

The Thermodynamic Continuum of Jet Engine Performance: The Principle of Lost Work due to Irreversibility in Aerospace Systems

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Abstract

The performance continuum for air-breathing engines is formally developed and illustrated in terms of fundamental thermodynamic quantities including heat and work interactions and the irreversibility occurring in the flow-path of the engine. The thermodynamically consistent base-line from which performance losses due to irreversibility must be measured is clearly defined based on this analysis. Issues and problems with conventional flow availability (flow exergy) in terms of the assessment (design and optimization) of jet engines are discussed. The formal analytical relationship between lost thrust work and the irreversible generation of entropy in a jet engine is then reviewed in terms of underlying principle and methodology used to quantify this lost thrust work. This relationship is then extended based on the same underlying principle to the more general concept of lost thermodynamic work across a jet engine. It is then proposed that this concept of lost thermodynamic work as measured between the actual and the reversible device (rather than as referenced to a thermodynamic dead state) can, in fact, be extended to encompass other sub-systems and ultimately can be applied across the overall aerospace vehicle.

Key words: Jet propulsion, (aerospace) propulsion, hypersonics, availability, exergy, fluid dynamics, aerodynamics

1. Introduction

The optimized design of an aerospace vehicle formally requires the integration of a large number of inter-locking and dependent sub-systems, usually involving all of the broad categories of avionics, weapons and counter-measures, thermal protection, aerodynamics, structures, control, and propulsion (engine selection and design). However, within the context of overall system optimization, integration aspects between the engine and the vehicle generally have not been dominant drivers in either engine or airframe design and optimization. This *de facto* segregation of optimization between vehicle and engine has worked fairly well for conventional vehicles simply because the lower Mach number speed regimes (subsonic to low supersonic) have not usually required significant integration between airframe and engine. Therefore, the engine in such regimes has been designed primarily by meeting the dominant propulsive requirements

for specific thrust and specific fuel consumption (or its inverse, the specific impulse) and meeting weight requirements (itself related to specific thrust) while necessarily satisfying a host of other periphery sub-system requirements.

This engine design/optimization process has itself also been largely segregated in terms of individual engine components. For instance, combustor design is driven by specifying combustor performance measures of 'goodness' which, while not engine-based, nevertheless usually provide well-understood engineering figures of merits; similarly (but using different figures of merits) for the inlet, compressor, turbine, etc. In fact, by incremental improvements using such component-based design methods along with judicious experience-based integration of components, conventional gas turbine jet engines have evolved very high current second-law efficiencies of around 80% to 90% (since their inception in the middle of the last century). However, this evolution of

effectiveness in terms of working engine designs has largely occurred without any formalization of the optimization process. Surprisingly, other than early and seminal groundwork laid in such references as Foa (1951) and Builder (1964), this development has also generally occurred without a firm thermodynamic understanding of the relationships between engine thrust, (which drives engine specific impulse and specific thrust) heat, work, and irreversibility.

However, with the advent of vehicles operating in the hypersonic regime, both the impact of engine/air-frame/overall system integration and optimization in terms of minimizing losses as well as the need for detailed and fundamental understanding of integrated-system losses become critical issues. Furthermore, as new energy-based technologies mature, concepts and requirements for vehicle energy management and a formalization of system optimization become increasingly important (see Moorhouse (2000) and Rancruel and von Spakovsky (2003)).

There is broad consensus that there should be a single 'unit of currency' for the evaluation of all components and sub-systems within the overall vehicle. The value of correctly defining and using such a measure is enormous inasmuch as an aerospace vehicle can then be rigorously optimized with all sub-systems and components themselves truly optimized based on the overall system optimization. The application of such a parameter in design-based optimization (for high-speed vehicles in particular) could also identify thoroughly 'out-of-the-box' designs which out-perform current design paradigms. Although noted above specifically in terms of engine development, this idea of a 'single currency' has not historically been implemented (at least in a formal and thermodynamically fundamental manner) for the overall vehicle. For instance, optimization efforts based on minimizing take-off-gross-weight (TOGW) for a vehicle, although successful in general, are nevertheless not directly or consistently related back to fundamental second-law principles. This is despite the fact that, from the consistent thermo/fluid dynamic viewpoint, it is axiomatic that in order to optimize component/vehicle/mission performance, one must somehow go about minimizing the overall 'lost work potential' associated with irreversibilities occurring within the engine flow-path and within all other sub-systems within the vehicle. This process, of course, is accomplished while maintaining the necessary force, energy, and mass interactions to ensure that the overall design criteria are met for the vehicle/mission.

Methodology based on conventional flow availability (lost work associated with a thermodynamic dead state – usually based upon the ambient) has been suggested (and, less often, applied) in numerous references (e.g. see Clarke and Horlock (1975) and Brilliant (1995) for low-speed applications and Murthy (1994, 2000), Czysz (1991), and Moorhouse and Hoke (2002) for high-speed engines) for the design, evaluation, and optimization of aerospace jet engines and jet engine components. However, Riggins (1996, 1997) has conclusively shown engineering problems and issues with this same technique when applied to relatively simple flows and configurations. Investigations by Roth and Mavris (2000a, 2000b) and Roth (2001) have also identified issues in terms of comparative performance evaluations for more complex jet engine systems and point toward the need for a revised view of work potential, at least for aerospace systems.

In the context of an engine flow-path performance evaluation, thrust potential concepts (concepts first articulated by Curran and Craig (1973)) have been shown to yield optimized engines (see Riggins (1996) and Riggins et.al. (1997)). As discussed above, unformalized variations of this philosophy are, in fact, what have usually been used in engine design and optimization efforts. Furthermore, analytical work done in recent years including work presented in this paper (see Sections 2 and 3), has further formalized the relationship between engine thrust and irreversibility within the flow-path of the engine. This ongoing work has firmly established the following principal: In evaluating losses in an actual engine, the baseline from which performance losses must be measured is the performance of the same engine with identical energy/mass interactions but with all processes reversible. When this concept of loss evaluation is used in thrust-based optimization, optimal engine performance invariably results. Additionally, it has been shown (see Section 4) that if the loss in *thermodynamic work* (rather than the more 'restrictive' definition of lost *thrust* work) is suitably defined (i.e. strictly based on the same principle as stated above) – optimization and flow-path evaluation between both thrust-based methods and corrected conventional flow availability methods are indistinguishable for simplified engine-only evaluations.

The purpose of the present work is to 1) formally characterize and illustrate general aerospace jet engine performance (as measured by engine specific impulse and specific thrust) in terms of fundamental thermodynamic drivers and 2) discuss issues and suggestions concerning basic loss evaluation techniques and their impact

on both engine and vehicle design and optimization.

2. General Engine Performance In Terms Of Fundamental Thermodynamic Quantities

This section describes jet engine performance as measured in terms of specific thrust and specific impulse and provides the fundamental fluid/thermodynamic dependencies for these performance quantities. These fundamental dependencies are mass, work and heat interactions with the engine surroundings and irreversibility occurring within the flow-path.

Consider the specific thrust (uninstalled) of an air-breathing engine with the assumptions of i) constant specific heats throughout the engine, ii) adiabatic inlet, adiabatic work-interaction devices, and adiabatic nozzle, iii) steady flow and iv) uniform flow at engine entrance (i) and engine exit (e). This last restriction does not preclude multi-dimensionality inside the engine itself. See Figure 1 for a generalized schematic of the jet engine. The general expression for the specific thrust of an air-breathing engine (non-dimensionalized by the ambient speed of sound) with or without work interactions (i.e. compressors, turbines, deceleration modules, acceleration modules, etc.) can be written as follows (note that the conditions at free-stream, 0, are the same as at the engine entrance, i):

$$\frac{F/\rho u A}{a_0} = M_0 \left[\frac{(1+f) \frac{u_e}{u_i} - 1 + (1+f) \frac{1}{\gamma M_e^2} \frac{u_e}{u_i}}{-\frac{1}{\gamma M_0^2} \frac{A_e}{A_i}} \right] \quad (1)$$

$$\text{where } \frac{u_e}{u_i} = \frac{M_e}{M_0} \sqrt{1 + \frac{Q}{C_p T_{ti}}} \sqrt{\frac{1 + \frac{\gamma-1}{2} M_0^2}{1 + \frac{\gamma-1}{2} M_e^2}} \quad (2)$$

and M_e is solved in the following expression:

$$M_e \left(1 + \frac{\gamma-1}{2} M_e^2 \right)^{\frac{-(\gamma+1)}{2(\gamma-1)}} = M_0 (1+f) \left(1 + \frac{\gamma-1}{2} M_0^2 \right)^{\frac{-(\gamma+1)}{2(\gamma-1)}} \sqrt{1 + \frac{Q}{C_p T_{ti}} \frac{A_i P_{ti}}{A_e P_{te}}} \quad (3)$$

Here the fuel-air mass ratio is f and the non-dimensional term, $Q/C_p T_{ti}$, describes the ratio of the heat (per unit mass of air) input into the combustor to the total enthalpy (per mass) entering the engine.

For

$$f \leq f_{\text{stoichiometric}}, \quad \frac{Q}{C_p T_{ti}} = \frac{f h}{C_p T_0 \left(1 + \frac{\gamma-1}{2} M_0^2 \right)} \quad (4)$$

$$f > f_{\text{stoichiometric}}, \quad \frac{Q}{C_p T_{ti}} = \frac{f_{\text{stoichiometric}} h}{C_p T_0 \left(1 + \frac{\gamma-1}{2} M_0^2 \right)} \quad (5)$$

where $f_{\text{stoichiometric}}$ is the stoichiometric fuel-air mass ratio (approximately .029 for H_2 - air) and h is the heating value of the fuel (approximately 1.2×10^8 J/kg(fuel) for H_2 in air).

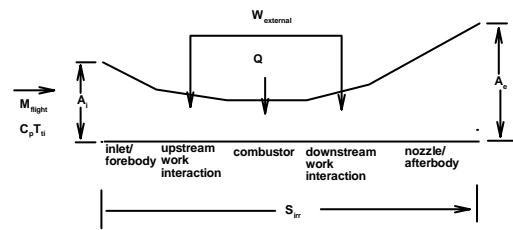


Figure 1. Single-stream air-breathing engine schematic

Furthermore, for engines with equal external work interaction (of either sign and with or without irreversibility) both upstream and downstream of the combustor, the following expression can be readily derived for the total pressure ratio across the engine (P_{te}/P_{ti}) required in Equation (3):

$$\frac{P_{te}}{P_{ti}} = \left[\frac{\left(1 + \frac{W_{up}}{C_p T_{ti}} \right) \left(1 + \frac{Q}{C_p T_{ti}} \right)}{1 + \frac{W_{up}}{C_p T_{ti}} + \frac{Q}{C_p T_{ti}}} \right]^{\frac{\gamma}{\gamma-1}} e^{\frac{-s_{irr}}{R}} \quad (6)$$

This expression for the total pressure ratio across the engine for simplicity assumes that the (non-dimensional) upstream work interaction (per unit mass of fluid), $W_{up}/C_p T_{ti}$, is equal in magnitude to the downstream work interaction (per unit mass of fluid) and that the fuel-air ratio, f , is small relative to 1.0. Here the work term is defined as positive to the flow. (For so-called inverse cycle engines, this term refers directly to the bypass ratio and would be negative in these equations, i.e. it corresponds in such a case to upstream work extraction from the flow-path.) The term s_{irr} is the entropy generated per unit mass of fluid inside the engine (from i to e) due solely to irreversibility within the flow-path (i.e. it does not include any entropy increment associated with reversible heat transfer across the engine boundary).

The specific impulse, I_{sp} , (non-dimensionalized by a_0/g_0) is then given as

$$\frac{I_{sp}}{a_0/g_0} = \left[\frac{F/\rho u A}{a_0} \right] \frac{1}{f} \quad (7)$$

These general relationships imply the fundamental functional dependence of non-dimensional specific thrust and specific impulse, I_{sp} , as summarized in the following expressions:

$$\frac{F/\rho u A}{a_0} = G \left(M_0, \gamma, \frac{h}{C_p T_{ti}}, \frac{A_e}{A_i}, \frac{Q}{C_p T_{ti}}, \frac{W_{up}}{C_p T_{ti}}, \frac{s_{irr}}{R} \right) \quad (8)$$

$$\frac{I_{sp}}{a_0/g_0} = H \left(M_0, \gamma, \frac{h}{C_p T_{ti}}, \frac{A_e}{A_i}, \frac{Q}{C_p T_{ti}}, \frac{W_{up}}{C_p T_{ti}}, \frac{s_{irr}}{R} \right) \quad (9)$$

where G and H are functions.

For $f > f_{stoichiometric}$ (i.e. fuel-rich condition), f decouples from $h/C_p T_{ti}$ and $Q_{stoichiometric}/C_p T_{ti}$ such that the functional dependencies given above then include f . Also note that for a given fuel (fixed heating value, h), the altitude dependency exists implicitly (through T_i) in the calculation of the $h/C_p T_{ti}$ parameter.

Equations (8) and (9) form the basic fluid-thermodynamic foundation for engine analysis from the standpoint that they clearly demonstrate the functional dependence of engine performance on the following three fundamental thermodynamic parameters; 1) the effective heat added inside the engine, 2) the work interaction within the engine, and 3) the degree of irreversibility occurring in the engine. The three-dimensional functional performance space (in terms of specific thrust and specific impulse) defined by these three non-dimensional quantities will be discussed in the next section as defining the performance continuum for air-breathing jet engines. In addition, from the standpoint of performance, the importance of the exit to inlet area ratio of the engine is demonstrated (i.e. no assumption of ideal – and non-physical – expansion to ambient pressure is made).

3. The Thermodynamic Continuum Of The Performance Of Air-Breathing Jet Engines

Based on the derived performance relationships in Section 2, the full performance characterizations of all single-stream jet engines (turbo-jets, ram-scrumjets, and inverse-cycle engines) can be shown in terms of three-dimensional surfaces of normalized constant specific impulse and normalized specific thrust. These performance surfaces are located within

the thermodynamic three-space continuum as sketched in Figure 2. The three independent axes are therefore defined by; 1) added heat over the total free-stream enthalpy and 2) the upstream work interaction to the engine flow over the total free-stream enthalpy, and 3) the non-dimensional entropy due to irreversible mechanisms within the engine flow-field. This characterization of performance is done at fixed flight Mach number, a given exit to inlet area ratio, and given fuel heating value (non-dimensionalized by inflow total enthalpy per mass).

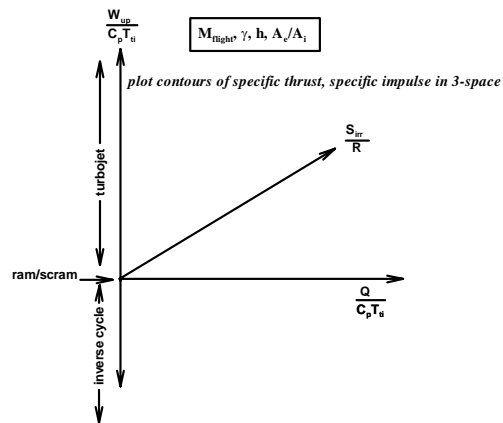


Figure 2. Three-dimensional thermodynamic space for performance characterization (air-breathing performance continuum)

Note that only positive heat and irreversibility terms are relevant and possible, respectively (i.e. only the positive axes for these two parameters are shown whereas the axis for the work interaction parameter needs to cover both negative upstream work interactions (inverse turbo-jet type cycles) as well as conventional turbojet type cycles (positive upstream work interactions). As one moves up the positive work axis, effectively the compression ratio (amount of upstream work transferred to the fluid) is being increased within the turbojet engine. Conversely, as the negative work interaction is increased (inverse-cycle side of the axis), the amount of upstream work transferred from the fluid is being increased. The physical limit ('floor') on this (negative) side of the upstream work axis is when the $W_{up}/C_p T_{ti}$ parameter is -1, i.e. when all of the total enthalpy per unit mass entering the inlet is removed within the work device upstream of the burner.

Ram/scramjet systems (no work interaction within the flow) are defined completely in terms of performance by the zero work-interaction

plane in Figure 2, i.e. performance is defined simply in terms of the balance of s_{irr}/R and $Q/C_p T_{ti}$ occurring in the engine.

It is also possible and very useful to superimpose non-dimensional combustor exit (commonly designated as engine station 4) total temperature surfaces (τ_λ surfaces) throughout this three-space region where

$$\tau_\lambda = \frac{T_{t4}}{T_0} = \left(1 + \frac{\gamma - 1}{2} M_0^2\right) \left(1 + \frac{W_{up}}{C_p T_{ti}} + \frac{Q}{C_p T_{ti}}\right). \quad (10)$$

The combustor exit total temperature, T_{t4} , is significant in the analysis and design of all types of jet engines. It usually corresponds to the highest (static) temperature within the engine and hence often is viewed as the limiting/driving parameter in engine selection and design in terms of determining the allowable balance of heat and work interactions. It can also be viewed as a throttling parameter.

The critical significance of the zero irreversibility plane ($s_{irr}/R=0$) for all engine types is easily seen in Figure 2. Specifically, this zero irreversibility performance plane serves as the 'reversible engine' performance base-line for measuring performance losses in an actual engine due to internal flow-path irreversibility. This reversible performance base-line is generated with the given engine constraints (exit to entrance area ratio, flight Mach number, etc.) and identical energy and mass interactions from the surroundings to the flow-field. However, in this base-line flow, all internal processes are reversible. (For engine flow-fields with multiple (and varying) species, this also entails identical chemical composition distributions between actual and reversible base-line engines). The difference in performance as measured between the actual and the reversible base-line is the lost performance, whether measured in terms of thrust or an availability parameter. This will be discussed in greater detail in Section 4 of this paper.

The following series of figures illustrate selected planes of performance contours within the 3-space performance continuum as described above, at an altitude of 30 kilometers. Contours of actual specific impulse (I_{sp}) are shown (rather than the non-dimensional I_{sp} term) in these figures for ease of interpretation. The plots also show contours (surfaces) of combustor exit total temperature rather than τ_λ for the same reason. Both specific impulse and specific thrust values which are actually negative are simply shown as zero in all of the following plots.

3.1 Scramjet performance

Figure 3 is a plot of specific impulse contours for no work interaction (i.e. the ram/scramjet performance plane) at a fixed flight Mach number of 8, hydrogen fuel, and unit area ratio across the engine. The maximum value of the heat parameter on the x axis corresponds to the stoichiometric fuel-air ratio; the fuel-lean fuel-air ratio, f , is then directly proportional to the heat parameter along the x axis in this figure. Maximum possible specific impulse is seen to occur at very low irreversibilities and also lower heating. However, note that a realistic value for s_{irr}/R for an operational scramjet at this Mach number is in the range of 4 to 5; hence for a scramjet operating at close to stoichiometric, an I_{sp} of around 2000 is achievable. As heat (injected fuel mass flow rate) is reduced in an operational scramjet, dominant irreversibilities associated with injection, mixing, and combustion in the combustor also decrease rapidly such that the I_{sp} does not plummet as would be the case at constant s_{irr}/R .

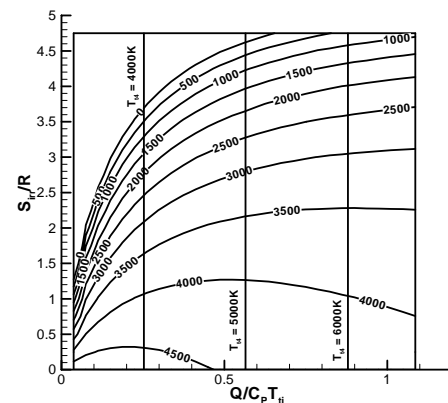


Figure 3. Specific impulse contours for ram/scramjet performance plane (flight Mach = 8, hydrogen fuel, $A_e/A_i=1.0$).

Also shown on this figure are (vertical) lines corresponding to the total temperature at the combustor exit based on the heat input into the engine. The magnitudes of the numbers are relatively high due to the constant specific heat assumption made for this demonstration analysis. Taken together, the performance curves and the total temperature lines define the possible growth potential (and limitations) for high-temperature materials. In other words, the development of high temperature materials may not be feasible if the inevitable cost of irreversibility within the flow in actually reaching those temperatures (more fuel, etc.) is too large (such that performance actually declines). The figure also shows that this issue will become continually

more challenging as technology is improved since the I_{sp} performance rapidly flattens with increasing heat release at high temperatures.

Figure 4 is a plot of the non-dimensional scramjet engine specific thrust for identical constraints as above. Engine specific thrust (thrust per air mass flow rate) is not a measure of fuel economy (such as I_{sp}) but is a measure of the thrust delivered to engine size; this parameter is necessary in engine design and evaluation. Maximum specific thrust is seen to occur at high (stoichiometric) heat release and the smallest irreversibility (recall that the maximum I_{sp} occurred at low heat release and smallest irreversibility).

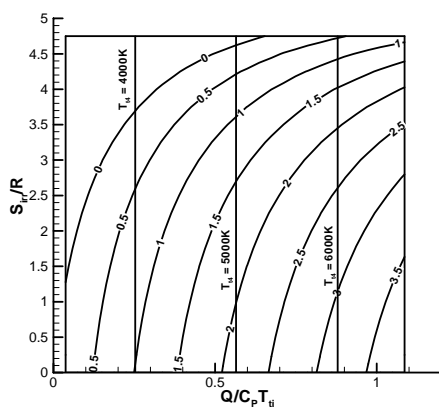


Figure 4. Non-dimensional specific thrust contours for ram/scramjet performance plane (flight Mach = 8, hydrogen fuel, $A_e/A_i=1.0$).

The character of the performance surfaces shown in these figures sheds new light on the diminishing influence of additional progressive heat release on scramjet engine performance (and ultimately can provide numerical guidance regarding related design rules). It has long been maintained in scramjet combustor design that attempting to achieve a mixing/combustion efficiency above 80-85 % (as measured with respect to the maximum heat release possible) is simply not feasible or desirable, especially in terms of simply lengthening the combustor. This is usually attributed to the asymptotic nature of the heat release in a scram combustor caused by fuel mixing and chemical kinetics – the argument being (quite logically) that the irreversibility generation continues apace even as heat release winds down in the latter part of the combustor. This is certainly a real effect.

However, Figures 3 and 4 show a significant additional thermodynamic effect which will drive the design to even shorter combustors. As apparent in the figures for moderate to high heat release and reasonable irreversibility, even were additional heat release

somehow obtained completely ‘free-of-charge’ (i.e. with no penalty in terms of increasing s_{ir}/R), the impact on scramjet engine performance of that additional heat release is negligible in terms of specific impulse and small in terms of specific thrust. This trend worsens significantly as actual irreversibilities mount in an actual engine.

The critical impact of correctly assessing the trade between irreversibilities and combustion in realistic scramjet design is also clearly demonstrated by examining these plots. It is apparent that design changes in a component must take into account the overall irreversibility environment that the engine actually operates in.

Figure 5 shows the ram/scramjet performance spectrum (in terms of I_{sp}) as impacted by the fuel-air ratio, f , rather than released heat. Recall that f scales linearly with released heat (up to the maximum heat release shown) on the previous charts but for fuel-rich (greater than stoichiometric) conditions, the heat-release becomes fixed at the stoichiometric even though f above stoichiometric will positively impact the thrust (due to mass addition). Its impact on the specific impulse will be generally negative since no direct benefit is obtained other than mass addition (at least in this simplified analysis which does not account for fuel injection momentum, etc.). The region of $f > f_{stoichiometric}$ is clearly shown to be dominated by decreasing specific impulses. Generally, this region of high f is encountered at Mach numbers around 9 to 10 and above due to the need to cool the scramjet engine and the increasing importance of injected fuel momentum.

3.2 Jet engine performance with work interactions (turbo-jet/inverse-cycle)

Figures 6 and 7 illustrate I_{sp} contours and total temperature surfaces for jet engines at a flight Mach number of 2.0 with work interactions on the ‘y’ axis. Figure 6 is for s_{ir}/R equal to 0 (i.e. the reversible engine) while Figure 7 is for s_{ir}/R equal to 1.0. Keep in mind that ‘ramjet’ performance at the given engine irreversibility is itself collapsed on the zero work-interaction lines in these figures.

Typical turbo-jets at this flight Mach number operate with an irreversibility parameter approximately in the range of 0.5 to 1.0 and non-dimensional work interaction parameter between 1 and 2. The rapid decrease of performance with increasing irreversibility can be seen by examining these figures sequentially. With increasing irreversibility, the region of maximum specific impulse moves from low heat input and

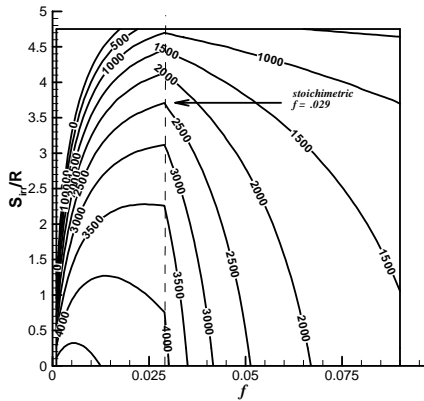


Figure 5. Specific impulse contours for the ram/scramjet performance plane as a function of fuel-air ratio (flight Mach = 8, hydrogen fuel, $A_e/A_i=1.0$).

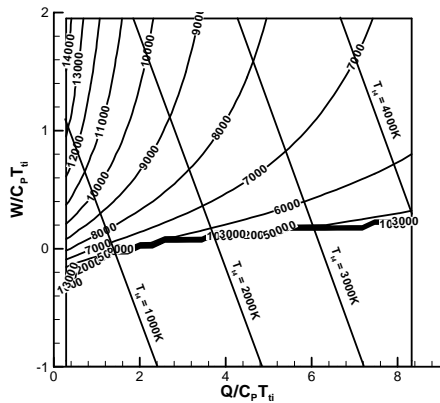


Figure 6. Specific impulse contours for the engine performance plane at $s_{irr} / R = 0$ (flight Mach = 2, hydrogen fuel, $A_e/A_i=1.0$).

high positive work to larger input heat such that a performance ‘island’ exists in terms of heat input (or fuel-air ratio) at about 25% of stoichiometric fuel flow rate. The cycle-driven performance penalty for inverse cycle engines (even in the complete absence of irreversibility) is obvious across the heating range. However, in these figures, the slope of the total temperature lines is negative (as compared to the vertical for the ram/scramjet performance plane). The magnitude of this negative slope increases with flight Mach number. The reason for the development of this slope is obvious. For instance if work is extracted upstream of the burner (corresponding to a negative work interaction parameter), a larger heat release can be realized while maintaining the same combustor exit temperature. Furthermore, the larger the flight Mach number, the steeper the slope will be. Therefore there may be some benefit for the inverse cycle in terms of the ability to add more heat while keeping the temperature under control. Additionally, at higher Mach numbers, there should be some

favorable effects in the combustor in terms of lessening of the irreversibility and in increasing allowable heat release, off-set to a greater or lesser degree by losses in the work-interaction devices. In general, the use of the inverse cycle at such low Mach numbers is not of actual interest and is shown here for information purposes only.

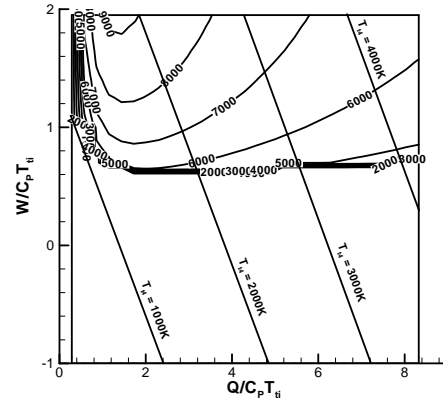


Figure 7. Specific impulse contours for the engine performance plane at $s_{irr} / R = 1.0$ (flight Mach = 2, hydrogen fuel, $A_e/A_i=1.0$).

Figure 8 is a plot of non-dimensional specific thrust contours for the same flight Mach number and for zero irreversibility (the reversible engine). The maximum thrust occurs at maximum heat release and large positive work interaction. However, this figure when compared with Figure 6 (the specific impulse contours) clarifies the familiar trade between high I_{sp} (usually characterized in terms of the closely related specific fuel consumption) and high specific thrust. If maximum combustor temperature is fixed for conventional turbojets (low-flight Mach numbers), the specific impulse is maximized by increasing the positive work

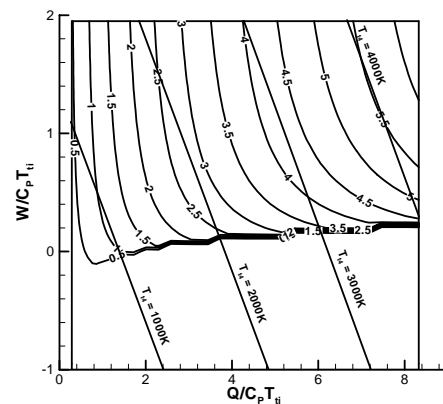


Figure 8. Non-dimensional specific thrust contours for the engine performance plane at $s_{irr} / R = 0$ (flight Mach = 2, hydrogen fuel, $A_e/A_i=1.0$).

interaction (i.e. the compressor pressure ratio) while the specific thrust is lowered and vice-versa. This trend can be easily seen by inspecting the performance and total temperature lines in these two figures.

Figures 9 and 10 show I_{sp} contours at a flight Mach of 5.0 and for irreversibility parameters of 0 and 2 (note engines in this flight regime typically have values of this parameter around 2). In addition, the range of permissible positive work interaction has been somewhat arbitrarily decreased to a non-dimensional value of 1.0 due to the larger total temperatures experienced in the flow-field associated with the increased flight

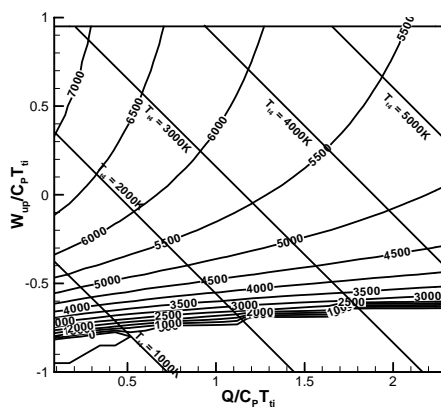


Figure 9. Specific impulse contours for engine performance plane at $s_{irr} / R = 0$ (flight Mach = 5, hydrogen fuel, $A_e/A_i = 1.0$).

Mach number. Similar trends and conclusions are seen in this series of plots as for the flight Mach 2 results. The performance drops rapidly with increasing irreversibility. The increased negative slope for the engine total temperature (combustor exit) lines indicates a possible increase in the feasibility of the inverse cycle concept - provided that irreversibilities associated with such an engine can be minimized to the point that it overcomes its inherent cycle disadvantage. Additionally, it must be emphasized that the actual fluid dynamics in terms of required area ratios and choking phenomena through the engines (of all types) are not reflected on these very fundamental charts. Unfortunately, the inverse cycle engine is especially prone to design problems featuring these issues.

Figure 11 (for $s_{irr} / R = 0$) and Figure 12 (for $s_{irr} / R = 4$, i.e. actually representative of the loss history of a high-speed engine) show (for flight Mach of 8) specific impulse contours for a range of work interactions (negative) associated

with the inverse cycle. Positive work interactions (turbo-jet cycles) simply are not feasible due to

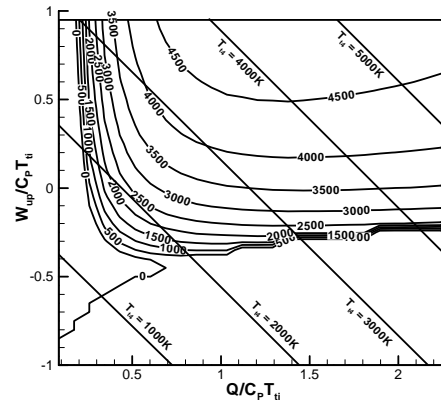


Figure 10. Specific impulse contours for the engine performance plane at $s_{irr} / R = 2.0$ (flight Mach = 5, hydrogen fuel, $A_e/A_i = 1.0$).

the high temperatures that result at combustor exit and hence are not shown. Recall that the zero work interaction line reflects the scramjet performance line at the same conditions. The tremendous impact on performance due to irreversibility is again seen in comparing Figure 11 to Figure 12. For the inverse cycle engine, there is a possible benefit due to temperature limits in the engine but similar caveats as mentioned for the flight Mach 5 case are again noted.

4. Thrust, Lost Work, and Availability in Jet Engines

Sections 2 and 3 of this paper described and characterized the fundamental relationships for aerospace jet engine specific impulse and specific thrust in the most basic thermodynamic sense. Of particular relevance to the current section is the description and development of the reversible engine performance base-line from which performance losses for the actual engine can consistently be examined. While most propulsion analysts certainly accept the two parameters (specific thrust and I_{sp}) as dominant performance criteria for engine flow-path evaluation, design, analysis, and optimization, they of course cannot by themselves provide any absolute information concerning the evaluation of other non-propulsion sub-systems on-board an aerospace vehicle. As discussed in the Introduction, there remains the need for a single 'unit of performance currency' for the evaluation of all components and sub-systems within the overall vehicle. The candidate most frequently cited and featured in past work is conventional flow exergy or flow availability. Exergy enjoys a sound thermodynamic track record (both

theoretical and applied) in many ground-based mechanical systems.

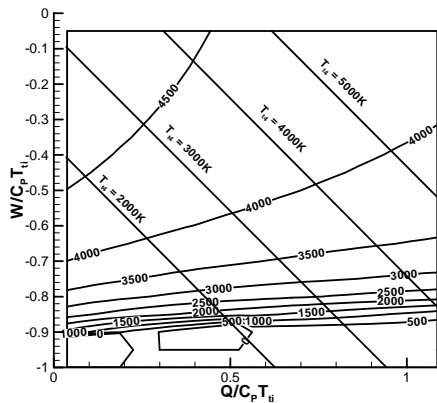


Figure 11. Specific impulse contours for the engine performance plane at $s_{irr} / R = 0$ (flight Mach = 8, hydrogen fuel, $A_e/A_i=1.0$).

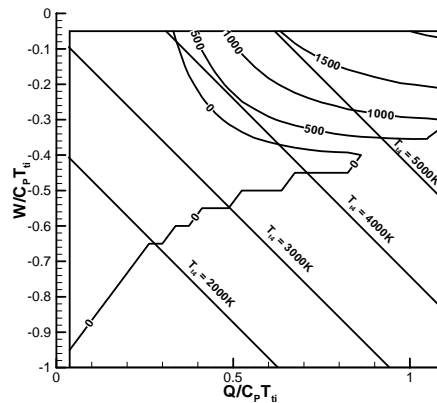


Figure 12. Specific impulse contours for the engine performance plane at $s_{irr} / R = 4.0$ (flight Mach = 8, hydrogen fuel, $A_e/A_i=1.0$).

This section, however, provides a simple constrained problem that shows the deficiency of ‘conventional’ flow availability in providing a measure-of-goodness for a highly simplified aerospace jet ‘engine’ flow-field. This is a deficiency that can only be ignored at peril as it violates the following logical statement of principle which must be maintained for aerospace system optimization (if one truly believes in the concept of a common thermodynamic currency):

The accounting or auditing methodology used in optimizing an overall system composed of sub-systems must also be able to give optimal solutions/configurations when any sub-system is optimized in isolation – i.e. when it is itself the overall system.

This principle does not mean that the optimal design for a sub-system in isolation is necessarily identical or even similar to its final design within an overall-optimized system. It simply demands that the single currency should

work uniformly. What this means is that the unit of currency must automatically take into account the functionality of a system or sub-system and all the engineering constraints upon it. In the case outlined here and in other examples published elsewhere, high-speed air-breathing engine problems can be constructed with significant (but permissible and reasonable) constraints in which the engine is the vehicle and optimization of the engine implies optimization of the vehicle. No method, however successful in other areas, can hide behind the inarguable complexity of an actual aerospace vehicle – the method, or methods of choice must surely work on simple and highly constrained problems before they can be relied on for complex systems.

The analytical foundation of the relationship between thrust losses and irreversibility is reviewed next because it has been observed that this methodology does in fact yield optimal engines. This analytical work is directly related to the previous Section 3 in which the performance continuum of aerospace jet engines was developed. The methodology is then extended to the description of lost thermodynamic work (not just thrust work) due to irreversibilities in engine flows. It is then proposed here that the philosophy of measuring lost work can be formally applied to individual sub-systems within an aerospace system for system-level optimization.

4.1 Availability issues in jet engine flow-fields

Availability is usually defined as the maximum (reversible) work that can be produced by a flow at a given state as measured from some reference ‘dead’ state at ambient temperature, T_0 . For single species perfect gas flow in a streamtube, the change in conventional flow availability from an upstream station to a downstream station is defined as follows:

$$\Delta Ex = C_p T_e + \frac{u_e^2}{2} - C_p T_i - \frac{u_i^2}{2} - T_0(s_e - s_i) \quad (11)$$

This can be written in terms of work and heat interactions crossing the fluid boundary as

$$\Delta Ex = \int_i^e \delta w_{\text{bound}} + \int_i^e \delta q_{\text{bound}} - T_0(s_e - s_i) \quad (12)$$

Consider flow in an adiabatic ($\delta q_{\text{bound}}=0$) constant area duct but with an asymptotic progressive work (δw_{bound}) interaction schedule (to the flow) between i and e such that the maximum supplied work is reached asymptotically at the end of the duct. Let there also be the progressive generation of entropy due to internal

irreversibility, i.e. perhaps associated with boundary friction. For the sake of argument, let the entropy generation be approximately linear with distance along the duct. Neither of these assumptions (asymptotic work interaction and linear irreversibility) is necessary but as stated provides a useful thought experiment for examining the impact of reference state (T_0) selection on device performance evaluation using availability analysis.

The design problem for this simple problem is to simply identify the optimal length of the duct. It is obvious from this constructed (but thermodynamically permissible) example that as $\Delta Ex(x)$ is evaluated along x using equation (12), ΔEx increases rapidly initially (with small x) since the work interaction is rapid in that region. However, as ΔEx is evaluated at larger and larger x , the (beneficial) work interaction term reaches a limiting (asymptotic) value whereas the frictional (Δs) term continues to increase. Hence ΔEx will reach a maximum at some axial location and then begin to drop. In fact, at some point along the duct (if the duct is made long enough and choking does not occur), the frictional lost availability term ($T_0(s(x) - s_i)$) will become greater than the cumulative work term, $\int_0^x \delta w_{\text{bound}}$, such that ΔEx will become negative. The critical point here is that the axial location corresponding to the maximum ΔEx (i.e. the 'optimal' length) is **dependent on the selection of the reference temperature**. In other words, the 'design' of the duct utilizing availability is plainly driven by the choice of the dead state.

For an aerospace engineer who is interested in producing the maximum thrust (axial force) using such a constrained device (with the given external work schedule), this in fact makes flow availability as conventionally defined problematic for evaluating performance. Note that the example flow-field as constructed (with asymptotic work interaction and progressive wall friction) would also result in an optimal length at which axial force produced is maximized – this location however is fixed by the constraints of the problem and is entirely independent of T_0 . Although this problem is highly constrained and simplified, it demonstrates the need to 'revisit' the availability concept especially in terms of the analysis of aerospace systems, specifically jet engines.

4.2 Loss thrust work and the principle of lost work

Work has been done in recent years to formalize the relationship between engine performance and irreversibility. This work has firmly established the following useful principle:

In evaluating losses in an actual engine, the base-line from which performance losses due to irreversibilities occurring in the flow-path MUST be measured is the performance of the same engine with identical energy/mass interactions but with all processes reversible.

When this principle of loss evaluation is used in thrust-based optimization, optimal engine performance inevitably results. Note that in this statement there is no comparative measurement, either direct or implied, of performance from a non-vehicle based 'dead state'. The lost thrust work due to irreversibilities is correctly measured between the actual engine and the same engine except with all processes reversible. Furthermore, when this term is minimized, a thrust-optimized engine is guaranteed.

The principle stated above is entirely consistent with the original intent of general availability analysis in terms of lost thermodynamic work potential and not necessarily just lost thrust work in an engine (see Haywood (1974) and Lewis (1976)). It simply describes the correct reference (the reversible device) from which work potential lost due to irreversibility should be measured for the actual device. Hence the same principle is restated from a broader thermodynamic viewpoint:

In evaluating losses in an actual device (sub-system or overall system), the base-line from which lost work potential due to irreversibilities occurring in the device MUST be measured is the same device with identical energy/mass interactions but with all processes reversible.

This principle of lost work should then provide the correct 'common currency' for aerospace systems and sub-systems. Consistently applied, it enables the true optimization of an overall system composed of individual sub-systems. Each sub-system is continually evaluated within the optimization process based on this common principle. The minimization of the summation of all lost work potential across all sub-systems is then the goal of the optimization strategy.

4.3 Analysis of lost thrust in jet engines

Consider the flow at the exit plane of an actual jet engine with known fluid dynamic property and internal irreversibility distributions within the flow-path (i.e. the engine flow-path is sufficiently modeled or determined). For simplicity, assume calorically perfect gas and

quasi-one-dimensional steady flow throughout the engine flow-field from inlet plane i to exit plane e . There are specified externally provided heat and work interactions inside the engine (i.e. energy as heat is added across the boundary in the 'combustor' in a Rayleigh-type process). These given assumptions simplify the following analysis since there is no mass addition through the engine and species variation is not allowed. However, it is important to note that the following analysis remains entirely valid for complex engine flow-fields with three-dimensionality, fuel addition, fuel-air mixing and reaction, etc.

For this actual engine, the net axial force on the internal wetted surfaces of the engine from i to e is known (here this is termed the thrust) and is equal to

$$F_{\text{eng}} = \dot{m}(u_e - u_i) + P_e A_e - P_i A_i \quad (13)$$

It is useful to ask the following question: what would be the net axial force on the internal wetted surfaces of the same engine from i to e if all processes inside the engine were reversible (i.e. there were no irreversibility within the flow-path) This reversible engine thrust would be written as

$$F_{\text{eng-rev}} = [\dot{m}(u_{er} - u_i) + P_{er} A_e - P_i A_i] \quad (14)$$

where P_{er} and u_{er} represent the static pressure and velocity at the exit plane of the reversible engine. This reversible engine is in fact the $s_{\text{irr}} = 0$ performance base-line in equation (8); hence it corresponds to a point on the zero-irreversibility plane in Figure 3.

A competent propulsion engineer would have no difficulty computing this reversible engine thrust directly with given heat/work inputs – such a computation simply entails modeling isentropic ducts, adding the scheduled heat reversibly (corresponding to zero Mach number heat addition at the local total temperature of the fluid) and enforcing isentropic work interactions as required in order to generate the fluid dynamics – and hence the thrust – representative of the reversible engine. Note that this requires the analytical assumption of a 'flexible engine' in terms of required internal area ratios between inlet and exit for the reversible engine (i.e. in order to slow the flow isentropically to zero velocity in order to add the heat reversibly at the total temperature of the flow followed by an isentropic expansion back to the original area). This, however, is routine in ideal cycle analysis and is not an issue in terms of the performance characterization of the ideal (reversible) flow-path.

Certainly $F_{\text{eng-rev}}$ is the limiting (maximum) thrust performance theoretically available to the engineer for this engine operating with the given constraints (given area ratio of exit to inlet, given externally provided heat/work interactions and distributions, etc.) The thrust lost due to irreversibilities occurring in the flow-path of the engine is hence simply the following:

$$F_{\text{lost}} = F_{\text{rev-eng}} - F_{\text{eng}} \quad (15)$$

When this lost thrust is minimized as much as possible, the given engine is thrust-optimized.

A critical question remains however: what exactly is the formal relationship between the irreversible entropy generation in the engine flow-path and this lost thrust? This question is answered by considering essentially the same problem posed above for calculating the difference between the actual engine thrust and the reversible engine thrust. However, in order to correctly relate lost thrust to irreversibility, the lost thrust between the actual engine and a differentially more reversible engine must be calculated in terms of entropy generation. In other words, consider the exit plane of the actual engine. Since the entropy generation due to irreversibilities is considered known in detail throughout the engine flow-field, it is possible to 're-compute' the exit flow-field of the engine with all upstream irreversibilities unchanged however with the fluid dynamics over the last differential step in the engine (before the exit) computed assuming all processes in that single final step reversible. Again, as discussed before for the entire reversible engine, reversible heat addition across a differential step implies the process of an isentropic compression to zero Mach number, reversible heat addition at the local total temperature of the flow, and followed by isentropic expansion to the original area. Using this methodology, it is possible to directly relate the differential lost thrust work per unit mass to the differential entropy increase associated with irreversibility in that last step, δs_{irr} .

To show this relationship, note that the change in the resultant axial force between the actual engine and the differentially more reversible engine (change in realized thrust) is by definition

$$dF = \dot{m} du + A dP \quad (16)$$

The energy and entropy balances can be written in terms of the differential changes between the two exit planes such that

$$C_p dT + u du = 0 \quad (17)$$

$$C_p dT - \frac{dP}{\rho} = T \delta s_{\text{irr}} \quad (18)$$

Note that the exit areas of the two engine flow-fields are the same while du , dP and dT are the differential changes in exit fluid velocity, static pressure, and static temperature between the two flow-fields.

These relationships can then be manipulated to yield the following expression for the differential lost thrust work per unit mass:

$$\frac{udF}{\dot{m}} = T\delta s_{irr} \quad (19)$$

where T and u are (within the differential approximation) the averaged exit temperature and velocity between the actual engine flow-field and the differentially more reversible engine flow-field. δs_{irr} is the differential increase in entropy at given axial stations throughout the engine due to all internal irreversible mechanisms.

This same process of measuring lost thrust work at the exit plane is then repeated but now between the differentially more reversible engine flow-field as the base-line and the same engine flow-field with the next upstream differential step taken reversibly with the appropriate δs_{irr} , T and u being used. This represents in some sense a loss-stripping (loss-deconstruction) process across the actual engine flow-field, moving from engine exit to entrance. (It is also possible to reverse the philosophy and move from reversible engine to actual engine via a loss-construction process from entrance to exit. The result is, of course, identical.)

In any event, the lost differential thrust increments are then summed, i.e. integrated, along the defined path line linking the actual and reversible engines such that

$$F_{lost} = \dot{m} \int_{er}^e \frac{T\delta s_{irr}}{u} \quad (20)$$

It must be emphasized that the integration path represents the path necessary for consistent fluid/thermodynamic recovery of lost thrust work between the actual engine and the reversible engine. The integration path for lost thrust work recovery is illustrated in Figure 13 on a temperature-entropy diagram.

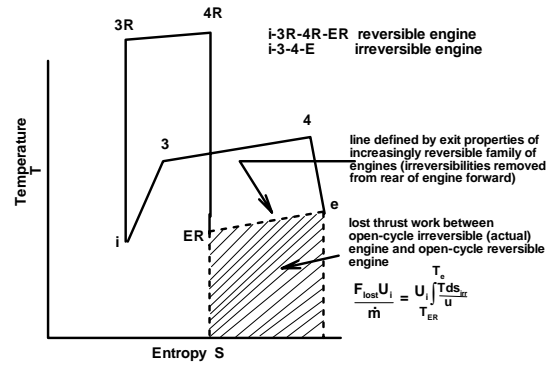


Figure 13. Schematic of temperature-entropy diagram showing lost thrust work integration path between the actual and reversible engines.

4.4 Lost thermodynamic work in an aerospace jet engine

A similar procedure as outlined above can be described for the (more general) engine-specific lost work, w_{lost} , (not the lost thrust work) between the actual engine and the reversible engine. For this quantity, the same integration path is specified as for the lost thrust work since the same methodology of recovering losses is followed, namely,

$$w_{lost} = \int_{er}^e T\delta s_{irr} \quad (21)$$

Two important points should be made at this time: 1) for relatively simple engine-only flow-field optimization problems in which thrust-based optimization strategies yield absolute optimal engine configurations, optimization strategies using the engine-specific lost work yield identically optimized configurations and 2) the concept (philosophy) of engine-specific lost work can be extended to other sub-systems such that there is indeed a single (common) thermodynamic currency across an entire aerospace system which works as well on a single component as it does on the system. (Recall that conventional flow availability analysis does not provide optimal engine designs even in very simple problems.) This philosophy of minimizing the summation of the various lost works for various sub-systems within a large system is, in fact, familiar. The difference between this philosophy and conventional flow availability analysis is that here the sub-system lost work terms are measured from the standpoint of the actual-to-reversible performance of the sub-systems (rather than referenced to an external dead state without due consideration of device functionality). Hence minimizing their collective losses yields true optimization of the overall vehicle within the context of the specific function and form of that vehicle.

5. Summary

Section 2 of this paper provides the development of the expressions for jet engine specific thrust and specific impulse in terms of fundamental thermodynamic quantities, including the irreversibility occurring within the actual engine flow-field. The third section then explores the continuum of jet engine performance established from this basic analysis. The continuum of performance provides a powerful tool for understanding, optimizing, and assessing engine types, regimes, and performance issues from a thrust-based performance perspective. It also provides the natural base-line for measuring the impact of irreversibility on engine performance.

The fourth section of this work provides a very simple fluids problem in which conventional flow availability as suggested in numerous references fails to yield the optimized configuration. The conclusion is drawn that conventional flow availability analysis needs to be revisited, at least in terms of the functionality or purpose of the vehicle and how an availability analysis should be applied to the optimization of the vehicle. Also noted is that, for consistency, any proposed candidate 'thermodynamic currency' for vehicle optimization must be robust in terms of also achieving optimization for isolated sub-systems with given constraints. This section continues by reviewing the meaning and evaluation of lost thrust work from fundamental thermodynamic principals and then argues for an extension of the concept to the broader principle of the minimization of lost work between actual and reversible devices in system optimization efforts. This argument is based on the successful linkage established between engine performance and lost thermodynamic work for engine-only applications. It is also in the original spirit of availability. Several important principles regarding loss evaluation and optimization of sub-systems and systems are stated.

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Nomenclature

A cross-sectional area (m²)
a speed of sound (m/s)

C_p specific heat at constant pressure (1005 J/kgK for air)
F thrust (N): axial force developed on internal surfaces of stream-tube
f fuel/air mass ratio
I_{sp} engine specific impulse (s): thrust per unit weight flow rate of fuel
M Mach number of fluid
R gas constant (287 J/kgK for air)
P pressure of fluid (N/m²)
P_t fluid total pressure $\left[P_t = P \left(1 + \frac{\gamma - 1}{2} M^2 \right)^{\frac{\gamma}{\gamma - 1}} \right]$
Q overall heat interaction per unit mass crossing engine boundary in combustor (positive to flow) (J/kg)
q heat interaction per unit mass (J/kg)
s entropy per unit mass of fluid (J/kgK)
s_{irr} entropy per unit mass generated by irreversibilities across entire engine flow-field (i to e)
T temperature of fluid (K)
T_t fluid total temperature $\left[T_t = T \left(1 + \frac{\gamma - 1}{2} M^2 \right) \right]$
U,u velocity of fluid (m/s)
W,W_{up} overall work interaction per unit mass crossing engine boundary in upstream work-interaction component (positive to flow) (J/kg)
w work interaction per unit mass (J/kg) (positive to flow)
ΔEx change in flow availability per unit mass (measured from a thermodynamic dead state) between two stations in a flow-field (J/kg)
γ ratio of specific heats (1.4 for air)
ρ density of fluid (kg/m³)

Subscripts

bound designates quantity crossing fluid boundary
e engine exit station
g₀ gravitational acceleration (m/s²)
i engine entrance station (conditions at i,0 assumed same in this work)
r reversible engine
ṁ engine/stream-tube mass flow rate (kg/s)
0 free-stream station
4 combustor exit station
η work interaction second-law effectiveness

τ_λ combustor exit total temperature/ambient temperature (ratio) (positive to flow)

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