

Single-Component Optimal Heat Exchanger Effectiveness Using Specific Exergy Costs And Revenues

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Abstract

In order to understand the relationship between capital costs and the cost rate of exergy destruction in the heat exchangers of a combined cycle power plant (CCPP), the economic optimum design of a heat exchanger considered as a single component is explored. Expressions for time rates of profit are written using specific exergy revenues and costs. These expressions are non-dimensionalized, and their derivatives are taken to find the optimum heat exchanger effectiveness. This optimum is shown to be a function of several dimensionless groups. Three of the variables contained within these groups are both of the stream entrance temperatures and the reference temperature. Results of numeric optimization of heat exchangers confirm the validity of the dimensionless groups.

Keywords: Exergoeconomics, thermoeconomics, heat exchanger optimization

1. Introduction

To date, methods have been developed to aid in finding the thermodynamic optimum heat exchanger network, such as pinch analysis (Bejan et al., 1996). Additionally, there has been significant research in optimizing a heat exchanger as a component. (See, for example, Bejan and Errera, 1998 and Vargas et al., 2000.) These methods provide the engineer with valuable information. However, they do not take into account economic information. Normally, the economic optimum of a design will differ from the thermodynamic optimum design.

The thermoeconomic factor (f-factor) is defined as

$$f = \frac{\dot{Z}}{\dot{Z} + \dot{C}_D} \quad (1)$$

It expresses the ratio of the capital cost rate of the component to its total cost, the sum of its capital cost rate and its cost rate of exergy destruction (Bejan et al., 1996). The capital cost rate is found by amortizing the component's capital cost, for example, through the use of a capital recovery factor or by proportionally assigning the carrying charges calculated through the revenue-required method (Bejan et al., 1996). The f-factor can serve as a tool during iterative optimization by helping to judge whether it is desirable to reduce exergy destruction at the

expense of additional capital, or to save capital at the expense of increased exergy destruction.

A possible complication to the use of the f-factor in the optimization of a heat exchanger network, such as a heat recovery steam generator (HRSG), was observed by Paulus and Tsatsaronis (2004). The capital costs in order to achieve a given heat exchanger exergetic efficiency increase as hot stream temperatures decrease. This fact was used to explain the observation that in an HRSG the f-values of the heat exchangers in a CCPP optimized for minimal product costs tend to rise as the hot stream temperatures decrease (assuming that the hot stream is the fuel stream). This temperature dependency can make it very difficult to judge if an f-factor shows a good balance between capital and total costs, i.e. if the relative capital investment in the heat exchanger is correct for the system. The author has encountered instances while experimenting with hypothetical plants where the product costs of a two-pressure CCPP were decreased by lowering the pinch temperature (and *increasing* the capital investment) of the low pressure evaporator, although the f-factor of this component was judged quite high (which would suggest *decreasing* the capital investment). The author has similarly observed the opposite behavior with respect to the high-pressure evaporator.

The following work originated as an attempt to quantify the influence of temperatures on the f-factors in an HRSG. Initially, a heat

exchanger, specifically an evaporator in an HRSG, was numerically optimized as a single component, with the goal of maximizing the time rate of profit (for that single component), as defined by:

$$\dot{P} = r_p \dot{E}_P - c_F \dot{E}_F - \dot{Z} \quad (2)$$

In the above equation, r_p is the specific revenue of the exergetic product (Paulus and Tsatsaronis, 2004) and c_f is the specific cost of the exergetic fuel (Lazzaretto and Tsatsaronis, 1999). Revenues differ from costs as they are determined from the “market value of the products backwards”, as opposed to those from the fuel costs forward in traditional exergy cost accounting. In order to find the specific revenues, monetary balances are written as in the traditional case. However, the auxiliary equations are formulated differently, in essentially a “mirror image” (Paulus and Tsatsaronis, 2004) of the principles given by Lazzaretto and Tsatsaronis (1999)².

Observation of the f-factors at the optimal single-component effectiveness showed temperature influences, but the exact relationship was not readily quantifiable. Therefore, the expression for profit was rewritten as a function of the stream entrance temperatures and mass flows, along with the cost of the heat exchanger area. As the resulting expression was a function of many variables, it was non-dimensionalized. This was done for an evaporator, where it was assumed that the exergy increase in the cold stream was the product, and for a heat exchanger without a phase change, again with the cold stream as the product. All calculations assumed a constant overall heat transfer coefficient, constant stream heat capacities and no occurrence of pressure drop in either stream.

2. Non-Dimensional Profit for an Evaporator

If the product of an evaporator (see *Figure 1*) is the exergy increase of the cold stream, and it is assumed that no pressure drop occurs in the hot stream, the exergetic fuel can be expressed as

$$\dot{E}_F = C_h (T_{hi} - T_{hii}) - T_0 C_h \ln \left(\frac{T_{hi}}{T_{hii}} \right) \quad (3)$$

² The principles for formulating auxiliary equations for revenues are:

F-Principle: Each exergy unit, of any type of exergy, which is supplied by any fuel stream to a device, has the same specific revenue associated with it.

P-Principle: The revenue associated with a unit of exergy, supplied to a stream, is the same as the revenue of a unit of exergy removed from the same stream in a downstream component.

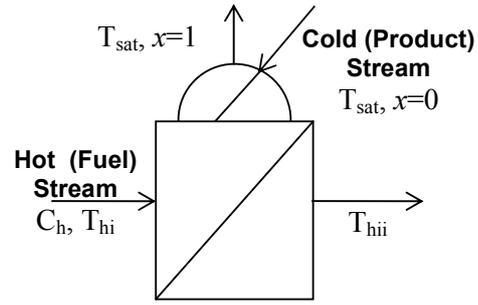


Figure 1. Schematic representation of an evaporator.

The term $C_h (T_{hi} - T_{hii})$ is equal to the duty of the heat exchanger, Q . The duty, in turn, is equal to the effectiveness times the maximum possible heat transfer with the given inlet conditions, $\dot{Q}_{max} = C_h (T_{hi} - T_{sat})$.

Equation 3 can thus be rewritten as

$$\dot{E}_F = \varepsilon \dot{Q}_{max} - T_0 C_h \ln \left(\frac{T_{hi}}{T_{hi} - \varepsilon \dot{Q}_{max} / C_h} \right) \quad (4)$$

If the cold stream enters as a saturated liquid, and leaves in a saturated state at the pressure with which it enters, the exergetic product is

$$\dot{E}_P = (1 - T_0 / T_{sat}) \dot{Q} = (1 - T_0 / T_{sat}) \varepsilon \dot{Q}_{max} \quad (5)$$

With the assumption that the cost of the heat exchanger varies linearly with its area (or that the average cost per unit area is known), the capital cost of the heat exchanger is given by

$$\dot{Z} = c_A A \quad (6)$$

From the definition of NTU (Kayes and Crawford, 1993), $A = NTU \cdot C_h / U$ and $NTU = -\ln(1 - \varepsilon)$. Equation 6 may be expressed as

$$\dot{Z} = -c_A \frac{C_h}{U} \ln(1 - \varepsilon) \quad (7)$$

With the given assumptions, Equation 2, after manipulation, can be rewritten as

$$\begin{aligned} \frac{\dot{P}}{r_p T_0 C_h} = & \varepsilon \left[\left(\frac{T_{hi} - T_{sat}}{T_0} \right) - \left(\frac{T_{hi}}{T_{sat}} - 1 \right) \right] - \\ & \frac{c_F}{r_p} \left[\varepsilon \left(\frac{T_{hi} - T_{sat}}{T_0} \right) + \ln \left(1 + \varepsilon \left(\frac{T_{sat}}{T_{hi}} + 1 \right) \right) \right] + \\ & \frac{c_A}{r_p U T_0} \ln(1 - \varepsilon) \end{aligned} \quad (8)$$

Inspection of Equation 8 shows that all of the variable groups are now dimensionless, and

the following dimensionless variables may be defined³:

Dimensionless profit:

$$\Pi = \dot{P} / r_p T_0 C_h \quad (9)$$

Fuel cost-product revenue ratio:

$$\chi_F = c_F / r_p \quad (10)$$

Dimensionless entrance temperature difference:

$$\theta = \left(\frac{T_{hi} - T_{sat}}{T_0} \right) \quad (11)$$

Entrance temperature ratio:

$$\tau = T_{hi} / T_{sat} \quad (12)$$

Dimensionless heat exchanger area cost:

$$\chi_A = c_A / r_p U T_0 \quad (13)$$

Equation 8 becomes

$$\begin{aligned} \Pi = \varepsilon [\theta - (\tau - 1)] - \\ \chi_F \left[\varepsilon \theta + \ln \left(1 + \varepsilon \left(\frac{1}{\tau} - 1 \right) \right) \right] + \\ \chi_A \ln(1 - \varepsilon) \end{aligned} \quad (14)$$

and the profit is now a function of four variables instead of nine.

The first derivative of Equation 14 with respect to effectiveness is

$$\frac{\partial \Pi}{\partial \varepsilon} = \theta - (\tau - 1) - \chi_F \left(\theta + \frac{\frac{1}{\tau} - 1}{1 + \varepsilon \left(\frac{1}{\tau} - 1 \right)} \right) - \chi_A \quad (15)$$

$$\frac{\chi_A}{1 - \varepsilon}$$

Equation 15, when solved for effectiveness set equal to zero, may have no solution, or it may yield either a local or global maximum in the range from $\varepsilon = 0$ to 1. *Figure 2* illustrates this with plots of Π as a function of ε at several values of θ . When it yields a global maximum, Equation 15 allows the optimal effectiveness to be expressed as a function of four variables instead of eight. There are regions for which the dimensionless profit is negative. This means that the time rate of profit is negative as well; the combined capital costs and fuel costs exceed the revenue flow of the product stream. (Moreover, this implies that in the economic point of view, the heat exchanger should either be eliminated,

or the network of which it is part should be re-designed.) *Figure 3* shows the effect of dimensionless variables containing temperatures (over a typical range found within an HRSG) on the optimal effectiveness for one set of χ_A and χ_F .

Interestingly, while a greater entrance temperature *ratio* favors a less effective heat exchanger, a greater entrance temperature *difference* favors a more effective. (Not shown in the figures is that the optimal effectiveness decreases linearly with increasing χ_A and decreases approximately parabolically with increasing χ_F .) Although this may seem counterintuitive, consider that if the entrance temperature ratio is very large, and the heat exchanger effectiveness very low, all heat transfer occurs across large temperature differences, with correspondingly low exergetic efficiency. Increasing effectiveness causes additional heat transfer to occur, and this additional heat transfer occurs across smaller temperature differences. The larger the inlet temperature difference, the greater effectiveness is required to achieve a given exergetic efficiency.

Equation 1, after substitution of appropriate expressions for the capital cost and heat exchanger area, becomes

$$f = \frac{-c_A \frac{C_h}{U} \ln(1 - \varepsilon)}{-c_A \frac{C_h}{U} \ln(1 - \varepsilon) + r_p \dot{E}_D} \quad (16)$$

The exergy destruction can be found with $\dot{E}_F - \dot{E}_P$, and with Equations 4 and 5 is found to be

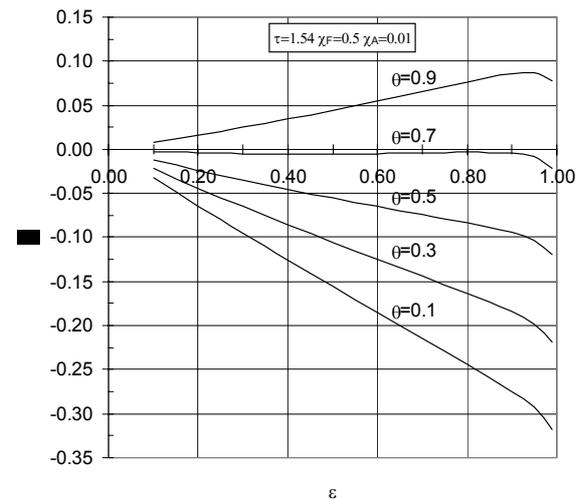


Figure 2. Non-dimensional profit Π as a function of effectiveness and the non-dimensional temperature difference θ .

³ It is worth noting that the ratio τ also appears in the equations for dimensionless entropy production in heat exchangers as defined by Sekulic (2000). The dimensionless entropy production is given the symbol σ and is equal to \dot{S}_π / C_{min} .

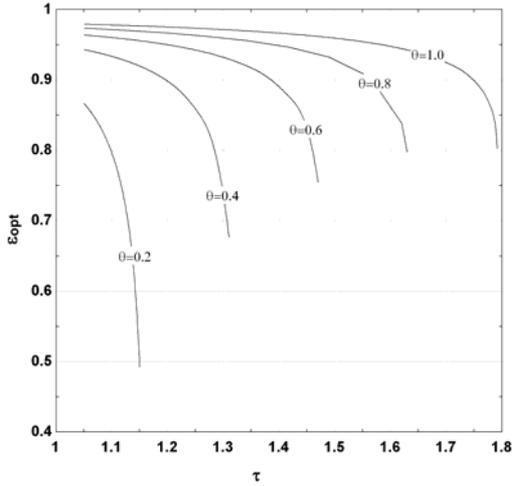


Figure 3. Influence of τ and θ on optimal effectiveness for an evaporator, constant χ_A and χ_F .

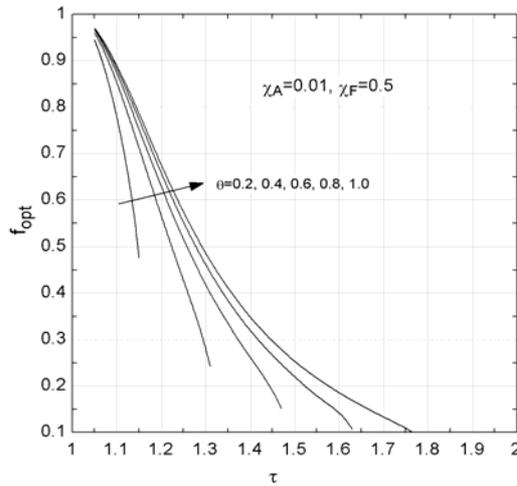


Figure 4. Influence of τ and θ on the f -factor at optimal effectiveness for an evaporator, constant χ_A and χ_F .

$$\dot{E}_D = \varepsilon \dot{Q}_{\max} - T_0 C_h \ln \left(\frac{T_{hi}}{T_{hi} - \varepsilon \dot{Q}_{\max} / C_h} \right) - (1 - T_0 / T_{\text{sat}}) \varepsilon \dot{Q}_{\max} \quad (17)$$

Equation 17 may then be substituted into Equation 16. The resulting expression can be written in terms of the non-dimensional groups above

$$f = \frac{-\chi_A \ln(1 - \varepsilon)}{-\chi_A \ln(1 - \varepsilon) + \ln \left[1 + \varepsilon \left(\frac{1}{\tau} - 1 \right) \right] - \varepsilon(1 - \tau)} \quad (18)$$

This is equivalent to

$$f = \frac{\chi_A NTU}{\chi_A NTU + \sigma} \quad (19)$$

where σ is the dimensionless entropy production, \dot{S}_π / C_{\min} .

The f -factor itself is thus a function of NTU , dimensionless heat exchanger area cost and dimensionless entropy production (itself also a function of the entrance temperature ratio and effectiveness). Because the optimal effectiveness of an evaporator is a function of χ_A , χ_F , θ and τ , and the f -factor is a function of χ_A , effectiveness and τ , it follows that the optimal f -factor is dependent on the same dimensionless groups upon which the optimal effectiveness is dependent. Figure 4 shows the f -factor at optimal effectiveness for various τ and θ differences at fixed values of χ_A and χ_F . The entrance temperature ratio shows a decided effect on the f -factor at the optimal single-component effectiveness; the dimensionless entrance temperature difference has a much smaller effect.

3. Non-Dimensional Profit for a Heat Exchanger with no Phase Change

For an economizer or a superheater (see Figure 5), Equation 5 does not describe the exergetic product. This is instead given by

$$\dot{E}_P = C_C (T_{cii} - T_{ci}) - T_0 C_C \ln \left(\frac{T_{cii}}{T_{ci}} \right) \quad (20)$$

The introduction of the effectiveness and maximum heat transfer, $\dot{Q}_{\max} = C_C (T_{hi} - T_{ci})$, yields

$$\dot{E}_P = \varepsilon \dot{Q}_{\max} - T_0 \ln \left(\frac{T_{ci} + \frac{\varepsilon \dot{Q}_{\max}}{C_c}}{T_{ci}} \right) \quad (21)$$

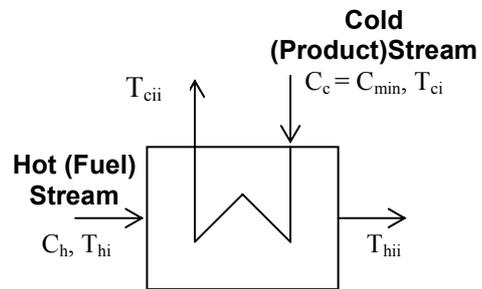


Figure 5. Schematic representation of a heat exchanger.

In order to express the maximum heat transfer as a function of the entrance temperatures, it is necessary to specify which stream has the

smaller heat capacity, C_{\min} . In an HRSG, the cold stream commonly has the lower heat capacity (when no phase change is present), so this derivation will proceed from this assumption. Moreover, in order to express it, it is necessary to specify the geometry of the heat exchanger. Because of the many tube passes in one of the HRSG's superheaters or economizers, counterflow geometry will be assumed. With these two assumptions, and using $NTU = UA/C_{\min}$ along with the appropriate effectiveness-NTU relation for a counterflow heat exchanger,

$NTU = \ln\left(\frac{\varepsilon - 1}{\varepsilon C_r - 1}\right) / (C_r - 1)$, the heat exchange area may be expressed as

$$A = \frac{C_c \ln\left(\frac{\varepsilon - 1}{\varepsilon C_r - 1}\right)}{U(C_r - 1)} \quad (22)$$

where, for this case, $C_r = C_c/C_h$.

Now, Equation 2 may be rewritten as

$$\begin{aligned} \dot{P} = r_p & \left[\varepsilon \dot{Q}_{\max} - T_0 \ln\left(\frac{T_{ci} + \frac{\varepsilon \dot{Q}_{\max}}{C_c}}{T_{ci}}\right) \right] - \\ c_F & \left[\frac{\varepsilon \dot{Q}_{\max}}{T_0 C_h} \ln\left(\frac{T_{hi}}{T_{hi} - \varepsilon \dot{Q}_{\max}/C_h}\right) \right] - \\ c_A & \frac{C_c \ln\left(\frac{\varepsilon - 1}{\varepsilon C_r - 1}\right)}{U(C_r - 1)} \end{aligned} \quad (23)$$

with $\dot{Q}_{\max} = C_c(T_{hi} - T_{ci})$. This can be non-dimensionalized, reducing an expression with ten independent variables to one with six:

$$\begin{aligned} \Pi = (1 - \chi_F) \varepsilon C_r \theta - \chi_F \ln\left[1 - \varepsilon C_r \left(1 - \frac{1}{\tau}\right)\right] - \\ C_r \ln\left[1 + \varepsilon(\tau - 1)\right] - \chi_A \frac{C_r \ln\left(\frac{\varepsilon - 1}{\varepsilon C_r - 1}\right)}{C_r - 1} \end{aligned} \quad (24)$$

The dimensionless groups remain defined as before with the exception that T_{ci} replaces T_{sat} and the addition of C_r ⁴.

⁴ In this expression, τ is the inverse of that defined by Sekulic (2000). A decision was made to be consistent between the case of an evaporator and that of an economizer or superheater, rather than Sekulic's of the ratio of temperature of the stream with the smaller heat capacity to the temperature of the stream with the larger capacity. Sekulic retained consistency between

The derivative of Equation 24 with respect to ε is

$$\begin{aligned} \frac{\partial \Pi}{\partial \varepsilon} = (1 - \chi_F) C_r \theta + \chi_F C_r \frac{1 - \frac{1}{\tau}}{1 - \varepsilon C_r \left(1 - \frac{1}{\tau}\right)} - \\ C_r \frac{\tau - 1}{1 + \varepsilon(\tau - 1)} - \\ \chi_A C_r \frac{\frac{1}{\varepsilon C_r - 1} - \frac{\varepsilon - 1}{(\varepsilon C_r - 1)^2} C_r}{\varepsilon - 1} \frac{\varepsilon C_r - 1}{C_r - 1} \end{aligned} \quad (25)$$

Graphing of Equation 24 (see *Figure 6*) shows that setting Equation 25 to zero and solving for ε will either yield no solution, or a local or global maximum for Π for rational values of effectiveness. The number of variables upon which the optimal effectiveness depends is reduced to five. *Figures 7 and 8* show the influence of τ , θ and C_r over a typical range for the economizers and superheaters of an HRSG. Again, while a greater entrance temperature *ratio* favors a smaller heat exchanger, a greater entrance temperature *difference* favors a larger.

The dimensionless variables χ_A and χ_F have similar effects as for the case of an evaporator.

The f-factor is related to the effectiveness and other dimensionless groups with

$$\begin{aligned} f = \frac{\chi_A \ln\left(\frac{\varepsilon - 1}{\varepsilon C_r - 1}\right) / (C_r - 1)}{\chi_A \ln\left(\frac{\varepsilon - 1}{\varepsilon C_r - 1}\right) / (C_r - 1) +} \\ \frac{1}{C_r} \ln\left[1 - \varepsilon C_r \left(1 - \frac{1}{\tau}\right)\right] + \ln(1 + \varepsilon(\tau - 1)) \end{aligned} \quad (26)$$

This is again equivalent⁵ to

$$f = \frac{\chi_A NTU}{\chi_A NTU + \sigma} \quad (27)$$

The optimal f-factor for an evaporator or economizer is thus not only dependent on the stream entrance temperatures, but also on the ratio of heat capacities of the streams. *Figure 9* shows the temperature influences on the f-factor at optimal effectiveness.

The effects of the various dimensionless groups on optimal effectiveness and the optimal f-factor are summarized in TABLE I.

the heat capacity ratio and the temperature ratio, which was the logical choice for his work.

⁵ The difference between the term for dimensionless entropy production σ in Equation 26 and that of Sekulic (2000) is due to the inverse of definitions of τ .

These derivations are straightforward to repeat for other cases (i.e. different geometry, the case where the hot stream has the lower heat capacity, etc.).

TABLE I: SUMMARY OF EFFECTS OF DIMENSIONLESS VARIABLES.

Variable increased	Effect on ε_{opt}	Effect on f_{opt}
θ	increased	increased
τ	decreased	decreased
χ_A	decreased	increased
χ_F	decreased	decreased
C_r	increased	increased

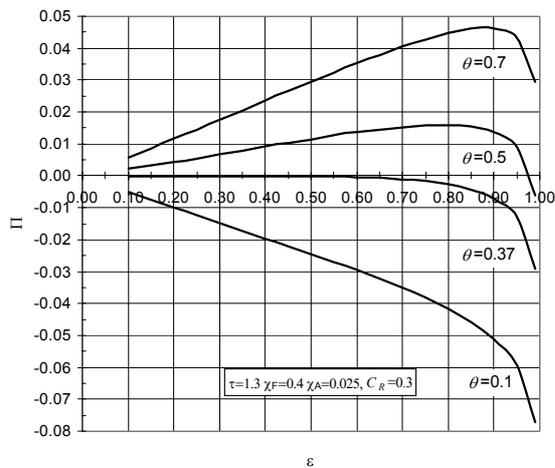


Figure 6. Non-dimensional profit Π of a heat exchanger as a function of the effectiveness and dimensionless temperature difference θ .

4. Numerical Testing

In order to test the validity of the dimensionless groups, numerical optimization was undertaken. A set of governing equations for heat exchanger performance was written, assuming that the overall heat transfer coefficient was constant. Engineer Equation Solver (EES) software was used to simultaneously solve the equations. For an evaporator, the component profit was maximized (with EES's optimization routine) for several different hot stream entrance temperatures. The pressure of the cold stream and the reference temperature were varied to keep the entrance temperature ratio, τ , and the dimensionless entrance temperature difference, θ , constant. This was done for a variety of values for τ and θ . The hot stream was modeled as air, using ideal gas property relations with varying specific heat. The cold stream was modeled using real properties.

It was found that (a) the resulting optimal effectiveness remained nearly constant when the

relevant dimensionless groups remained constant and (b) the resulting optimal effectiveness matched the optimal effectiveness predicted by setting equation 15 equal to zero. The maximum discrepancy between the two calculated values over a range of hot stream entrance temperatures from 550 to 800K was 0.2%.

The calculations were repeated for a heat exchanger without a phase change. To maximize any possible effects of real fluid behavior, the heat exchanger was modeled as a superheater with the cold stream entering as a saturated vapor. The same observations were made as for an evaporator – nearly constant effectiveness resulted from the numerical optimization, and it agreed well with the theoretical, in this case predicted by setting Equation 25 to zero and solving for effectiveness. The maximum discrepancy in this test was 0.4%. When the calculations were repeated at lower pressures and away from the saturation point, the optimal effectiveness remained even more constant and matched the predicted value to three significant figures.

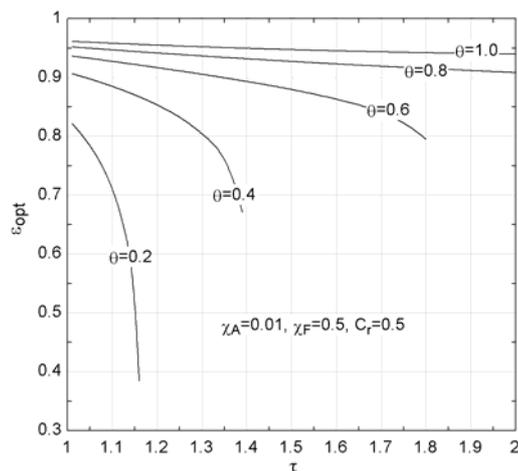


Figure 7. Influence of τ and θ on the optimal effectiveness for a heat exchanger with the cold stream as product and having minimum C , constant χ_A , χ_F , and C_r .

5. Discussion

5.1 The f-factor along the gas path of a HRSG

It was shown before that greater entrance temperature ratios favor lower values for the f-factors, while greater entrance temperature differences favor higher values. Thus, the optimum f-factor of an evaporator depends on far more than the hot stream gas temperature alone. Paulus and Tsatsaronis (2004) observed the effect of only one of the important independent variables.

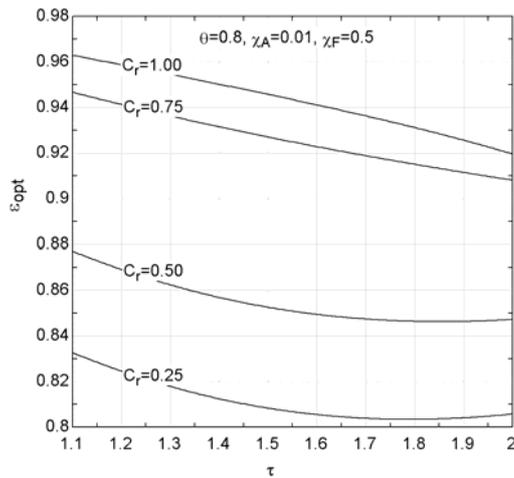


Figure 8. Influence of τ and C_r on the optimal effectiveness for a heat exchanger with the cold stream as product and having minimum C , constant θ , χ_A and χ_F .

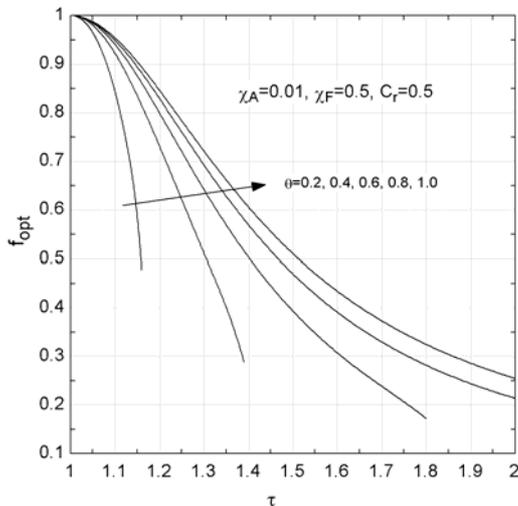


Figure 9. Influence of τ and θ on the f -factor at optimal effectiveness for a heat exchanger with the cold stream as product and having minimum C , constant χ_A , χ_F and C_r .

For a superheater, the situation is complicated by the effect of the heat capacity ratio, and it is difficult to state any trends of the f -factors based solely on hot-stream temperatures. The effect for the economizers is even more difficult to assess, as these are often constrained by either the maximum cold stream exit temperature (below the saturation temperature) or the hot stream exit temperature (to prevent condensation).

The work contained in this paper shows that the optimal capital investment in a heat exchanger, when investigated as a single component, depends on more than just the hot stream entrance temperature. The cold stream entrance temperature, the ratio of heat capacities and even the dead state temperature play roles as

well. And, if these variables influence the optimal size of a heat exchanger considered as a single component, they are extremely likely to play a role in the optimum sizing of a heat exchanger within a system.

5.2 The optimal effectiveness and optimization

Most types of energy system components show a common relationship between exergetic efficiency and exergy destruction. As the efficiency increases, the exergy destruction *decreases*. The increased efficiency comes at the price of a greater capital investment in the component. With such components, the f -factor is a valuable tool for iterative optimization. For components showing this relationship between efficiency and exergy destruction, the f -factor will increase as the capital investment is increased and the component's efficiency improves.

Heat exchangers are unique in that an increase in exergetic efficiency can be accompanied by an *increase* in exergy destruction. When the inlet conditions are fixed, it is only possible to increase the efficiency of a heat exchanger by changing (normally increasing) the UA value. Consider the case when the cold stream has the lower heat capacity: Because additional heat is transferred across a smaller temperature difference than the average for the heat exchanger, to cold stream temperatures higher than the average, the efficiency of the heat exchanger increases. However, because additional heat transfer occurs, the total exergy destruction *can* increase. This could lead to problems with the use of f -factors for optimization of a heat exchanger network: depending on the relative costs of heat transfer area and exergy destruction, the f -factor may actually decrease with increasing efficiency. Moreover, if the inlet conditions are even partially fixed, given the fact that increased efficiency comes with increased fuel exergy use, product exergy delivery and exergy destruction renders the assumption made when calculating the cost of exergy destruction – the assumption that either the exergetic fuel or product remains constant – incorrect.

The resulting dimensionless expressions derived here for optimal effectiveness might well serve as a replacement for the f -factor. Indeed, it would not be unreasonable to hypothesize that an optimally designed heat exchanger network would find all heat exchangers at their optimal component effectiveness (unless constrained by other design parameters), *as long as the fuel costs and product revenues were properly calculated.*

The question might arise, “Why not use exergetic efficiency instead of effectiveness?” It is clear that just as the f-factor is fixed at optimal effectiveness, so is the exergetic efficiency. The advantage of the effectiveness, in the eyes of the system designer, is that it quickly specifies the heat exchanger without having to (again) resort to the use of calculations involving the entrance temperatures.

6. Conclusion

By non-dimensionalizing the expressions for the component profit of two types of heat exchangers, the number of independent variables upon which these expressions depend was dramatically reduced. It was then possible to take the derivative of these expressions for dimensionless profit with respect to effectiveness, and from these derivatives the optimal heat exchanger effectiveness, considering the heat exchanger as a single component, could be found. It was found that, among other dimensionless groups, the dimensionless profit and optimal effectiveness depend on the entrance temperature ratio and the dimensionless entrance temperature difference.

Subsequently, the f-factor for the investigated heat exchanger was related to its effectiveness, the dimensionless cost of the heat exchanger area and the entrance temperature ratio. Therefore, the optimal f-factor for a heat exchanger depends upon the same groups as the optimal effectiveness. As two of these groups contain the stream entrance temperatures, it is clear that the f-factor of heat exchangers in an HRSG of a cost-optimized system will vary with these temperatures. The analysis points to a situation more complicated than just a dependency of thermodynamic fuel potential, as not only the hot stream temperature, but also the cold stream entrance temperature and the ratio of heat capacities influence at least the optimal capital investment for a heat exchanger.

Beyond temperature dependency, other shortcomings in the use of the f-factor for optimizing heat exchanger networks were highlighted. These difficulties suggest that optimal effectiveness might well be an improvement on the f-factor in the iterative optimization of a heat exchanger network. Future work will center on this application.

Nomenclature

c	specific exergy cost, \$/kJ
c_A	average heat exchange area cost, \$/m ²
C	heat capacity of a stream, kW/K

C_{\min}	heat capacity of the stream with the smaller heat capacity, kW/K
C_r	heat capacity ratio
\dot{C}_D	cost rate of exergy destruction, \$/s
\dot{E}	rate of exergy flow/destruction, kW
F	f-factor
NTU	number of transfer units
\dot{P}	profit per unit time, \$/s
\dot{Q}	heat transfer rate, kW
r	specific exergy revenue, \$/kJ
\dot{S}_π	rate of entropy production, kW/K
T	Temperature, K
U	overall heat transfer coefficient, kW/K
\dot{Z}	capital cost per unit time, \$/s

Greek Variables

χ_A	dimensionless specific cost of area
χ_F	fuel cost/product revenue ratio
ε	heat exchanger effectiveness
Π	dimensionless profit
σ	dimensionless entropy production
θ	dimensionless temperature difference
τ	entrance temperature ratio

Subscripts

0	reference
c	cold
F	fuel
h	hot
i	in
ii	out
opt	optimal
p	product

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