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Exergy Efficiency Calculation of Energy Intensive Systems by Graphs

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Abstract

In the design and operation of energy intensive systems the problem of improving its efficiency is very important. The main way to solve this problem is thermodynamic analysis. This paper describes the general approach for calculating the exergy efficiency of complex energy intensive systems with arbitrary structure. A novel general equation of systems exergy efficiency is provided. An example of the method applied to a nuclear power plant analysis is given.

Key words: exergy, efficiency, thermodynamic analysis, power plant

1. Introduction

The processes taking place in thermal power and other complex energy intensive systems are characterized by mutual transformation of quantitatively different power resources. The thermodynamic analysis of such systems requires combined application of both laws of thermodynamics and demands the exergy approach Fratzsher et al. (1986); Szargut et al. (1988); Kotas (1995); Bejan et al. (1996); Moran (1998).

In contrast to the traditional thermodynamic analysis, the new exergetic method takes into account not only quantity but also quality of energy flows. The quality of energy is as important as the quantity. The first feature of the exergetic methods is their universality. It is possible to estimate the fluxes and balances of all kinds of energy for every element of the system using a common criterion of efficiency. Therefore, the exergetic methods are meaningful in analysis and calculations. The second very important feature of the exergetic methods is their direct ties with the technical-economical characteristics of system. The economical investigations that base on exergy cover wide area of problems from systems optimization to costs of products. This approach is known as thermoeconomic analysis. The use of exergy permits an easy way to choose the objective criterion for the estimation and optimization of systems. As result, exergy and its functions in systems analyses provide a very good guide to practical engineers.

Despite its usefulness, the exergetic approach was not realized fully for a long time. One reason for this situation is underestimating the capability of exergetic functions for mathematical modeling, synthesis and optimization of flow sheets. Another reason is its mathematical difficulty in thermodynamic

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analyses. Meanwhile, increasing complexity of optimization problems requires more effective and powerful mathematical methods. Hence, during the last few years in this area, many papers with different applications (see for example Cornelissen et al. 2000; Falsetta and Sciubba 1998; Sciubba, 1999; Finnveden and Ostlund 1997; Graverland and Gisolf 1998; Oh et al. 1996) have been published. The scope of these papers and our investigations (see Wu and Nikulshin 2000; Nikulshin and Wu 2000; Nikulshin and Klemes 1998; Nikulshin and Andreev 1999) showed that one of the most effective mathematical methods is the method of graph theory (Harary 1995). The approach of graph theory is very effective in systems investigations because the binary connection between the elements of any multitude can be conveniently displayed by graphs.

The usefulness of the graph model can also be demonstrated by its flexibility and in its wide variety of applications. Consequently, the most effective way for solving the problems of thermodynamic analyses and optimization of the flowsheets is to combine the methods of exergetics analyses with mathematical methods of graph theory. Hence, the unit exergy-graph method developed by the authors (Andreev and Nikulshin 1992) as exergy-topological is herewith described.

The exergy topological method is based on the combination of the exergy flow graph, the exergy losses graph and the thermoeconomic graph. In this article the exergy flow graph is used.

The models described in (Nikulshin and Wu 2000, Nikulshin and Andreev 1999) can be employed to find degrees of thermodynamic perfection and exergy losses for separate elements as well as for the whole system. However, these methods do not taken into account the technological aim of the system activities. Such an aspect can be taken into account with the help of exergy efficiency of the system. A simple and clear example of the difference between the degree of thermodynamic perfection of element and its exergy efficiency can be given by a pipeline. In the best case the exergy losses in it can be very small (approximately zero) and the degree of thermodynamic perfection will be near 100%. But this element has no useful efficiency from the thermodynamic point of view so its efficiency in all cases will equal zero.

2. General Equation for Exergy Efficiency (EEF) of Complex Systems

It is well known (Fratzsher et al. 1986, Szargut et al. 1988) that the exergy efficiency of the system on the whole can be calculated as the ratio of total useful exergy effect of the system (product) to available exergy (fuel).

But in this case the system is considered a "black box" and it is impossible to make any conclusion about the ways of system's improvement. Such a conclusion can be made by analysis of the system's structure, taking into consideration the influence of exergy efficiency of separate elements on the exergy efficiency of the system as a whole (Nikulshin and Andreev 1999).

The suggested equation is:

$$\eta_{ex}^{\Sigma}=\frac{E_{\Sigma}^{u}}{E_{\Sigma}^{a}}=\sum_{i=1}^{m_1}\eta_{ex}^{i}\beta_i-\sum_{i=1}^{m_2}(1-\eta_{ex}^{i})\beta_i \qquad (1)$$

where;

$\eta_{ex}^{i}=\frac{E_i^u}{E_i^a}$ - exergy efficiency of i th-element

$\beta_i=\frac{E_i^a}{E_{\Sigma}^a}$ - influence coefficient of *i* th-element

E_i^u, E_{Σ}^u - *i* th-element and whole system useful used exergy (below following Bejan et al. 1996, this exergy will be called exergy of product)

E_i^a, E_{Σ}^a - i th--element and whole system available exergy (below following Bejan et al. 1996, this exergy will be called exergy of fuel).

m_1 - the number of head elements of the system.

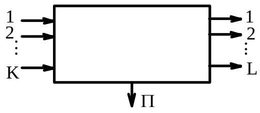
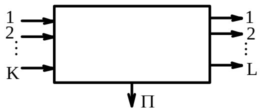
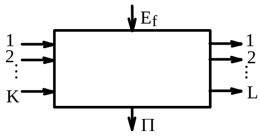
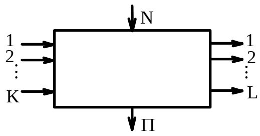
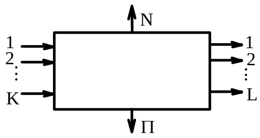
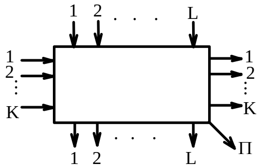
m_2 - the number of other (not head) elements of the system.

$m = m_1 + m_2$ - the total number of elements of the system.

It is easy to see that suggested influence coefficients are quite different from the same named coefficients given in Kotas (1995). They do not request calculating any derivatives and for this reason can be applied to discrete variables as well as to continuous ones.

Elements that use energy resources from external sources are to head elements. Notice that on one hand, the calculation of the values of inlet - E_i^{in} and outlet - E_i^{out} exergies are independent of the type of the element. On the other hand, the calculation of E_i^u and E_i^a is closely associated with the concrete type of exergy conversion in that element. In the analysis of different thermal power systems, six types (groups) of elements (TABLE I) are allotted and the exergies of product and fuel of the allotted elements are calculated by the given formulas. In other energy-intensive systems such as chemical-technological ones, the number of types of elements may be increased if necessary.

TABLE I. TYPES OF ELEMENTS OF THERMAL-POWER SYSTEMS

	Name of element	Principal scheme of exergy flows	Exergy flow rate of product E_i^u	Exergy flow rate of fuel E_i^a
1	Reservoir, pipeline, pressure regulator, mixer or separator of flows		0	$\sum_{k=1}^{K} E_k^{in} - \sum_{l=1}^{L} E_l^{out}$
2	Electric motor, electric generator		$\sum_{l=1}^{L} E_l^{out}$	$\sum_{k=1}^{K} E_k^{in}$
3	Combustion chamber, nuclear reactor		$\sum_{l=1}^{L} E_l^{out} - \sum_{k=1}^{K} E_k^{in}$	E_F - exergy flow rate of fuel
4	Pump, compressor		$\sum_{l=1}^{L} E_l^{out} - \sum_{k=1}^{K} E_k^{in}$	N -power of pump
5	Turbine		N -power of turbine	$\sum_{k=1}^{K} E_k^{in} - \sum_{l=1}^{L} E_l^{out}$
6	Multiflow recuperative heat-exchanger		$\sum_{k=1}^{K} \left(E_k^{out} - E_k^{in} \right)$	$\sum_{l=1}^{L} \left(E_l^{in} - E_l^{out} \right)$

where K - number of cold flows and L - number of hot flows for elements of type 6.

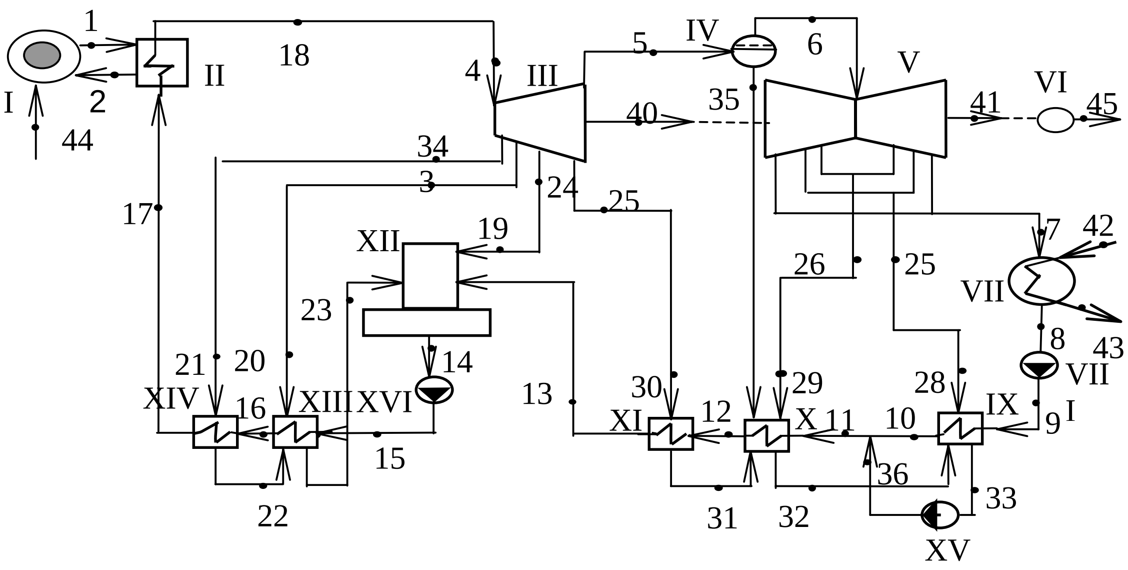


Figure 1. Flowsheet of nuclear power plant

TABLE II. PARAMETERS OF FLOWS IN FLOWSHEET OF NUCLEAR POWER PLANT IN FIGURE 1

Number of flow	Temperature t, ^{o}C	Pressure P, MPa	Specific enthalpy h, kJ/kg	Specific entropy s, kJ/(kg·K)	Mass flow rate M, kg/s	Specific exergy e, kJ/kg	Exergy flow rate E, MW
1	2	3	4	5	6	7	8
1	544.1	10.0	1189.6	2.9736	3738	378.0	1412
2	525.0	9.80	1095.6	2.7977	3738	332.0	1240
3	-	0.90	2628.8	6.2997	8.428	909.0	7.661
4	504.0	2.85	2801.8	6.2032	178.3	1108	197.6
5	-	0.201	2436.0	6.4399	139.4	677.9	94.50
6	-	0.181	2679.9	7.1071	77.30	739.6	57.18
7	300.6	0.004	2250.0	7.4681	67.09	211.2	14.7
8	300.6	0.004	116.90	0.4027	67.09	6.963	0.4476
9	300.8	1.20	117.40	0.4056	67.09	6.671	0.4671
10	340.3	1.00	282.80	0.9126	67.09	33.52	2.249
11	339.0	1.00	279.60	0.9116	150.3	30.73	4.619
12	359.0	0.80	360.90	1.1448	150.3	48.36	7.269
13	394.5	0.60	510.30	1.5432	150.3	89.01	13.38
14	411.2	0.35	581.40	1.7194	178.3	112.0	19.97
15	411.6	4.00	585.50	1.7205	178.3	115.8	20.65
16	439.8	3.80	707.00	2.0058	178.3	159.4	28.42
17	465.0	3.60	830.90	2.2798	178.3	208.5	37.18
18	507.0	3.05	2801.9	6.1832	178.3	1114	198.6
19	-	0.465	2548.2	6.4135	8.428	797.3	6.719
20	-	0.843	2628.8	6.3000	8.428	908.9	7.660
21	-	1.50	2703.7	6.2600	11.20	994.7	11.14
22	467.6	1.50	828.50	2.2782	11.20	206.7	2.310
23	442.0	0.84	714.90	2.0311	19.63	160.4	3.100
24	-	0.50	2548.2	6.3501	8.420	814.6	6.865
25	-	0.27	2472.8	6.4132	10.89	722.0	7.862
26	-	0.088	2578.2	7.1454	4.270	527.5	2.679
27	-	0.038	2474.5	7.2278	5.973	501.3	2.994
28	-	0.035	2474.5	7.2281	5.973	501.2	2.993
29	-	0.081	2578.1	7.1454	4.270	627.5	2.678
30	-	0.241	2472.8	6.4135	10.89	721.9	7.861
31	389.5	0.240	488.90	1.4892	10.89	82.35	0.897
32	363.2	0.080	376.90	1.1925	77.26	51.35	3.967
33	339.3	0.034	278.80	0.9121	83.23	29.79	2.480
34	-	1.60	2703.7	6.2556	11.20	995.9	11.15
35	-	0.20	504.70	1.5301	62.10	86.98	5.401
36	339.3	1.00	277.00	0.9047	83.23	29.82	2.487
37	-	-	-	-	-	-	0.100
38	-	-	-	-	-	-	0.812
39	-	-	-	-	-	-	0.100
40	-	-	-	-	-	-	47.32
41	-	-	-	-	-	-	24.41
42	293.0	0.30	84.100	0.2962	3415	3.230	11.03
43	303.0	0.30	126.00	0.4384	3415	6.860	23.43
44	-	-	-	-	-	-	253.0
45	-	-	-	-	-	-	70.00

The model EEF (see below) works with the exergy flow graph (Nikulshin and Wu 2000; Nikulshin and Klemes 1998). As was shown in (Nikulshin and Wu 2000), the exergy flow graph of a system with arbitrary structure is a graph E =(A, Γ)= (A, U) whose nodes multitude A={a_1, a_2, ..., a_i, ..., a_m} corresponds to the system elements and arcs multitude U={a_i ,a_l}; $i \neq l$; i = 1, 2, ..., m; l = 1, 2, ..., m; to the distribution of exergy flows in the system, while Γ represents a multivalued display of multitude A into itself.

The numbering of nodes (elements) of graph is arbitrary. For the effective calculation of E_i^u and E_i^a for elements of the sixth type, exergy flows should be numbered in a special order: the number of the flow at the exit of the element should be a unit larger than that of the flow at the inlet to the element. The numbering of exergy flows for other types of elements are arbitrary.

The method and algorithm of calculation of exergy efficiency works with a matrix of incidence (Harary 1995) of exergy flow graph. These elements can have one of three meanings: 0, 1, -1; where 0 means that jth-flow and ith-element are not tied; 1 means that jth-flow enters ith-element; and -1 means that jth-flow leaves ith-element, respectively.

3. Outline of a Model EEF

A model EEF for the determination of exergy efficiency for a system of arbitrary structure consists of four main blocks.

In the first block, the exergy flow graph E = (A, U) and its matrix of incidence are built corresponding to the system under consideration (see example given below).

In the second block, the procedure for recognizing the types of flows on arcs on graph E = (A, U) and for calculation of their exergies is carried out.

For example, the exergy flows of five types: exergy of mass-flow, exergy of heat-flow, exergy of radiation, exergy of work and exergy of fuel are considered in a thermal power system. The calculation method for these five types is given in

Fratzsher et al. (1986); Szargut et al. (1988); Kotas (1995); Bejan et al. (1996); Moran (1998).

Particularly the specific exergy flow rate of radiation can be calculated by formula:

$$e = C(3T^4 + T_0^4 - 4T_0T^3),$$

where C - coefficient of radiation of surface, T - absolute temperature of surface, T_0 - absolute temperature of environment.

In the third block, according to TABLE I, the procedure for computer calculation of exergies of product and fuel of elements by formulas of TABLE I are realized. In the fourth block, the exergy efficiency of i th-element $\eta_{ex}^i = E_i^u / E_i^a$, coefficient of influence $\beta_i = E_i^a / E_\Sigma^a$ and the exergy efficiency of the whole system are calculated by formula (1).

4. Example: Determination of Exergy Efficiency for a Nuclear Power Plant

The model EEF described above is applied to the exergy efficiency calculation of a nuclear power plant (*Figure 1*). The exergy flow graph for this flowsheet is given in *Figure 2*, and the part matrix of incidence is given in *Figure 3*.

The parameters of the flows are calculated in (Andreev and Nikulshin 1992) and shown in TABLE II (parameters of environment were taken: P_0 = 0.1 MPa, T_0 = 273.15 K).

Applying the model EFF, the calculation results of the exergetics characteristics of a power plant are given in TABLE III.

From TABLE III, it is observed that the largest exergy losses are in the nuclear reactor I because of the large irreversibility of the process that converts the exergy of radiation to the exergy of the steam. The large irreversibility explains the low exergy efficiency of the nuclear reactor (η_I = 0.680).

Exergy losses in boiler II are due to the transfer of heat from the high temperature level (flows 1, 2) to the lower temperature level (flows 17, 18).

The exergy efficiency of the heat exchangers are in a wide range - from η_{IX} = 0.397 (for low pressure heater IX) to η_{XIV} = 0.943 (for high pressure heater XIV). The exergy losses in heat exchangers are caused by high irreversibility of heat exchange with large temperature difference between “hot” and “cold” fluid flows. The larger the temperature difference, the larger is the exergy loss and the less is the exergy efficiency. Another factor on the exergy efficiency of the heat exchangers is the absolute temperature level of “hot” and “cold” flows. The bigger these are, the larger the exergy efficiency and the fewer the exergy losses.

Exergy losses in turbines and the pumps are the result of dissipation of the expansion and compression processes in real installations. Exergy losses in the generator are the result of mechanical (friction) and electrical dissipations. Exergy efficiency of these devises also depends on the quality of its construction.

TABLE III. THERMODYNAMIC CHARACTERISTICS OF NUCLEAR POWER PLANT IN FIGURE 1

No	Name of Element	Number of corresponding node of graph	Used exergy (product) flow rates E_i^u, MW	Available exergy (fuel) flows rates E_i^a, MW	Coefficient of influence β_i	Rates of exergy losses, Π_i, MW	Exergy efficiency η_i
1	2	3	4	5	6	7	8
1	Reactor	I	172	253	1.0	81	0.680
2	Steam generator	II	161	172	0.677	11	0.938
3	Turbine of high	III	47.3	69.5	0.274	22.2	0.680
4	Separator	IV	0	31.9	0.126	31.9	0
5	Turbine of low	V	24.4	37.3	0.147	12.9	0.654
6	Generator	VI	70.0	71.7	0.282	1.70	0.975
7	Condenser	VII	12.4	13.7	0.0539	1.31	0.905
8	Condenser pump	VIII	0.0195	0.10	$7.6\cdot10^{-5}$	0.0805	0.195
9	Low pressure heater	IX	1.78	4.48	0.0176	2.70	0.397
10	Low pressure heater	X	2.65	5.00	0.0197	2.35	0.529
11	Low pressure heater	XI	6.11	6.96	0.0274	0.85	0.878
12	Deaerator	XII	0	3.23	0.0127	3.23	0
13	High pressure heater	XIII	7.77	8.83	0.0347	1.06	0.879
14	High pressure heater	XIV	8.76	9.23	0.0363	0.47	0.943
15	Pump of low pressure heaters	XV	0.007	0.10	0.000394	0.093	0.07
16	Feed pump	XVI	0.68	0.81	0.0032	0.13	0.837
17	Pipeline between turbine of low pressure and heater of low pressure IX	XVII	0	0.001	$3.94\cdot10^{-6}$	0.001	0
18	Pipeline between turbine of low pressure and heater of low pressure X	XVIII	0	0.001	$3.94\cdot10^{-6}$	0.001	0
19	Pipeline between turbine of low pressure and heater of low pressure XI	XIX	0	0.001	$3.94\cdot10^{-6}$	0.001	0
20	Pipeline between turbine of low pressure and deaerator	XX	0	0.146	$5.75\cdot10^{-4}$	0.146	0
21	Pipeline between turbine of high pressure and heater of high pressure XIII	XXI	0	0.001	$3.94\cdot10^{-6}$	0.001	0
22	Pipeline between turbine of high pressure and heater of high pressure XIV	XXII	0	0.001	$3.94\cdot10^{-6}$	0.01	0
23	Mixture of flows	XXIII	0	0.117	$4.61\cdot10^{-4}$	0.117	0
24	Pipeline between steam generator and turbine	XXIV	0	1.0	$3.94\cdot10^{-3}$	1.0	0

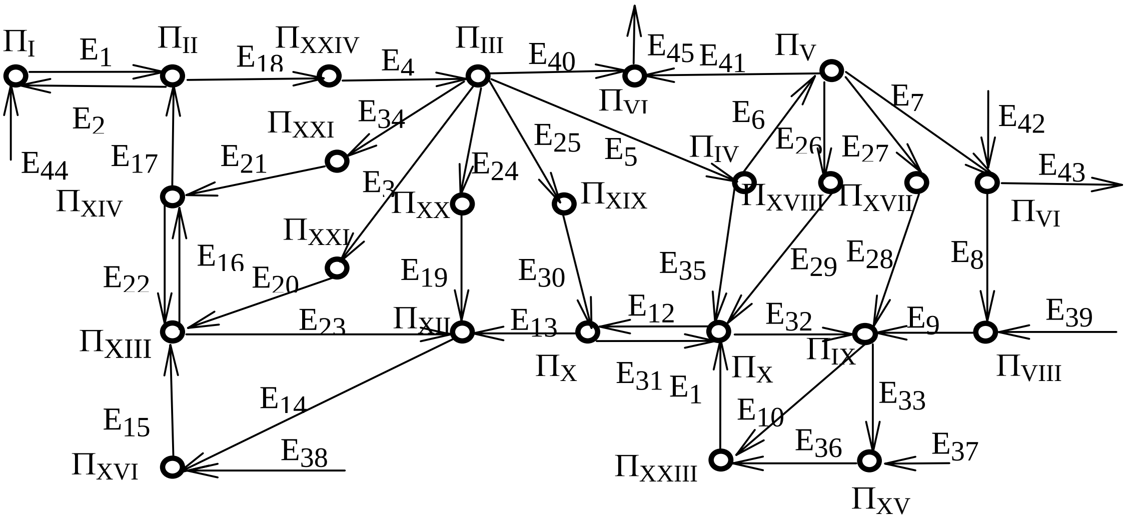


Figure 2. Exergy flow graph corresponding to the flowsheet in Figure 1

	1	2	3	4	5	6	7	8	9	10	11	…	40	41	42	43	44	45
1	-1	1															1	
2	1	-1																
3			-1	1	-1								-1					
4					1	-1												
5						1	-1							-1				
6													1	1				-1
7							1	-1							1	-1		
8																		
9									1	-1								
10											1							
…																		
40																		
41			1															
42																		
43										1	-1							
44				-1														

Figure 3. Matrix of incidence of the exergy flow graph shown in Figure 2

Exergy losses in other elements of the system (pipelines and mixture) are caused by dissipation of the flow. All these elements have a zero exergy efficiency ($\eta = 0$).

The dominant influential elements in the system are the nuclear reactor, the boiler and the turbines, because these devises transform the main exergy flows in the flowsheet.

For the system on the whole.

Exergy losses:

$$\Pi_{\Sigma} = \sum_{i=1}^{24} \Pi_i = 171.6 \text{ MW}$$

Exergy efficiency:

$$\eta_{ex}^{\Sigma} = \frac{E_{\Sigma}^{u}}{E_{\Sigma}^{a}} = \frac{82.4}{253} = 0.326$$

The exergy efficiency $\eta^{\Sigma}{}_{ex}$ is less than the same characteristic for the majority of elements of the system (excluding those which have zero efficiency). There are a few elements that have less exergy efficiency than the system on the whole is characterized (for example condensate pump $\eta_{VIII} = 0.195$). These elements have a very small coefficient of influence (β_{VIII}=0.000076) and little impact on the exergy efficiency of the whole system.

5. Conclusion

A novel calculation method of exergy efficiency of complex systems is described in this paper. This method is based on a general equation for systems of arbitrary structure and on special properties of exergy-topological models. It can be constructed for every energy-intensive system. The method is invariant of the technological aim and structure of the system. It can be applied for the investigation of various energy intensive systems in different branches of industry. The illustrative example given above demonstrates the application of the proposed method for thermodynamic analysis of a nuclear power plant.

Nomenclature

A	multitude of nodes
A	node
C	coefficient radiation of surface
E	exergy flow rate
$E = (A, \Gamma) = (A, U)$	- exergy flow graph
e	specific exergy
h	specific enthalpy
M	mass flow rate
m_1	the number of head elements of the system
m_2	the number of other (not head) elements of the system
m	the total number of elements of the system
N	power
s	specific entropy
p	pressure
T	absolute temperature
t	temperature
U	arcs multitude
u	arc
Γ	multivalued display of multitude A into itself
Π	rate of exergy losses
β	influence coefficient
η	efficiency

Subscripts

A	available (fuel)
ex	exergy
f	fuel
j	number of flow
i	number of element (node)
in	inlet
o	point of equilibrium with the environment
out	outlet
u	useful used (product)
Σ	for system on the whole

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