics, together with a convention regarding the values of the intensive properties of the environment.

More recently the thermodynamic procedure described in (Torres-Reyes et al., 2001), allows establishing a guideline for optimal design of the solar collector's surface according to the application. It was based on the Entropy Generation Minimization Method, proposed by Bejan (1996). Minimization of the entropy generation rate was carried out in a system operating between the apparent temperature of the sun and the environmental temperature taking into account the real operating conditions and the characteristics of the equipment (Torres-Reyes et al., 2001).

In this work a new concept of a thermoeconomic model is developed. It integrates, along with typical economic concepts, a cost function that includes optimal parameters of design for the performance of the solar device, with the minimal entropy generation. It is applied to estimate costs of a drying process with a heated airflow stream. The total cost involves the efficiency of the solar-to-thermal energy conversion.

The general method proposed for the design of the solar device here is established by means of Mass Flow Number and Entropy Generation Number as a function of the Number of Transfer Units. In this paper the Number of Transfer Units, besides being defined by means of Second Law analysis explicitly, is used in an implicit way in the thermoeconomic model.

2. Thermodynamic Optimization and Thermoeconomic Analysis

2.1 Entropy generation minimization

According to Bejan (1996), the total rate of entropy generation in a non-isothermal solar energy collector of area Ap operating at temperature T, can be written as:

$$\dot{S}_{gen} = \dot{m} c_p \ln \frac{T_{out}}{T_{in}} - \frac{\dot{Q}'}{T'} + \frac{\dot{Q}_0}{T_0} \tag{1}$$

where the first term on the right hand side is the non-isothermal heat transfer to the working fluid between inlet and outlet, Q' is the solar radiation heat transfer, and Q_0 is the heat loss to the ambient. In getting this expression, the pressure drop that may occur between inlet and outlet has been neglected.

On the other hand, an overall energy balance gives, for the total heat loss to the ambient:

$$\dot{Q}_0 = \dot{Q}' - \dot{m} c_p (T_{out} - T_{in}) \tag{2}$$

Substituting Eq. (2) in Eq. (1), one can obtain a dimensionless form by using T_0 as the temperature scale, $\theta = T/T_0$, and the absorbed solar energy on the collector, $\dot{Q}' = GA_c(\tau\alpha)$, where the value of $(\tau\alpha)$ is considered constant, as the heat scale. Thus:

$$\frac{\dot{S}_{gen} T_0}{GA_c(\tau\alpha)} = \frac{\dot{m} c_p T_0}{GA_c(\tau\alpha)} \left(\ln \frac{\theta_{out}}{\theta_{in}} - \theta_{out} + \theta_{in} \right) - \frac{1}{\theta'} + 1 \tag{3}$$

The left-hand side of Eq. (3) is defined as the dimensionless Entropy Generation Number, N_s, a parameter that states the exergy loss during solar collection in a solar collector (Bejan, 1996):

$$N_s = \frac{\dot{S}_{gen} T_0}{GA_c(\tau\alpha)} \tag{4}$$

A second dimensionless parameter can also be defined in Eq. (3) that represents the thermal energy of the fluid in relation to the solar energy collection, at the ambient temperature. This parameter is called the Mass Flow Number, M (Bejan, 1996):

$$M = \frac{\dot{m} c_p T_0}{GA_c(\tau\alpha)} \tag{5}$$

Substituting the definitions (4) and (5) in Eq. (3), the resulting equation becomes:

$$N_s = M \left(\ln \frac{\theta_{out}}{\theta_{in}} - \theta_{out} + \theta_{in} \right) - \frac{1}{\theta'} + 1 \tag{6}$$

If $\theta_{in} = 1$ (that is, $T_{in} = T_0$) in Eq. (6), the entropy generation rate is a function of M and θ_{out}. These variables are not independent of each other, as for a fixed total solar input, $GA_c(\tau\alpha)$, higher flow rates yield lower outlet temperatures and vice versa. The relationship between these variables can be obtained by integrating the steady-state energy balance at any point on the surface A_p, of a non-isothermal solar collector, assuming local thermal equilibrium between the collector surface and the working fluid. This means that the fluid and collector surfaces are at the same temperature level, which vary at any given position along the collector as:

$$G\frac{A_c}{A_p}(\tau\alpha) = U_c(T - T_0) + \dot{m}c_p\frac{dT}{d\sigma} \quad (7)$$

Rearranging the solution in terms of dimensionless variables and after a few algebraic steps, the following is obtained:

$$M = \frac{\dot{m}c_pT_0}{GA_c(\tau\alpha)} = \frac{1}{\theta_{max}-1}\frac{1}{\ln\frac{\theta_{max}-\theta_{in}}{\theta_{max}-\theta_{out}}} = \left[(\theta_{max}-1)\ln\frac{\theta_{max}-\theta_{in}}{\theta_{max}-\theta_{out}}\right]^{-1} \quad (8)$$

In the definition, given by Eq. (8), the relation between M and θ_{out} is clearly observed. The maximum collector temperature or "Sun-Air" temperature (Torres-Reyes et al., 1998) (also known as stagnation temperature), in dimensionless form, θ_{max}, is expressed as:

$$\theta_{max} = 1 + \frac{GA_c(\tau\alpha)}{U_cA_pT_0} \quad (9)$$

2.2 Mass Flow Number and Entropy Generation Number as a function of Number of Transfer Units

The Number of Transfer Units widely known as a performance parameter of heat exchangers is defined as:

$$NTU = \frac{U_cA_p}{\dot{m}Cp} \quad (10)$$

This group can be also expressed only as a function of maximum collector temperature and outlet fluid temperature in their dimensionless forms. Simplifying and rearranging Eqs. (8) and (9), the NTU becomes:

$$NTU = \frac{U_cA_p}{\dot{m}Cp} = \left[\ln\frac{\theta_{max}-\theta_{in}}{\theta_{max}-\theta_{out}}\right] \quad (11)$$

The Mass Flow Number as a function of the Number of Transfer Units can be represented by rearrangement and combination of Eqs. (8) and (11), as follows:

$$M = [(\theta_{max}-1)NTU]^{-1} \quad (12)$$

The relation between N_s and NTU is now obtained simply by substituting Eq. (12) in Eq. (6), and can be expressed as:

$$N_s = [(\theta_{max}-1)NTU]^{-1}\left(\ln\frac{\theta_{out}}{\theta_{in}} - \theta_{out} + \theta_{in}\right) - \frac{1}{\theta'} + 1 \quad (13)$$

The irreversibility obtained through N_s in Eq. (4) can be employed to estimate the Second Law Efficiency, given by the following equation, based on the concept of exergy destruction:

$$\eta_{II} = 1 - \frac{I}{\dot{E}_{x,in}} \quad (14)$$

An optimization procedure to determine the optimal mass flow rate and the optimal outlet fluid temperature, based on physical characteristics of the collector and geographical localization is suggested in (Torres-Reyes et al., 2001). Taking T' = 6000 K for the apparent sun temperature in Eq. (1), for solution of Eq. (7), varying θ_{max} and θ_{out} an approximate relationship, $\theta_{out} = \theta_{max}^{0.7}$, for the minimum Entropy Generation Number condition can be obtained. This relationship can be used as an easy way to estimate the outlet fluid temperature for an operation with minimum entropy generation.

2.3 Entropy Generation Number as a basis of an economic model for flat-plate solar collectors design

The avoidable irreversibilities should be taken into account for efficient energy use. In this way, a thermoeconomic model based on dimensionless design parameters, Entropy Generation Number and Mass Flow Number can be developed. Costs function combines operating cost, investment cost and maintenance cost; total cost is obtained adding fixed and variable costs. Fixed costs are required investment for construction and installation of solar collectors and the estimated annual maintenance. Variable costs, according to the system, occur by entropy generation during collector-air heat transfer when stream of a working fluid flows through the collector. For these costs, Entropy Generation Minimization Method provides optimal parameters.

The following cost function is proposed:

$$Costo = \underbrace{z_1'A_c + z_2'A_p}_{investment} + \underbrace{k'}_{maintenance} + \underbrace{y'M_{opt} + x'\frac{G_TN_{s,min}}{\eta_{II}} + rL'}_{operation} + \underbrace{D}_{others} \quad (15)$$

where x', y', z_1', z_2', are unit costs for different elements of the thermal system, k' is the cost of equipment maintenance, r'L is the cost of auxiliary energy (electricity, gas) for hybrid systems and D is considered for auxiliary equipment or additional costs from the solar collector application. N_s can be expressed as a function of collector conductance U_cA_p, as follows:

$$N_s = \frac{\dot{S}_{gen}\,T_0}{GA_c(\tau\alpha)} = \frac{1}{(\theta_{max}-1)}\frac{\dot{S}_{gen}}{U_cA_p} \tag{16}$$

The function in Eq. (15) incorporates cost and product quality: cost by solar energy conversion to thermal useful energy considering equipment costs and operating costs explicitly and it also considers thermal energy quality through Second Law Efficiency.

The operation cost terms are described as follows: the cost ($y'M_{opt}$) is obtained estimating required power for a necessary extractor for air forced convection through chosen area. Entropy generated cost, ($x'G_TN_s/\eta_{II}$), is evaluated as the amount of kW-h received by the collector during the operation period. The Second Law Efficiency for established temperatures and flows is obtained by multiplying the Entropy Generation Number obtained as above and the cost x, assuming that solar energy has the same cost as electrical energy. Accordingly the entropy generation cost for solar energy conversion at this device is obtained. Although the evaluation of this parameter is based on an assumption, it is nevertheless important because this cost indicates that it should be paid by solar energy destruction in accordance with the system's efficiency. The entropy generation cost ($x'G_TN_s/\eta_{II}$), uses the economical term represented by the unit cost x because there does not exist a method for measuring the collected solar energy cost. Therefore, it is assumed that the electrical energy cost (exergy 100%) is a near substitute of solar energy, which has a high level of exergy. In the proposed model, the cost expressed by $/kW-h to calculate the total annualized cost of heated airflow rate production with solar energy as a function of the Entropy Generation Number N_s, will be used as the datum point.

If the operation is carried out solely with solar energy, then the last term, referring to alternative energy (gas, electricity), does not exist. Finally the total cost is annualized, considering a useful life of the system collector-process of 10 years and an annual tax of 10%, (TABLE I). The recovery capital factor (Koutsoyiannis, 1979) is evaluated by multiplying the total cost. It can then be taken as an indicator to compare estimated investment with other options. When the operation of the solar collector does not reach the optimum outlet temperature, and operates underneath the point N_s, the Second Law Efficiency diminishes.

In summary, the cost criterion for an optimal design of a solar air heater can be expressed as:

$$C = z'A + k' + y'\left(\left(\frac{\dot{m}}{A_c}\right)\frac{CpT_0}{G(\tau\alpha)}\right)_{opt} + \\ + x'\frac{G_T}{\eta_{II}}\left(\frac{\dot{S}_{gen}\,T_0}{GA_c(\tau\alpha)}\right)_{min} + D \tag{17}$$

where D is assigned a value based on investment and operating costs from the solar collector application. In this equation, an important relation for design $\dot{m}/A_c$, may be distinguished. It is constant when the Mass Flow Number is fixed to a level temperature, and is obtained from thermodynamic optimization and evaluated with Eq. (5). The optimization of a thermal system is a complex process that involves generally a variety of geometric and economic variables, justifying its capital cost with thermodynamic efficiency. Integration in a thermoeconomic analysis results in a thermodynamic optimization as was proposed in the model given by Eq. (15).

3. Results and Discussion

The thermodynamic optimization procedure was evaluated to determine the optimal performance parameters of an experimental solar collector (Torres-Reyes, et al., 2001). This apparatus, fully described elsewhere (Torres-Reyes, 1983), was built with a covered set of cylindrical cavities to receive and capture the solar radiation. The whole experimental rig was fully instrumented and a special allowance was provided to measure the solar radiation properties and environmental properties. Experimental tests were run to determine the optimal performance parameters for different airflow rates. The values obtained were dimensionless outlet fluid temperature, θ_{out} = 1.12; dimensionless maximum collector temperature, θ_{max} = 1.17; Minimum Entropy Generation Number, N_s = 0.91; Mass Flow Number, M = 4.76 and required collection area, for minimum entropy generation, A_c =32.8 m^2.

An optimal airflow rate of 0.02 kg/s was obtained (Torres-Reyes et al., 2001), with global radiation of G = 600 W/m^2, ambient temperature of 298 K for the cylindrical cavities of the solar collectors. However, in the experimental test, the lowest possible airflow rate was of 0.05 kg/s. F*igure 1* shows the Mass Flow Number as a function of the dimensionless temperature. The outlet dimensionless temperature varied from 1.04 to 1.2 as a function of the climatological conditions. The data correspond to an airflow rate of 0.05 kg/s obtained through a long day.

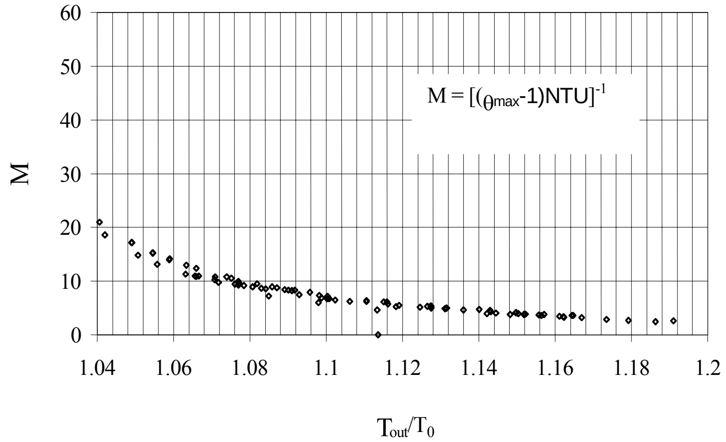

Figure 1. Mass Flow Number M of a solar collector (NTU Eq. 12) as a function of the dimensionless temperature for an airflow rate of 0.05 kg/s in a cylindrical cavity air solar heater.

TABLE I. COSTS FOR THE EQUIPMENT AND OPERATION OF A SOLAR DRIER FOR 90 KG OF MANGOS, CALCULATED AT CLIMATOLOGICAL CONDITIONS OF GUANAJUATO CITY. *(1 MX peso = 0.1 US $).*

	Cost (MX peso)	Annualized cost
Cylindrical cavities solar collector, 32.8m^2	$ 21,320.00	
Annual maintenance	$ 200.00	
Auxiliary equipment and equipment for drying	$ 16,000.00	
Investment total cost	$37,520.00	
Forced convection, (y'M_{opt})	$ 1,117.00/3 months	
TOTAL COST	$ 38,637.00/3 months	$ 6,282.5/year
Exergy loss apparent cost		
Cost expression	*Operating cost during 3 months*	***Annualized Cost***
$x' G_T \frac{N_S}{\eta_{II}}$	$ 123.55 x10^4	$ 19.8 x10^4/year

In TABLE I the cost estimated for a cylindrical cavities solar collector with 32.8 m^2 are shown. This installation is applied for drying 90 kg of mangos; the operation can be carried out during 9 hours through the months of May, June and July at Guanajuato State, Mexico.

From the definition of Mass Flow Number in Eq. (5) an important relationship can be proposed for design, since once the required temperature level is established for the given conditions, the ratio $\dot{m}/A_c$ is a constant value and the flow rate can be chosen corresponding to the minimum collection area. Another useful relationship that can be obtained for design proposes through Eqs. (10) to (12) is the Mass Flow Number as a function of thermal losses of the solar collector. This relationship involves the collection area, the heat transfer area and the overall heat transfer coefficient, and is shown in *Figure 2* for an airflow rate of 0.05 kg/s in the experimental device.

4. Comments

Although solar energy does not have a price, the cost for its thermal application shows the conversion equipment cost. On the other hand, when a solar collector operates in an inefficient way, the entropy generation is significant. If solar energy would have an equal cost compared to electrical energy, the cost due to the entropy generation would be considerably high.

The thermoeconomic model based on Second Law analysis, presented in this paper, contains elements that allow a preliminary economical evaluation of the design alternatives that are considered optimum from a thermodynamic point of view. It is taking into account the cost estimation as an important component for the feasibility of a project.

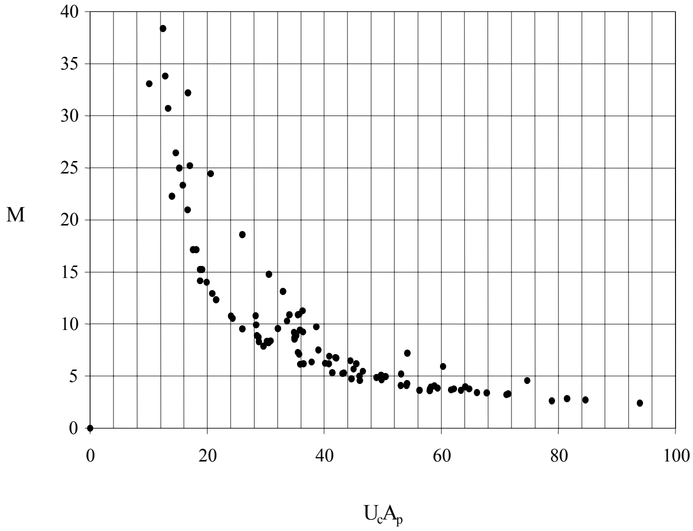

Figure 2. Mass Flow Number M of a solar collector as a function of conductance UcAp, Eqs. (11) and (12), for an ai flow rate of 0.05 kg/s in the cylindrical cavities of the air solar heater

The model compares availability losses due to energy conversion to useful thermal energy in the solar collector. From a thermodynamic viewpoint, inefficient performance of a solar collector means high entropy generation; then, if solar energy would have an equal cost to electric energy, cost magnitude of entropy generated would be considerably higher. On the other hand, since this cost represents energy that cannot be used, the engineering challenge is to design devices with a minimum cost per produced exergy unit.

The Second Law may be applied to implement changes to thermodynamic systems in order to diminish exergy losses by means of thermoeconomic optimization methods. The entropy generation minimization concept can be used in this method like a parameter integrated directly to the economics of the energy system, where the savings potential given by minimization is related to the environmental impact and the energy conversion cost.

Nomenclature

A	Area, m^2
A_c	Collection area, m^2
A_p	Collectors area, heat transfer area, m^2
C	Total annualized cost, \$/year
Cp	Specific heat, kJ/kg K
D	Equipment auxiliary cost, or additional costs, \$/year
$E_{x,in}$	Inlet exergy flow, kJ/s
G	Incident solar radiation, W/m^2
G_T	Total solar radiation in a time period, kW-h/m^2
I	Irreversibility, W
k'	Maintenance, \$
$\dot{m}$	Mass flow rate, kg/s
M	Mass Flow Number
N_s	Entropy Generation Number
r'	Alternative energy cost, i.e. gas, \$/kg
S_{gen}	Entropy generation rate, W/K
T	Temperature, K
T'	Apparent sun temperature, K
T_p	Solar collector temperature, K
T_{s-a}	Sun-air temperature, K
T_o	Environment temperature, K
U_c	Overall heat loss coefficient, W/m^2 K
x'	Electrical energy cost as substitute of solar energy cost, \$/kW-h
y'	Electrical energy cost, forced convection, \$/kW-h
z'_1	Collection area cost, \$/m^2
z'_2	Collector area cost, \$/m^2
z'	Integrated cost, \$/m^2

Greek letters

$(\tau\alpha)$	Absorptance-transmittance product
η_{II}	Second Law Efficiency
θ	Dimensionless temperature, T/T_0

Subscripts

c	Collector, collection
in	Inlet flow
max	Maximum
min	Minimum
out	Outlet flow
opt	Optimum
p	Heat transfer area, collector area
0	Environmental reference

References

Altfeld, K., Leiner, W. and Fiebig, M., 1988, "Second Law Optimization of Flat-Plate Solar Air Heaters", *Energy-The International Journal*, 41, (2), pp. 127-132.

Altfeld, K., Leiner, W. and Fiebig, M., 1988, "Second Law Optimization of Flat-Plate Solar Air Heaters. Part 2: Results of Optimization and Analysis of Sensibility to Variations of Operating Conditions", *Energy-The International Journal*, 41, (4), pp. 309-317.

Bejan, A., 1988, *Advanced Engineering Thermodynamics*, Wiley-Interscience, USA, pp. 501-514.

Bejan, A., 1996, *Entropy Generation Minimization, The Method of Thermodynamic Optimization of Finite-size Systems and Finite-time Process*, CRC Press, New York, Florida, USA, pp. 7, 249-253, 264-267.

Bejan, A., Tsatsaronis G., and Moran, M., 1996, *Thermal Design and Optimization*, John Wiley & Sons, Inc., pp. 405, 463.

Koutsoyiannis, A., 1979, *Modern Microeconomics, (In Spanish), Amorrortu Editores*, Argentina, pp. 31, 88.

Kwong, W. H. and Chung, Ch. L., 1991, "A Second Law Concept of an Equivalent Solar Collector", *Energy- The International Journal*, 16, (5), pp. 859-865.

Sieniutycz, S. and Salamon, P., 1990, *Finite-Time Thermodynamics and Thermoeconomics, Advances in Thermodynamics* Vol. 4, (Eds.) Taylor & Francis, New York, pp. 249-260.

Torres E., 1983, “Cylindrical cavities a solar collector design (In Spanish), *In Proceedings of 9th Congress, National Academy of Engineering of Mexico*, León, Gto., México, pp.1-5.

Torres-Reyes E., Cervantes-De Gortari J. G., Ibarra-Salazar B. A. and Picón-Núñez M., 2001, “Design method of flat-plate solar collectors based on minimum entropy generation”, *Exergy an-International Journal*, Vol 1, No. 1, pp. 1-7.

Torres-Reyes, E., Picón-Núñez, M. and Cervantes-de, G. J. G., 1998, *Energy, The International Journal*, (4), pp. 337-344.

Valero Capilla A., and Lozano Serrano, M. A., 1990, *Thermoeconomics (In Spanish), Centro de Investigación del Rendimiento de Centrales Eléctricas, Fundación de la Universidad de Zaragoza*, ENDESA, ERZSA, y Diputación General de Aragón, Zaragoza España, pp. 1-25.