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Thermoeconomic Optimization and Parametric Study of an Irreversible Ericsson Heat Engine Cycle

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Abstract

The thermoeconomic study of an irreversible Ericsson heat engine with finite heat capacities of the external reservoirs is presented in this paper. The external irreversibilities are due to finite temperature differences between the heat engine and external reservoirs while the internal irreversibility is due to the regenerative heat loss. The thermoeconomic function is defined as the power output divided by the total annual cost of the system. The thermoeconomic function is optimized with respect to the working fluid temperatures and at the optimal operating condition the values for various performance parameters are calculated. The effects of different operating parameters on the performance of the cycle have been studied. It is found that the effects of regenerative-side effectiveness are more than those of the other cycle parameters, not only on the objective function but also on the corresponding power output and thermal efficiency and can also be explained in terms of internal and external irreversibilities associated with the cycle for the same set of operating conditions. It is also found that the effects of the source- and sink-side parameters are nearly equal on all the performance parameters.

Keywords: Thermoeconomic function, irreversible Ericsson cycle, optimal operating condition, power output, thermal efficiency.

1. Introduction

Ericsson and Stirling engines have attracted the attention of several generations of engineers and physicists due to their potential to provide high conversion efficiency and utilize various types of working fluids. However, use of these engines has not proven to be successful due to the relatively poor material technology available at that time. As the world community has become much more environmentally conscious, further attention to these engines has again been revived because these engines are inherently clean. Moreover, as a result of advances in material technology, these engines are currently being considered for a variety of applications due to their many advantages such as low noise, less pollution and their flexibility to utilize a variety of fuels (Blank and Wu, 1995).

In recent years, a lot of work has been carried out on the cycles (Blank and Wu, 1995, Chen, 1997, Chen and Schouten, 1999, Kaushik, 1999, Tyagi, 2000, Kaushik and Kumar, 2000, Kaushik and Kumar, 2001 Kaushik, Tyagi, and Mohan, 2003,) using the concept of finite-time thermodynamics (Curzon and Ahlborn, 1975, Salamon and Nitzan, 1981). Some workers have applied the ecological criteria (He, Chen and Wu, 2001, Tyagi, Kaushik and Salhotra, 2002),

*Author to whom correspondence should be addressed while others have used the thermoeconomic approach (Sahin, and Kodal, 1999, Kodal, Sahin and Yilmaz, 2000, Sahin and Kodal, 2001, Antar and Zubair, 2001, Bandyopadhyay, Bera and Bhattacharyya, 2001, Kodal, Sahin and Erdil, 2002, Kodal, Sahin, Ekmekci and Yilmaz, 2003, Chen, Tyagi and Wu, 2003, Tyagi, Chen and Kaushik, 2004) based on energy analysis (Mirandola, Stoppato and Tonon, 2000) and exergy analysis (Moorhouse, Hoke and Prendergast, 2002) on different cycles for a typical set of operating conditions.

In this paper, we will discuss the effects of both the internal and the external irreversibilities on the maximum thermoeconomic function and on the corresponding power output and thermal efficiency of an irreversible Ericsson heat engine.

2. System Description

It is well known that the working substance of an Ericsson cycle may be a gas or magnetic material, and for different substances these cycles have different performance characteristics. When the working substance is an ideal gas, the Ericsson cycle consists of two isothermal and two isobaric processes as shown in *Figure 1* along with the T-S diagram. This cycle approximates the compression stroke of a real engine as the isothermal process 1-2 with an irreversible heat rejection at constant temperature (T_c) to a heat sink of finite heat capacity whose temperature varies from T_{L1} to T_{L2}. The heat addition to the working fluid during the regeneration is modeled as the isobaric process 2-3. The work producing expansion stroke is modeled as the isothermal process 3-4 with an irreversible heat addition at constant temperature T_h from a heat source of finite heat capacity whose temperature varies from T_{H1} to T_{H2}. Finally the heat rejection during the regeneration is modeled as the isobaric process 4-1, thereby, completing the cycle.

As mentioned earlier, the external heat transfer processes 1-2 and 3-4 in a real cycle must be carried out in finite time. This in turn requires that these heat transfer processes must occur through a finite temperature difference and are, therefore, externally irreversible. Similarly, there is some net heat loss per cycle through the regenerator as an ideal regenerator requires an infinite regeneration time or area, which is not the case in practice. Hence, it will be difficult to obtain correct results in the investigation of an Ericsson cycle, if the regenerative losses are not considered in the analysis. Thus, it is desirable to consider a real regenerator rather than an ideal one.

3. Thermodynamic Analysis

Let Q_h be the amount of heat absorbed from the heat source at temperature T_h and Q_c be the amount of heat released to the heat sink at temperature T_c during the two isothermal processes. They are given by

$$Q_h = U_H A_H (LMTD)_H t_H = T_h \Delta S = C_H (T_{H1} - T_{H2}) t_H \quad (1)$$

$$Q_c = U_L A_L (LMTD)_L t_L = T_c \Delta S = C_L (T_{L2} - T_{L1}) t_L \quad (2)$$

where

$$\Delta S = nR_0 \ln\lambda \quad (3)$$

and n is the number of moles of the working fluid, R_0 the universal gas constant, and λ the pressure ratio of the cycle. C_H and C_L are the heat capacitance rates of the source and sink reservoirs and t_H and t_L are the heat addition and rejection times. $U_H A_H$ and $U_L A_L$ are the heat transfer coefficient-area products and $(LMTD)_H$ and $(LMTD)_L$ are the Log Mean Temperature Differences on the source- and sink-sides, respectively. Solving equations (1) and (2), we obtain

$$Q_h = C_H \varepsilon_H (T_{H1} - T_h) \quad (4)$$

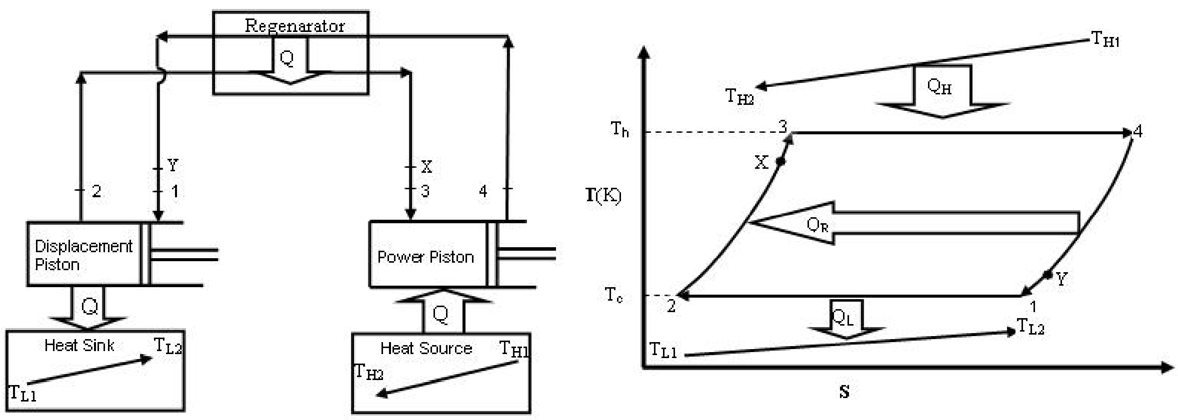

Figure 1. Schematic and T-S diagrams of an irreversible Ericsson heat engine.

$$Q_c = C_L\, \varepsilon_L\, (T_c - T_{L1})\, t_L \tag{5}$$

where

$$\varepsilon_H = 1 - \exp(-U_H A_H / C_H) \tag{6}$$

$$\varepsilon_L = 1 - \exp(-U_L A_L / C_L) \tag{7}$$

These latter two equations are for the effectiveness of the hot- and cold-side heat exchangers, respectively.

When the irreversibility of heat transfer is considered, these cycles, in general, do not possess the condition of perfect regeneration. It is reasonable to assume that the regenerative loss per cycle is proportional to the temperature difference of the two isothermal processes and is given by (Chen and Schouten, 1999, Kaushik, 1999, Tyagi, 2000, Kaushik and Kumar, 2000, Kaushik and Kumar, 2001, Kaushik, Tyagi and Mohan, 2003,.He, Chen and Wu, 2001, Tyagi, Kaushik and Salhotra, 2002),

$$\Delta Q_R = n\, C_f\, (1 - \varepsilon_R)(T_h - T_c) \tag{8}$$

where c_f is the molar specific heat of the working fluid and ε_R the effectiveness of the regenerator defined as

$$\varepsilon_R = \frac{Q_{regen,actual}}{Q_{regen,ideal}} = \frac{Q_{4Y}}{Q_{41}} = \frac{Q_{2X}}{Q_{23}} = \frac{T_h - T_Y}{T_h - T_c} = \frac{T_X - T_c}{T_h - T_c} = \frac{N_R}{1 + N_R} \tag{9}$$

where N_R is the number of heat-transfer units of the regenerator defined by

$$N_R = \frac{(UA)_R}{C_f} \tag{10}$$

Here C_f is the heat capacitance rate of the working fluid**.** As the regenerator is not the ideal one, there is some heat loss through the regenerator per cycle as mentioned in equation (8). The amount of heat ΔQ_R, is taken from the heat source and rejected to the heat sink, during the processes X–3 and Y–1, respectively, without any useful output.

When the regenerative irreversibility mentioned above is taken into account, the net amount of heat absorbed from the heat source and released to the heat sink is given by

$$Q_H = Q_h + \Delta Q_R \tag{11}$$

$$Q_L = Q_c + \Delta Q_R \tag{12}$$

Owing to the influence of the irreversibility of finite-rate heat transfer, the regenerative time should be finite and can be compared to that of the two isothermal processes. There are several ways to express the regenerative time. Among these, one is assumed to be proportional to the temperature difference between the hot- and cold-side of the regenerator (Chen and Schouten, 1999, Kaushik, 1999, Tyagi, 2000, Kaushik and Kumar, 2000, Kaushik and Kumar, 2001, Kaushik, Tyagi and Mohan, 2003, He, Chen and Wu, 2001, Tyagi, Kaushik and Salhotra, 2002), and it is defined as;

$$t_R = t_3 + t_4 = 2\,\alpha\,(T_h - T_c) \tag{13}$$

where α is the proportionality constant, independent of the temperatures of the two sides but dependent on the property of the regenerative materials, and t_3 and t_4 are the times taken during processes 2-3 and 4-1, respectively. The total cycle time is given approximately by $t_{cycle} = t_h + t_c + t_R$. It should be pointed out that the heat addition time from state X to 3 and the heat rejection time from state Y to 1 are usually very small compared with the time spent in other processes and may be neglected sometimes for the sake of simplification.

Using equations (1) and (2) and entropy and energy balances, we have that

$$\frac{Q_h}{T_h} - \frac{Q_c}{T_c} = 0 \tag{14}$$

$$W = Q_H - Q_L = Q_h - Q_c \tag{15}$$

Thus, the power output and the corresponding thermal efficiency will be:

$$P = \frac{W}{t_{cycle}} = \frac{(Q_h - Q_c)}{t_H + t_L + t_R} = \frac{T_h \Delta S - T_c \Delta S}{\left[T_h \Delta S / \varepsilon_H C_H (T_{H1} - T_h) + T_c \Delta S / \varepsilon_L C_L (T_c - T_{L1}) + 2\alpha (T_h - T_c)\right]} = \frac{(x-1)}{\left[x / k_H (T_{H1} - xy) + 1/k_L (y - T_{L1}) + a_1 (x-1)\right]} \tag{19}$$

$$\eta = \frac{W}{Q_H} = \frac{(Q_h - Q_c)}{(Q_h + \Delta Q_R)} = \frac{T_h \Delta S - T_c \Delta S}{T_h \Delta S + n c_f (1 - \varepsilon_R)(T_h - T_c)} = \frac{(x-1)}{x + a_2 (x-1)} \tag{17}$$

where $x = T_h / T_c$, $y = T_c$, $k_H = C_H \varepsilon_H$, $k_L = C_L \varepsilon_L$, $a_1 = 2\,\alpha / \Delta S$ and $a_2 = n\, C_f\, (1 - \varepsilon_R) / \Delta S$.

The objective function of thermoeconomic optimization as proposed by earlier workers (Salamon and Nitzan, 1981, Sahin, and Kodal, 1999, Kodal, Sahin and Yilmaz, 2000, Sahin and Kodal, 2001, Antar and Zubair, 2001, Bandyopadhyay, Bera and Bhattacharyya, 2001, Kodal, Sahin and Erdil, 2002, Kodal, Sahin, Ekmekci and Yilmaz, 2003, Chen, Tyagi and Wu, 2003, Tyagi, Chen and Kaushik, 2004) is as follows:

$$F=\frac{P}{C_i+C_e+C_m} \tag{18}$$

where C_i, C_e and C_m refer to the annual investment, energy consumption and maintenance costs, respectively. The investment cost was considered to be the costs of the main system components, the heat exchangers and the compression and expansion devices together. The investment cost of the heat exchangers is assumed to be proportional to the total heat exchanger/transfer area (Salamon and Nitzan, 1981, Sahin, and Kodal, 1999, Kodal, Sahin and Yilmaz, 2000, Sahin and Kodal, 2001, Antar and Zubair, 2001, Bandyopadhyay, Bera and Bhattacharyya, 2001, Kodal, Sahin and Erdil, 2002, Kodal, Sahin, Ekmekci and Yilmaz, 2003, Chen, Tyagi and Wu, 2003, Tyagi, Chen and Kaushik, 2004). On the other hand, the investment cost of the compression and expansion devices is assumed to be proportional to their compression/ expansion capacities or the power output of the cycle (Sahin and Kodal, 1999, Kodal, Sahin and Yilmaz, 2000, Sahin and Kodal, 2001, Chen, Tyagi and Wu, 2003, Tyagi, Chen and Kaushik, 2004). Thus, the investment cost of the system can be written as;

$$\begin{aligned} C_i &= a\,(A_H+A_L+A_R)+b_1P \\ &= a\,(A_H+A_L+A_R)+b_1(Q_h-Q_c)/t_{cycle} \end{aligned} \tag{19}$$

where the proportionality constant for the investment cost of the heat exchangers, a, is equal to the annual cost per unit heat exchanger area and its dimension is ncu/(year-m^2), and the proportionality constant for the investment cost for the compression and expansion devices, b_1, is equal to the annual cost per unit power output and its dimension is ncu/(year-kW). The unit ncu stands for the National Currency Unit. The average energy consumption and maintenance costs are, respectively, proportional to the energy input rate and power output, i.e.

$$C_e=b_2\dot{Q}_H=b_2(Q_h+\Delta Q_R)/t_{cycle} \tag{20}$$

$$C_m=b_3P=b_3(Q_h-Q_c)/t_{cycle} \tag{21}$$

where the coefficient b_2 is equal to the annual cost per unit energy input rate, the coefficient b_3 is equal to the annual cost per unit power output and the dimension of both the parameters is ncu/(year-kW). Substituting equations (19) - (21) into equation (18), we have that

$$F=\frac{(Q_h-Q_c)}{t_{cycle}}\left[a\left(A_H+A_L+A_R\right)+\frac{b(Q_h-Q_c)}{t_{cycle}}+\frac{b_2(Q_h+\Delta Q_R)}{t_{cycle}}\right]^{-1} \tag{22}$$

where $b=b_1+b_3$. Thus, from equations (4), (5), (8), (19) - (21), we have

$$bF=(x-1)\left\{k_1k_2\left[\frac{x}{k_H\,(T_{H1}-x\,y)}+\frac{1}{k_L(y-T_{L1})}+a_1(x-1)\right]+(1+a_2b_4)(x-1)+b_4x\right\}^{-1} \tag{23}$$

or

$$bF=(x-1)\left\{k_1k_2\left[\frac{x}{k_H\,(T_{H1}-x\,y)}+\frac{1}{k_L(y-T_{L1})}+a_3(x-1)+b_4x\right]\right\}^{-1} \tag{24}$$

where $k_1=a/b$,

$$k_2=\left(A_R+A_L+A_H\right)=\left(\frac{C_f\varepsilon_R}{U_R(1-\varepsilon_R)}-\frac{C_H}{U_H}\ln(1-\varepsilon_H)-\frac{C_L}{U_L}\ln(1-\varepsilon_L)\right),$$

and $b_4=b_2/b$ and $a_3=1+k_1k_2a_1+a_2b_4$.

It can be seen from equation (24) that bF is a function of two variables x and y (as other parameters are constant for a typical set of operating conditions). Thus, optimizing bF with respect to 'y' viz a vie $\partial bF/\partial y=0$ yields (please see the Appendix)

$$\begin{aligned} x(T_{L1}-y) &= \sqrt{k_H/k_L}\,(xy-T_{H1}) \\ y &= (xT_{L1}+k_3T_{H1})/x(1+k_3) \end{aligned} \tag{25}$$

where $k_3=\sqrt{k_H/k_L}$. Substituting equation (25) into equations (24) and (16) we have:

$$bF_{(x)}=\frac{(x-1)}{\left[x\,k_5/(T_{H1}-xT_{L1})+a_3(x-1)+b_4x\right]} \tag{26}$$

$$P_{(x)}=\frac{(x-1)}{xk_4/(T_{H1}-xT_{L1})+a_1(x-1)} \tag{27}$$

where $k_4=(1+k_3)\,(k_H^{-1}+k_L^{-1}k_3^{-1})$ and $k_5=k_1k_2k_4$.

4. Results and Discussion

In order to have a numerical appreciation of the results of a thermoeconomic optimization of an irreversible Ericsson heat engine, we continue to investigate the effects of the temperatures of the working substance, the effectiveness of the heat exchangers (ε_H, ε_L and ε_R), the economic parameters (k_1 and b_2), the overall heat transfer coefficients (U_H, U_L and U_R), and the heat capacitance rates (C_H and C_L). The effect of each one of these parameters is examined while the rest of the parameters are kept constant as ($\varepsilon_H = \varepsilon_L = \varepsilon_R = 0.75$, T_{H1}=1250K, T_{L1}=300K, k_1 = 0.50 kW/m^2, λ=2.5, C_H=C_L=1.0kW/K, α=0.001 ncu/(year–m^2), b_1 = 0.7ncu/(year–kW), b_2 = 0.5ncu/(year–kW), b_3=0.3ncu/(year–kW), γ=1.4, R_0=8.31kJ/(kmole-K), n = 0.01kmole, $U_H = U_L = U_R$=2.0 kW/ (m^{2}K). The results obtained appear in the following sections.

4.1 Effects of cycle temperature ratio (x)

The variation of the objective function, power output and thermal efficiency with respect to the cycle temperature ratio ($x=T_h/T_c$) for a typical set of operating parameters is shown in *Figure 2*. It is seen from *Figure 2* that the objective function and power output first increase and then decrease while the efficiency monotonically increases as the cycle temperature ratio (x) increases. These properties can be directly expounded by equations (26), (27) and (17), because the objective function and power output are not monotonic functions of *x* while the efficiency is a monotonically increasing function of *x*. It can also be clearly seen from the figure that both the objective function and the power output attain their maxima but at different values of *x* and there exists the following relation

$$(x_{opt})_P \le (x_{opt})_{bF} \tag{28}$$

where $(x_{opt})_{bF}$ and $(x_{opt})_P$ represent the two different optimal values of x, the former corresponding to the point of the maximum value of the objective function while the latter to the maximum value of the power output. It is seen from equations (26) and (27) that both the performance parameters, i.e. bF and P, are functions of a single variable, x, for a typical set of operating conditions. Thus, maximizing bF and P with respect to *x* yields

$$(x_{opt})_{bF} = \frac{B - \sqrt{(B^2 - AC)}}{A} \tag{29}$$

$$(x_{opt})_P = \sqrt{T_{H1}/T_{L1}} \tag{30}$$

where $A=b_4T_{L1}^2 - k_5T_{L1}$, $B=b_4T_{H1}T_{L1}$ and $C=b_4T_{H1}^2 - k_5T_{H1}$. Substituting the values of $(x_{opt})_{bF}$ into equations (26), (27) and (17), we can calculate the maximum value of the objective function and the corresponding power output and thermal efficiency, while the maximum power output and the corresponding thermal efficiency can be calculated by substituting $(x_{opt})_P$ into equations (27) and (17). On the other hand, the optimal values of y, for both the cases, i.e. $(y_{opt})_{bF}$ and $(y_{opt})_P$, can be calculated by substituting the values $(x_{opt})_{bF}$ and $(x_{opt})_P$ separately into equation (25) for a typical set of operating conditions.

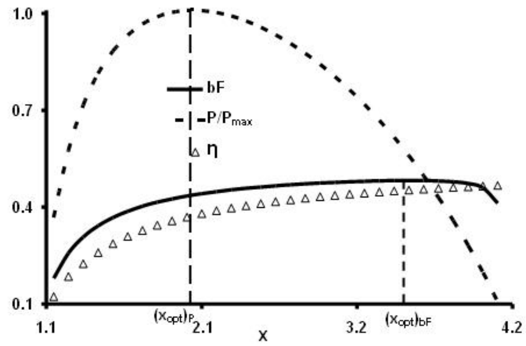

Figure 2. Effects of temperature ratio on different cycle parameters with $\varepsilon_H=\varepsilon_L=\varepsilon_R=0.75$, T_{H1} =1250K, T_{L1}=300K, k_1=0.50, λ=2.5, C_H = C_L =1.0kW/K, α =0.001s/K, b_1=0.7ncu/(year-kW), b_2=0.5ncu/(year-kW), b_3=0.3ncu/(year-kW), γ=1.4, R_0=8.31kJ/(kmole-K), n=0.01 kmole, $U_H=U_L=U_R$=2.0kW/m^{2}K.

4.2 Effect of overall heat transfer coefficients

Figures 3a-c show the effects of the overall heat transfer coefficients (U_H, U_L and U_R) on the maximum value of the objective function and the corresponding power output and thermal efficiency. It may be shown by numerical calculation that the larger the overall heat transfer coefficients are, the larger the $(x_{opt})_{bF}$ and the smaller the heat-transfer irreversibility between the cycle and the external reservoirs. Thus, the maximum value of the objective function and the corresponding thermal efficiency increase while the power output decreases as the overall heat transfer coefficient on any heat exchanger increases. The properties shown in *Figures 3a-c* are identical to those given in *Figure 2* for a typical region, resulting in a decrease in the power output and an increase in the maximum value of the objective function and the corresponding thermal efficiency. Again, this is because the internal irreversibility is more effective than the external irreversibility, not only from the point of view of thermodynamics but also from the point of view of economics. Since, the irreversibility associated with the regenerator is an internal one while the irreversibility associated with the hot- and cold-side heat exchangers is an external one, the effects of the regenerative-side overall heat transfer coefficient are more than those of the hot- and cold-side overall heat transfer coefficients on all the performance parameters for the same set of operating conditions. Also the effects of the hot- and cold-side overall heat transfer coefficients are almost the same for all performance parameters, so the two curves overlap.

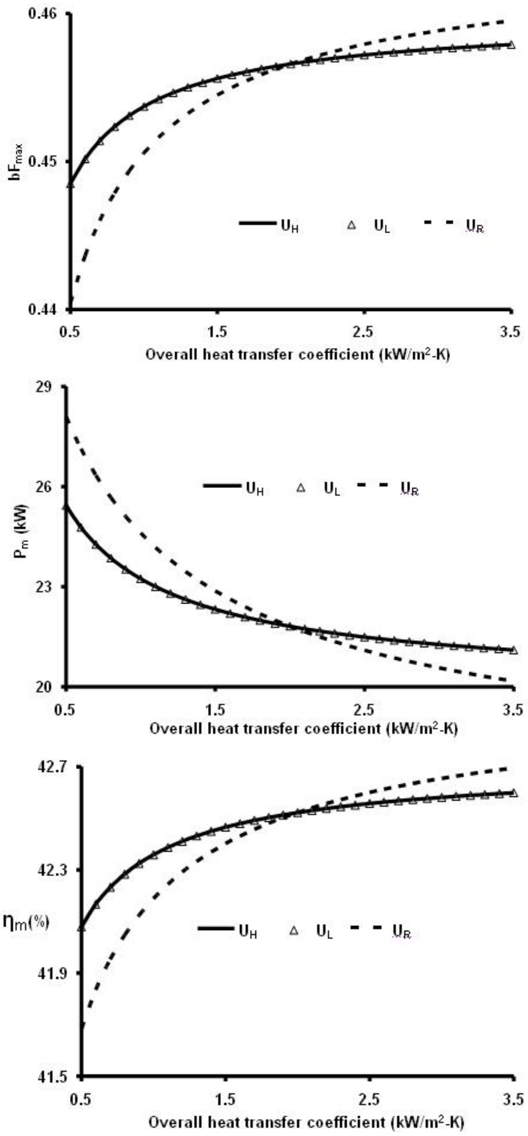

Figure 3. Effects of various overall heat transfer coefficients on (a) the maximum objective function, (b) the corresponding power output, and (c) the thermal efficiency, with other parameter values the same as those in Figure 2.

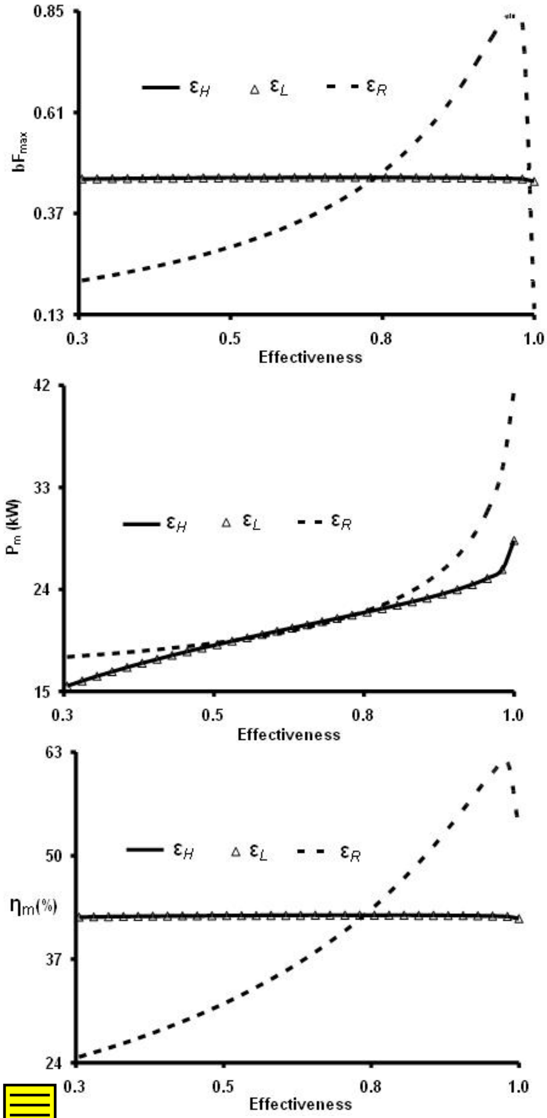

Figure 4. Effects of different effectiveness on (a) the maximum objective function, (b) the corresponding power output, and (c) the thermal efficiency, with other parameter values the same as those in Figure 2.

4.4 Effect of economic parameters

The effects of economic parameters (k_1 and b_2) on the maximum value of the objective function and the corresponding power output and thermal efficiency are shown in *Figures 5a-c*. It is seen from these figures that the maximum value of the objective function decreases as either parameter k_1 or b_2 increases. On the other hand, the corresponding thermal efficiency decreases and the power output increases as k_1 increases while the reverse is true for the case of b_2. Since the cost of the system increases by increasing either parameter, the maximum value of the objective function decreases with increasing economic parameters. However, the optimal value of the cycle temperature ratio $(x_{opt})_{bF}$ decreases with increasing k_1, resulting in an increase in the power output and a decrease in the thermal efficiency. On the other hand, the optimal value of the cycle temperature ratio $(x_{opt})_{bF}$ increases with increasing b_2, resulting in a decrease in the power output and an increase in the thermal efficiency for the same set of operating parameters. It is found that the effect of the input heat cost is more than that of the size cost of the system, as the former affects the overall performance of the system not only from the point of view of economics but also from the point of view of thermodynamics. Thus, it is found that the effect of b_2 is more than that of k_1, on all the performance parameters, i.e. on the maximum value of the objective function as well as on the corresponding power output and thermal efficiency for the same operating conditions as can be seen from these figures.

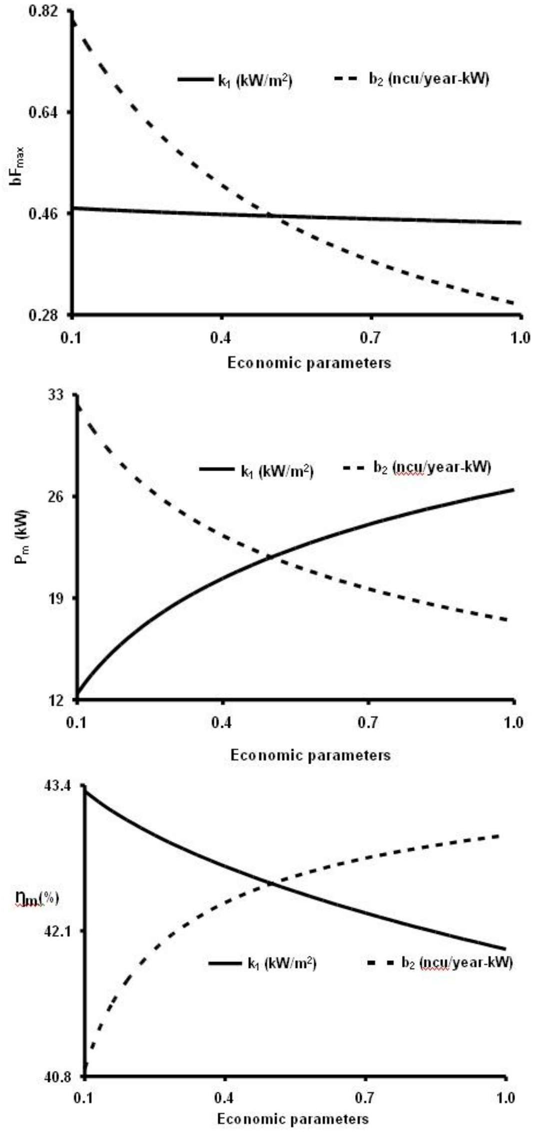

Figure 5. Effects of economic parameters (k_1 and b_2) on (a) the maximum objective function, (b) the corresponding power output, and (c) the thermal efficiency, with other parameter values the same as those in Figure 2.

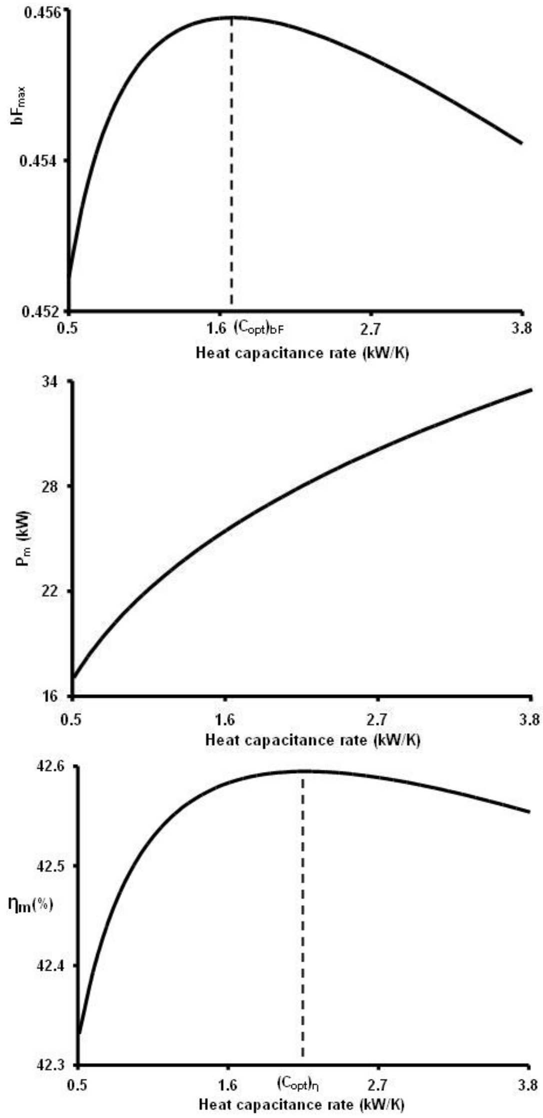

Figure 6. Effects of heat capacitance rate (C_H=C_L=C) on (a) the maximum objective function, (b) the corresponding power output, and (c) the thermal efficiency, with other parameter values the same as those in Figure 2.

4.5 Effect of heat capacitance rates

The effects of source/sink-side heat capacitance rates on the maximum value of the objective function and the corresponding power output and thermal efficiency are shown in Figures 6a-c. It is seen from these figures that the maximum value of the objective function as well as that for the corresponding thermal efficiency first increase and then decrease as the heat capacitance rate on the source/sink-side reservoir increases. On the other hand, the corresponding power output is found to be a monotonically increasing function of the heat capacitance rate on the source/sink-side. These results can be explained as follows. For the typical set of operating parameters given above, it implies that the larger the heat capacitance rates are, the larger the heat transfer areas required for a given effectiveness. This results in a larger value of the power output but at a higher cost of the system. Since the maximum objective function is not a linear function of the corresponding power output and the maximum objective function and its corresponding power output are functions not only of the heat capacitance rates but also of $(x_{opt})_{bF}$, which is also a function of the heat capacitance rate, there is a maximum value of the maximum objective function. On the other hand, the corresponding thermal efficiency is a monotonic function of $(x_{opt})_{bF}$, but $(x_{opt})_{bF}$ is not a linear function of the heat capacitance rate. It is found that the value of $(x_{opt})_{bF}$ first increases and then decreases as the heat capacitance rate on either side reservoir increases. As a result, there exists a maximum of the corresponding thermal efficiency.

6. Conclusions

A more realistic Ericsson heat engine cycle model including external and internal irreversibilities for the finite heat capacities of external reservoirs has been studied in detail. The thermoeconomic function is adopted as the objective function for maximization. The objective function is maximized with respect to the cycle temperatures. The corresponding power output and thermal efficiency are evaluated for different operating conditions. The maximum thermoeconomic function is found to be an increasing function of the overall heat-transfer coefficients while it is found to be a decreasing function of the economic parameters. On the other hand, it is found that there are optimal values of the cycle temperature ratio (x) and source/sink-side heat capacitance rates at which the objective function attains its maximum value for a typical set of operating condition. It is also found that the effects of the regenerative-side overall heat transfer coefficient and effectiveness are more than those of the other sides' overall heat transfer coefficients and effectiveness and the effect of the economic parameter (b_2) is more than that of k_1 for the same set of operating parameters. Again, the effects of the hot- and cold-side heat capacitance rates, overall heat transfer coefficients and effectiveness on all the performance parameters are found to be the same, hence the curves related to hot- and cold-side parameters overlap as can be seen from the figures given in this paper. Thus, the present cycle model gives some optimal results which will be useful for understanding the performance of a real cycle from the point of view of thermodynamics as well as from the point of view of economics. The results obtained here are also applicable to the Stirling cycle (Chen and Schouten, 1999, Kaushik, Tyagi and Mohan, 2003, Chen, Tyagi and Wu, 2003) in which the only difference is that the two isobaric processes will be replaced by two isochoric processes and the pressure ratio will be replaced by the volume ratio and hence, the specific heat at constant pressure will be replaced by the specific heat at constant volume.

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Nomenclature

A	Area (m^2)
a	Annual cost per unit heat exchanger area (ncu/year-m^{-2})
b_1	Annual cost per unit power output (ncu/year-kW)
b_2	Annual cost per unit energy input rate (ncu/year-kW)
b_3	Annual cost per unit power output (ncu/year-kW)
bF	objective function
C	Heat capacitance rates (kW-K^{-1})
C_i	Annual investment costs (ncu/year-kW)
C_e	Annual energy consumption cost (ncu/year-kW)
C_m	Annual maintenance cost (ncu/year-kW)
k_1	a/b (kW/m^2)
C_f	Molar specific heat (kJ/kmol-K)
N_R	Number of heat-transfer units
ncu	National Currency Unit
P	Power output (kW)
Q	Heat (kJ)
R_0	Gas constant (kJ/kmol-K)
S	Entropy (kJ-K^{-1})
T	Temperature (K)
t	Time (s)
U	Overall heat transfer coefficient (kW/K-m^2)
W	Work output (kJ)
1, 2, 3, 4	State points
X, Y	State points

Subscripts

C	Cold/sink-side
f	Fluid
H, h	Heat source/hot-side
L	Heat sink
max	Maximum
m	Optimum
R	Regenerator

Greek

α	Proportionality constant
η	Efficiency
ε	Effectiveness
λ	Pressure ratio
γ	Specific heat ratio

Appendix

The derivation for equation (25) is given as below:

$$bF = (x-1)\left\{k_1 k_2\left[\frac{x}{k_H(T_{H1} - xy)} + \frac{1}{k_L(y - T_{L1})}\right] + a_3(x-1) + b_4 x\right\}^{-1} \quad (24)$$

Using equation (24) and its extremal viz a vie $\partial bF/\partial y = 0$ yields

$$\frac{x^2}{k_H(T_{H1} - xy)^2} - \frac{1}{k_L(y - T_{L1})^2} = 0$$

$$\Rightarrow x(y - T_{L1}) = \sqrt{\frac{k_H}{k_L}}(T_{H1} - xy)$$

$$xy - xT_{L1} = k_3 T_{H1} - k_3 xy$$

$$\Rightarrow y(1 + k_3)x = xT_{L1} + k_3 T_{H1}$$

which on solving results in

$$y = \frac{xT_{L1} + k_3 T_{H1}}{(1 + k_3)x} \quad (25)$$

where $k_3 = \sqrt{\frac{k_H}{k_L}}$

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