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Constructal Optimization of Top Contact Metallization of a Photovoltaic Solar Cell

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Abstract

A top contact metallization of a photovoltaic solar cell collects the current generated by incident solar radiation. Several power-loss mechanisms are associated with the current flow through the front contact grid. The design of the top metal contact grid is one of the most important areas of efficient photovoltaic solar cell design. In this paper, an approach based on the constructal theory is proposed to design the grid pattern in a photovoltaic solar cell, minimizing total resistive losses. Constructal theory explains the geometric form (shape and structure) of most volume-to-point systems in nature. In this paper, the applicability of the constructal theory to design top contact metallization for a photovoltaic solar cell has been extended.

Keywords: Photovoltaic cell, constructal theory, top contact metallization, optimization

1. Introduction

In the era where conventional energy resources found on earth are on a decline, the need of the hour is to look for alternatives. Tapping energy from the sun, which drives most organic processes and life on earth, seems to be an obvious choice. One of the alternative ways to utilize this non-conventional source of energy is to convert it, using solar cells, directly into electricity.

The mechanism of photovoltaic solar cells involves knocking off electrons from a semiconductor material like crystalline silicon and allowing these electrons to flow through the material. A front / top contact grid, a high conductivity material, allows passage of electrons from the place of generation and thus collects the generated photocurrent. Designing this grid pattern (fingers and bus-bar arrangement) is one of the important aspects of solar cell design. Various power losses are associated with the top contact design. Structural details of the grid elements are important considering losses due to shadowing of the cell by the grid lines. A lateral current flow in the top diffused layer causes flow through a higher resistance material, increasing losses (Green, 1982). Losses are also present due to the series resistance of the metal grids and the contact resistance between the semiconductor surface and grid lines (Green, 1982). The efficiency of a solar cell is ratio of electricity produced to the incident radiation. The aim is hence to constantly increase electricity obtained for a given solar radiation. Losses need to be reduced. An efficient grid design would then be one that minimizes such power losses. The problem is to design a grid pattern that efficiently collects photocurrent generated over the entire solar cell and delivers it to a point.

Various methods are proposed to design these grid lines for minimum resistive losses under various constraints. These include determining optimum grid spacing for a one-dimensional cell having uni-directional current flow, multi-layer grid patterns, constant current flux in grids etc. Different approaches are based on the factors that are considered as playing a role. Serezze (1978) has proposed simultaneous optimization of the grid lines and leading bar while neglecting power loss in the active layer and due to contact resistance. Ellis and Moss (1970) have considered shadowing and active layer resistive losses while neglecting contact and metallization resistances. Heizer and Chu (1976) have proposed a method to get optimum spacing between fingers by modeling an incremental area. Flat and Milnes (1980) illustrate quantitative results of the influence of the aspect ratio of grid lines on minimizing the

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power loss. They also predict the use of multi-layer grid structures as a significant step to reducing losses. In this paper an approach based on the constructal theory is proposed to design the grid pattern in a photovoltaic solar cell, minimizing total resistive losses and neglecting other losses.

Constructal theory, proposed by Prof. A. Bejan, explains the geometric form (shape and structure) of most of the volume-to-point and point-to-volume systems (Bejan, 2000). The constructal theory emerged with an attempt to optimally design conduction paths for cooling of electronic packages (Bejan, 1997). The objective in this case was to keep the maximum temperature at any point of a cooling package within certain limits. Constructal methodology is an analytic technique / procedure to predict optimized structures. Interestingly, the constructal optimization technique has found applications in varied fields – typically those involving volume-to-point or point-to-volume systems. The constructal theory finds application in thermal-fluid engineering, design of transportation systems, traffic networks, spatial economics, man-machine systems, data flow systems, etc. (Bejan, 2000). The working of the photovoltaic solar cell is an example of volume-to-point flow. Here, current generated over a distributed volume is collected to a point. Hereby the applicability of the constructal theory to design the top contact metallization for a photovoltaic solar cell is extended.

2. The Smallest Element

The top contact grid design is posed as an optimization problem. The solar cell is illuminated over an area. We therefore consider a two-dimensional case, neglecting any current generation or flow normal to the illuminated cross section. The current density (*J*) is constant over the entire area. The grid material (having lower resistivity than the semiconductor) is available in limited quantity. We design a metallization pattern for a solar cell that minimizes the ohmic power losses inherent to the cell functioning.

We build up on the lines of constructal theory, starting from the smallest element possible (Bejan, 1997). Best elements are optimized elements which have a geometric form that minimizes the ohmic loss. The geometric structure of a system is built upon by moving to a higher level by assembling the immediate lower best elements.

The simplest form of a two-dimensional solar cell element is a rectangular area. We have a rectangular element whose dimensions are L_0 and H_0, and w is the depth in the direction perpendicular to the x-y plane. The length and breadth dimensions (namely L_0 and H_0) can vary, but the area $A_0 = H_0 \times L_0$ is fixed by manufacturing capabilities. The smallest element, thus, is the one having a fixed area A_0. This element is made up of the base material, whereas the grid lines are made of materials having a resistivity lower than that of the base material. The thickness of the grid material perpendicular to the x-y plane is assumed to be constant and equal to δ. Schematic of the rectangular element is shown in *Figure 1*. It may be noted that the path of minimum resistance is the one when the low resistivity material is placed along the larger axis of symmetry i.e., along the length (assuming that the length is greater than the breadth).

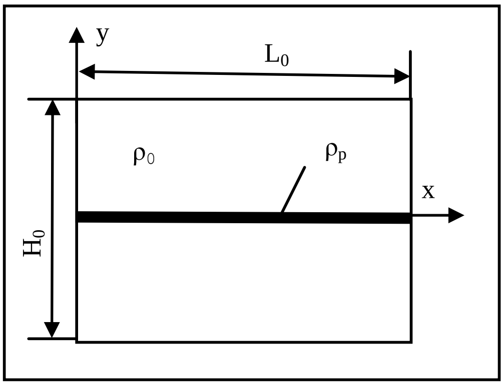

Figure 1. The smallest element.

The resistivity of the base material is ρ_0, whereas that of the metal grid is ρ_p. The resistivity of the base is comparatively very high ($\rho_0 >> \rho_p$). The dimensions of the grid line is thus (D_0, L_0), the width of the grid line being D_0. The current generation rate per unit area (J) over the entire element (and the solar cell) is assumed to be constant. The smallest element is a system in itself, current produced over the area being collected at a single point – the origin. There is no leakage of current from the area boundaries and also perpendicular to the x-y plane.

We assume the smallest element to be slender ($H_0 << L_0$). Hence the current flow along the lowest resistance path will be one-dimensional. The dimensions of this smallest element have to be optimized. Due to the resistance of the element, it is not at a constant potential. The flow of current causes a potential drop to occur. This causes power losses and a decrease in the efficiency of the solar cell. The ohmic losses due to the current flow have to be minimized.

We now formulate an expression for the current through the smallest element (see *Figure 2*):

$$I(y+dy) - I(y) + Jdxdy = 0 \qquad (1)$$

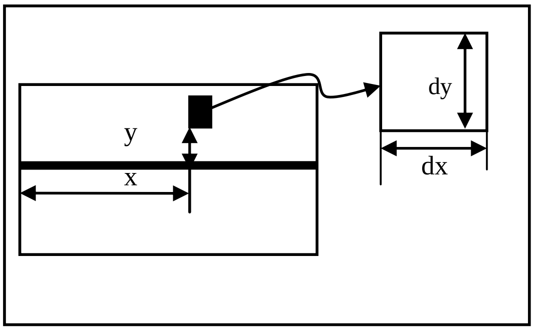

Figure 2. A differential element within the smallest assembly.

$$\Rightarrow \left(\frac{dI}{dy}\right)dy + Jdxdy = 0 \qquad (2)$$

By Ohm's law: $I = \left(\frac{\Delta V}{R}\right)$

$$dI = \frac{wdx}{\rho_0}\left(\frac{dV}{dy}\right) \qquad (3)$$

Therefore equation (1) now becomes:

$$\frac{d^2V}{dy^2} + \frac{J\rho_0}{w} = 0 \qquad (4)$$

Integrating equation (4) subject to the boundary conditions: $\left(dV/dy\right)_{y=H_0/2} = 0$ and $V(x,0) = V_0(x)$, we have:

$$V(x,y) = -\frac{J\rho_0 y^2}{2w} + \frac{J\rho_0 H_0 y}{2w} + V_0(x) \qquad (5)$$

For the high conductivity path shown in *Figure 3*, we now find an expression for the voltage. Similar to equation (4), the charge conservation along the grid finger results:

$$\frac{d^2V_0(x)}{dx^2} + \frac{J\rho_P H_0}{D_0\delta} = 0 \qquad (6)$$

Integrating equation (6) subject to the boundary conditions: $\left(dV_0/dx\right)_{x=L_0} = 0$ and $V_0(0) = V_{00}$ we obtain:

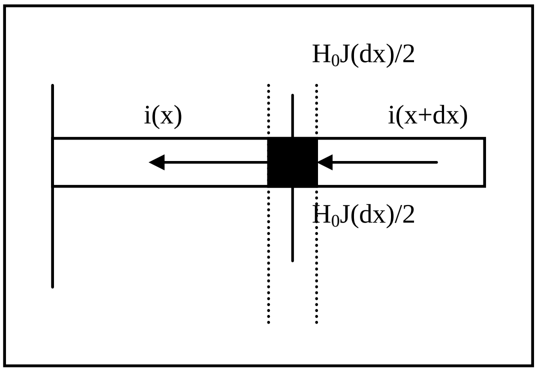

Figure 3: A differential element of the conductivity path.

$$V(x,0) = -\frac{J\rho_P H_0 x^2}{2D_0\delta} + \frac{J\rho_P H_0 L_0 x}{D_0\delta} + V_{00} \qquad (7)$$

Combining equations (5) and (7), we obtain the voltage at any point (x, y) as:

$$\begin{aligned} V(x,y) &= V_{00} + \frac{J\rho_0}{2w}\left(H_0 y - y^2\right) \\ &+ \frac{J\rho_P H_0}{D_0\delta}\left(L_0 x - \frac{x^2}{2}\right) \end{aligned} \qquad (8)$$

3. Smallest Element Optimization

Consider the smallest element (dx, dy). The current in the element is given by:

$$dI = \frac{wdx}{\rho_0}\left(\frac{dV}{dy}\right) = \frac{J}{2}\left(H_0 - 2y\right)dx \qquad (9)$$

The ohmic power loss in this element is:

$$dp_0 = d(I^2R) = \frac{\rho_0 J^2}{4w}\left(H_0 - 2y\right)^2 dxdy \qquad (10)$$

The power loss for the entire cell, then, is:

$$\begin{aligned} p_0 &= 2\iint dp_0 \\ &= 2\frac{\rho_0 J^2}{4w}\left(\int_0^{H_0/2}\left(H_0 - 2y\right)^2 dy\right)\left(\int_0^{L_0} dx\right) \\ &= \frac{\rho_0 J^2}{12w} H_0^3 L_0 \end{aligned} \qquad (11)$$

The power loss for the grid is as follows:

$$\begin{aligned} p_0' &= \sum i^2 R \\ &= \int_0^{L_0} J^2 H_0^2 \left(L_0 - x\right)^2 \frac{\rho_P}{D_0\delta} dx \\ &= \frac{\rho_P J^2}{3\delta}\frac{H_0^2 L_0^3}{D_0} \end{aligned} \qquad (12)$$

Therefore, the total power loss in the smallest element is given by:

$$\Delta P_0 = \frac{\rho_0 J^2}{12w} H_0^3 L_0 + \frac{\rho_P J^2}{3\delta}\frac{H_0^2 L_0^3}{D_0} \qquad (13)$$

Non-dimensionalizing the above equation, we have the total power loss as:

$$\begin{aligned} \frac{\Delta P_0}{\left(\frac{\rho_P J^2 H_0^2 L_0^2}{3\delta}\right)} &= \frac{1}{4}\frac{\delta}{w}\frac{\rho_0}{\rho_P}\frac{H_0}{L_0} + \frac{H_0}{D_0}\frac{L_0}{H_0} \\ &= \frac{1}{4}\frac{\delta}{w}\frac{\rho_0}{\rho_P} z_0 + \frac{1}{z_0\phi_0} \end{aligned} \qquad (14)$$

where $z_0 = H_0/L_0$ and $\phi_0 = D_0/H_0$. The extent of shading on the solar cell is represented by a constant, ϕ_0. Minimizing the total power loss in the elemental cell with respect to z_0 gives:

$$z_{0,opt} = 2\left(\frac{w}{\delta}\frac{\rho_P}{\rho_0}\frac{H_0}{D_0}\right)^{1/2} \tag{15}$$

$$\Delta P_{0,min} = \frac{1}{3}J^2 H_0^2 L_0^2\left(\frac{\rho_0\rho_P}{\delta w}\frac{1}{\phi_0}\right)^{1/2} \tag{16}$$

4. First Assembly

Here we consider the first assembly to be arrangements of the smallest constructs as shown in *Figure 4.* Let us, for the moment, consider the part of first construct made up of two smallest constructs placed on either side of the grid along the x-axis (*Figure 5*). Therefore, $H_1 = 2L_{0,opt}$. Current I_0 flows from the smallest element to the grid element along the x-axis of the smallest element through the area ($D_0 \times \delta$). The current if assumed to be flowing in from area ($H_0 \times \delta$), then the density of the current flowing in at any differential element (of length dx) along the *x*-axis grid is $2I_0/\delta H_0$. The concept of current diffusion is assumed here. Therefore, for the differential element dx, we may express:

$$\frac{d^2V}{dx^2} + \frac{J\rho_p H_1}{D_1\delta} = 0 \tag{17}$$

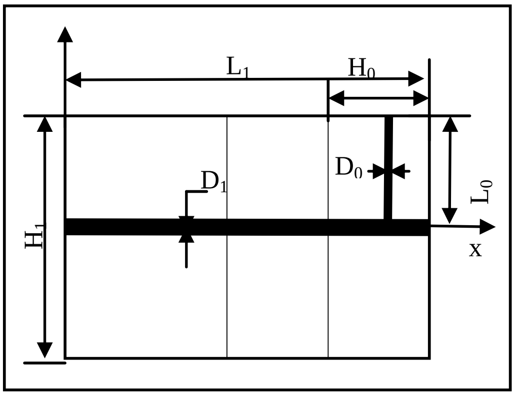

Figure 4. The first assembly.

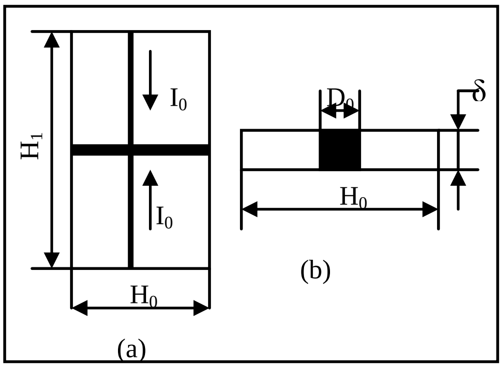

Figure 5. (a) Part of the first assembly (b) Cross-sectional view of the junction where the smallest element grid joins the first assembly grid.

with the boundary conditions being $\left(dV/dx\right)_{x=L_1} = 0$ and $V(0,0) = V_0$.

The above conditions give the current at any location *x* along the bus-bar:

$$I = JH_1\left(L_1 - x\right) \tag{18}$$

Integrating the power loss in the differential element along the bus-bar length, the power loss in the bus-bar (p_1') is given by:

$$p_1' = \frac{\rho_P J^2}{3\delta}\frac{H_1^2 L_1^3}{D_1} \tag{19}$$

Let the number of smallest elements making up the first assembly be n_1. Now:

$$n_1 = \frac{A_1}{A_0} = \frac{H_1 L_1}{H_0 L_0} = \frac{2L_1}{H_0} = \frac{4}{z_0 z_1} \tag{20}$$

where $z_1 = H_1/L_1$.

The power loss for the entire first construct may be expressed as:

$$\begin{aligned}\Delta P_1 &= n_1\frac{1}{3}J^2 H_0^2 L_0^2\left(\frac{\rho_0\rho_p}{\delta w}\frac{1}{\phi_0}\right)^{1/2} + \frac{\rho_p J^2}{3\delta}\frac{H_1^2 L_1^3}{D_1}\\ &= \frac{1}{3}\frac{J^2 A_1^2 z_0 z_1}{4}\left(\frac{\rho_0\rho_p}{\delta w}\frac{1}{\phi_0}\right)^{1/2} + \frac{\rho_p J^2}{3\delta}\frac{A_1^2 L_1}{D_1}\\ &= \frac{1}{6}\frac{J^2 A_1^2 z_1 \rho_p}{\delta\phi_0} + \frac{\rho_p J^2}{3\delta}\frac{A_1^2}{z_1\left(\phi_1 - \phi_0\right)}\end{aligned} \tag{21}$$

where $A_1 = H_1 L_1$ and $\phi_1 = (A_1\phi_0 + D_1 L_1)/A_1 = \phi_0 + D_1/H_1$.

Minimizing this power loss with respect to z_1:

$$z_{1,opt} = \left(\frac{2\phi_0}{\phi_1 - \phi_0}\right)^{1/2} \tag{22}$$

$$\Delta P_{1,opt(z_1)} = \frac{\sqrt{2}}{3}\frac{J^2 A_1^2 \rho_p}{\delta}\frac{1}{\sqrt{\phi_0(\phi_1 - \phi_0)}} \tag{23}$$

(opt(z_1) subscript denotes the optimized nature of the power loss with respect to z_1). Further minimizing the total power loss with respect to ϕ_0 for a fixed ϕ_1 gives:

$$\phi_{0,opt} = \phi_1/2 \tag{24}$$

$$z_{1,opt} = \sqrt{2} \tag{25}$$

$$\Delta P_{1,min} = \frac{2\sqrt{2}}{3}\frac{J^2 A_1^2 \rho_p}{\delta\phi_1} \tag{26}$$

$$n_{1,opt} = \frac{4}{z_0 z_1} = \left(\frac{\rho_0\delta\phi_1}{w\rho_P}\right)^{1/2} \tag{27}$$

For the case where we have equal bases of simplest construct and first construct, i.e. when $A_0 = A_1$ and $\phi_0 = \phi_1 = \phi$, we have: $\frac{\Delta P_{0,min}}{\Delta P_{1,min}} = \left(\frac{\rho_0 \delta\phi}{8w\rho_P}\right)^{1/2} >> 1$ for most practical solar cells. The first construct then is a better assembly over the simplest construct. We now move onto a higher order structure to examine the possibility of further reduction in the power loss.

5. Second Assembly

As can be seen in *Figure 6*, the second construct is essentially an arrangement of the immediate lower assemblies (i.e. the first assemblies). Proceeding in the manner similar to the first assembly, the total power loss in the second construct is given by:

$$\Delta P_2 = \frac{\sqrt{8}}{3}\frac{J^2 A_2^2 z_1 z_2 \rho_P}{4\delta\phi_1} + \frac{\rho_P J^2}{3\delta}\frac{A_2^2 L_2}{D_2} \qquad (28)$$

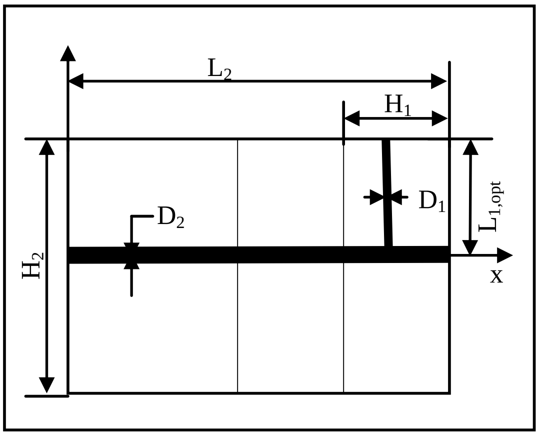

Figure 6. The second assembly.

At the optimum we have:

$$\phi_{1,opt} = \phi_2 / 2 \qquad (29)$$

$$z_{2,opt} = 1 \qquad (30)$$

$$\Delta P_{2,min} = \frac{4}{3}\frac{J^2 A_2^2 \rho_P}{\delta\phi_2} \qquad (31)$$

$$n_{2,opt} = 2\sqrt{2} \approx 2.8 \qquad (32)$$

The slenderness assumption, henceforth fails as *n* is very small, whereas at the onset we assume n to be a large enough number. The second construct may have an optimum geometry for $n_2 = 2$ or 4. We will analyze both cases separately.

6. Revised Second Construct

Case-I*:* $n_2 = 2$

Figure 7 shows the second construct made up of the two first assemblies. Note here the main grid along the x-axis may not be throughout the length. The current flowing through the grid along the x-axis is $I = 2 \times JA_2$. The power loss in the second construct will be equal to the sum of losses in the two first constructs and the bus-bar along the *x*-axis. It is given by:

$$\Delta P_2 = \frac{\sqrt{2}}{3}\frac{J^2 A_2^2 \rho_P}{\delta\phi_1} + \frac{\rho_P J^2}{2\delta}\frac{A_2^2 L_2}{D_2} \qquad (33)$$

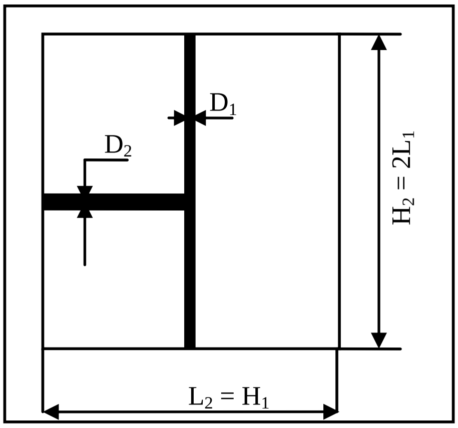

Figure 7. Second construct with n = 2.

Now, $\phi_2 = (A_2\phi_1 + D_2 L_2 / 2) / A_2 = \phi_1 + (1/2\sqrt{2}) D_2 / L_2$. Therefore:

$$\Delta P_2 = \frac{\sqrt{2}}{3}\frac{J^2 A_2^2 \rho_P}{\delta\phi_1} + \frac{\sqrt{2}}{8}\frac{\rho_P J^2 A_2^2}{\delta(\phi_2 - \phi_1)} \qquad (34)$$

Minimizing the total power loss with respect to ϕ_1 for a fixed ϕ_2, we get:

$$\phi_{1,opt} = \frac{2\sqrt{2}}{2\sqrt{2}+\sqrt{3}}\phi_2 \qquad (35)$$

$$\Delta P_{2,min} = \frac{11\sqrt{2}+8\sqrt{3}}{24}\frac{J^2 A_2^2 \rho_P}{\delta\phi_2} \approx 1.2255\frac{J^2 A_2^2 \rho_P}{\delta\phi_2} \qquad (36)$$

Case II: $n_2 = 4$

Proceeding in the same manner as we did for the $n_2 = 2$ case, define the parameter ϕ_2 as: $\phi_2 = \phi_1 + (3/2\sqrt{2}) D_2 / L_2$. Optimizing this geometry as done earlier gives the following results:

$$\phi_{1,opt} = \frac{4}{4+3\sqrt{3}}\phi_2 \qquad (37)$$

$$\Delta P_{2,min} = \frac{\left(4+3\sqrt{3}\right)^2 \sqrt{2}}{96}\frac{J^2 A_2^2 \rho_P}{\delta\phi_2} \approx 1.246\frac{J^2 A_2^2 \rho_P}{\delta\phi_2} \qquad (38)$$

Comparing equations (36) and (38), it is clear that the second construct made up of 2 first constructs is better than the one having 4 first constructs. The power loss taking place in Case I is nearly 1.65% less than in Case II.

For the case where we have equal bases of simplest construct and first construct, i.e. when $A_1 = A_2$ and $\phi_1 = \phi_2 = \phi$, we have: $\frac{\Delta P_{1,min}}{\Delta P_{2,min}} = \frac{16\sqrt{2}}{11\sqrt{2}+8\sqrt{3}} \approx 0.7693 < 1$. It may be noted that the second construct ($n_2 = 2$) is clearly sub-optimal and more disadvantageous than the first construct as 30% more power loss is experienced.

Higher order constructs can be formed in similar ways as described earlier. However, they will lead to mathematically sub-optimal and disadvantageous solutions. An increase in flow link does not necessarily decrease the global flow resistance (Ghodoossi, 2004). Therefore, for most of the practical cases of solar cells the geometric structure up to the first construct is best. However, it is possible to reduce the global resistance by introducing different materials with different resistivity for constructing grid patterns at different levels of construct. This approach is proven to reduce global resistance with increasing flow complexity (Ghodoossi, 2004; Bejan et al., 2000).

7. Conclusion

Resistive losses are a major hindrance to the useful energy available from the sun in the form of electricity. To make a maximum of solar energy available as useful energy, the mechanism of collecting solar energy and converting it to electricity should be such that minimum energy is lost. Thus it becomes imperative to design such a solar cell grid. As we have seen above, the constructal optimization technique provides an analytic approach to designing the optimized grid structure. We made a few assumptions in the beginning about neglecting the losses like those due to shading, contact resistance, etc. Considering these factors will introduce more variables, and this presents an area to delve into in more detail.

Using the step optimization process in our case, optimized constructs make up the next higher level of construct. As in the case of the second construct, two structures are possible if we tend to tolerate the level of loss difference between the two cases. We can proceed further by constructing higher levels using either of the two structures. Complexities can increase with a combination of the two constructs making up the next construct (the third construct). Selecting the better of these constructs then depends on the constraints like the amount of high conductivity material available, tolerable losses, cost, etc. Once we move to a higher level of constructs, a network of the high conductivity material similar to a tree structure can be observed. As in the case of the constructal theory applied to cooling electronic packages (Bejan, 1997), the tree structure is evident. The principle of equipartition also holds true. In this case we have used an approach where we assumed the concept of current diffusion for assemblies rather than the conductance diffusion considered by Bejan (1997). Various approaches to build up an assembly may be possible. Nevertheless, this approach to designing the grid pattern by constructal optimization gives a good design. As in most volume-to-point or point-to-volume systems, here too constructal theory offers an approach for designing an optimized structure.

Nomenclature

A_i	area of solar cell at i-th level
D_i	width of conductivity path
H_i	height of area A_i
I	current
i	order of assembly
J	current generation rate per unit area
L_i	length of area A_i
n_i	number of elements in i-th assembly
p'_i	power loss in the grid
p_i	power loss in semiconductor
R	resistance
V	voltage at a point
w	dimension perpendicular to A_i's cross section
x, y	Cartesian coordinates
z_i	aspect ratio
ΔP_i	total power loss
ΔV	voltage difference
δ	thickness of the grid
ϕ_i	shading factor
ρ_0	resistivity of the semiconductor
ρ_p	resistivity of the grid material

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