Comparative Analysis of the Entropy of Radiative Heat Transfer and Heat Conduction

Sean Wright

Department of Mechanical Engineering,
University of Colorado at Denver and Health Sciences Center (UCDHSC),
Campus Box 112, P.O. Box 173364, Denver, Colorado, 80217-3364, USA
Sean.Wright@cudenver.edu, Tel.: (303)556-8348, Fax: (303)556-6371

Abstract

Many thermodynamic texts incorrectly imply that the entropy flux of thermal radiation (TR) is the same as that for heat conduction, the heat flux divided by the local temperature (q/T). However, for blackbody radiation (BR) emission a 4/3 factor occurs. BR represents the maximum entropy for all radiation with the same emission temperature, as well as all radiation with the same energy radiance. However, using Planck's formulas it is shown that BR emission has the lowest entropy-to-energy ratio, and thus the lowest entropy factor, for all radiation with the same emission temperature or radiation with an enclosed energy spectrum. In practice, analysis of radiative transfer includes incident, reflected and emitted fluxes. Case-specific integration, based on Planck's entropy formula, can be used to determine the net radiative entropy flux. However, this net entropy flux can be put in the form $n(q/T)$, where n is a coefficient unique to the radiative fluxes involved. This allows the net entropy flux to be easily calculated given the energy flux and the temperature of the opaque absorbing material. This entropy coefficient can vary greatly, taking on values less than unity, and values greater than unity. This implies that the misuse of the heat conduction entropy flux expression can vary from overestimating ($n < 1$) to underestimating ($n > 1$) the net radiative entropy flux. Graphical tools and simplified approximate expressions are presented that allow the entropy coefficient n to be quickly determined in certain general scenarios of radiative transfer encountered in practice.

Keywords: Entropy, second law, radiation, heat transfer, conduction, blackbody, graybody

1. Introduction

All matter emits thermal radiation (TR) continuously, and consequently TR is an inherent part of our environment. Radiative heat transfer is important in system analysis particularly when high temperatures are involved, cryogenic systems are considered, when radiation is being utilized as a source flux, or when radiative transfer is the primary mode of heat rejection. Some application examples where TR transfer is of primary importance include solar collectors, boilers and furnaces, spacecraft cooling systems, and cryogenic fuel storage systems.

Second law analysis in general, is an important form of thermodynamic analysis that provides the basis for determining energy quality, as well as improved performance evaluation and optimization. However, when it comes to the entropy of radiative fluxes, thermodynamic texts often incorrectly state that the entropy flux of heat transfer is the ratio of the heat flux to the local temperature (q/T) with no restriction or exclusion for TR (e.g. Cengel and Boles 2006, p. 378; Moran and Shapiro 2004, p. 249). However, it is well known, at least among a number of physicists and solar engineers, that the entropy flux for TR emission is not calculated in the same manner as for heat conduction. For example, see Planck (1914), Petela (1964), Landsberg and Tonge (1979), Haught (1984), Landsberg (1984), De Vos and Pauwels (1986), Bejan (1987), Bejan (2006), Beretta and Gyftopoulos (1990), Gaggioli (1998), Wright et al. (2001), Wright et al. (2002), and Liu et al. (2006).

For BR emission the entropy flux is 4/3 times the energy flux divided by the emission temperature, while numerical integration or approximations are required to determine the entropy of non-blackbody radiation (NBR). See, for example, Wright et al. (2001). It is true that BR emission represents the maximum emission of

entropy for all materials with the same temperature, as well as the maximum entropy emission for all radiation with the same energy radiance. These facts were well known and utilized by Max Planck (1914) in determining the blackbody energy formula. In a recent paper, Santillan et al. (1998) verify that Planck's energy and entropy formula lead to these two conclusions using a mathematical Lagrange multiplier approach.

Based on the understanding that BR emission has the highest entropy one may conclude that NBR emission must have a coefficient less than that which occurs for BR emission, the 4/3 coefficient. However, NBR emission has a higher ratio of entropy-to-energy as can be demonstrated using Planck's formulas.

The net entropy flux involving incident, reflected and emitted radiative fluxes is often difficult to determine and depends on the character of incident radiation, as well as the character and temperature of the emission surface. Particularly troublesome is the fact that the entropy flux of radiation reflected and emitted from a surface is not independent. This occurs because these fluxes occupy the same hemisphere of directions and their frequency spectrums overlap. As a result, it is important to consider the ratio of the net energy transfer to the temperature, rather than just the entropy-to-energy ratio for emission alone. This is the primary focus of this paper.

2. Background

The energy and entropy of unpolarized[1] TR is correctly calculated using the spectral energy and entropy expressions derived by Planck (1914) using equilibrium statistical mechanics:

$$K_\upsilon = \frac{2h}{c^2} \frac{\upsilon^3}{e^{\frac{h\upsilon}{kT}} - 1}, \qquad (1)$$

and

$$L_\upsilon = \frac{2k\upsilon^2}{c^2}\left\{\left(1 + \frac{c^2 K_\upsilon}{2h\upsilon^3}\right)\ln\left[1 + \frac{c^2 K_\upsilon}{2h\upsilon^3}\right] - \left(\frac{c^2 K_\upsilon}{2h\upsilon^3}\right)\ln\left[\frac{c^2 K_\upsilon}{2h\upsilon^3}\right]\right\} \qquad (2)$$

where υ is frequency and T is the material emission temperature. The quantities K_υ and L_υ are the energy and entropy flow rates per unit frequency, area, and solid angle. Landsberg and Tonge (1980) used a non-equilibrium statistical mechanics approach to obtain the same result as

Planck. They concluded[2] "this result, usually obtained from equilibrium statistical mechanics, is therefore of wider significance and represents a non-equilibrium entropy." A plot of K_υ versus frequency υ for various values of temperature T gives a family of BR energy spectra. Upon substituting (1) into (2) one obtains a family of BR entropy spectra. For arbitrary TR, the entropy spectrum is found by substituting K_υ data, rather than Equation (1) for BR, into Equation (2).

The energy K and entropy L radiance of any TR spectrum can be calculated by integrating the K_υ and L_υ spectrums over frequency, respectively. That is, the energy radiance K and entropy radiance L are the area under the K_υ and L_υ spectrums, respectively. For BR Equations (1) and (2), integration over both frequency and solid angle provides the energy and entropy irradiances (fluxes), respectively:

$$H_{BR} = \pi K_{BR} = \sigma T^4, \; J_{BR} = \pi L_{BR} = \tfrac{4}{3}\sigma T^3 \;(3)$$

where $\sigma = 2\pi^5 k^4 / 15 c^2 h^3$ is the Stefan-Boltzmann constant. Note that irradiances H and J are the integration of the radiances K and L over solid angle and have the units of energy or entropy flow rate per unit area, respectively.

To compare the entropy flux of BR emission to that of heat conduction the entropy irradiance or flux can be expressed as:

$$J_{BR} = \frac{4}{3}\frac{H_{BR}}{T} \qquad (4)$$

By definition the spectral energy irradiance for isotropic gray radiation (GR) emission, or equivalently diluted blackbody radiation (DBR), is

$$H_{GR} = \pi K_{GR} = \varepsilon \sigma T^4 \qquad (5)$$

However, the entropy of GR is not as easily calculated because the spectral entropy is not a linear function of the spectral energy. For GR the entropy irradiance is

$$J_{GR} = \pi L_{GR} = \frac{2\pi k^4}{c^2 h^3} T^3 \int_0^\infty x^2 \{(1+f)\ln(1+f) - f\ln f\}dx \qquad (6)$$

---

[1] For simplicity it is assumed in this paper that the radiation is unpolarized and isotropic.

[2] Two assumptions were specified for this result to be exact: (1) the probability of finding N_j bosons in quantum state j is independent of the occupation numbers of the other quantum states, and (2) the probability of an additional particle occupying a state j is independent of the number already in that state.

where,

$$f = \frac{\varepsilon}{e^x - 1} \qquad (7)$$

and $x = h\nu/kT$. The entropy of GR (6) is a simple cubic function of the material emission temperature (Wright et al, 2001). The definite integral in (6) is a function of ε only, and was first recognized by Landsberg and Tonge (1979) in a slightly different form. The integral $I(\varepsilon)$ has not been solved in closed form. Stephens and Obrien (1993) presented an infinite series solution and Landsberg and Tonge (1980) presented an approximate limiting solution for low emissivity ($\varepsilon < 0.10$). Approximations with increased accuracy and over wider ranges of emissivity were presented by Wright et al. (2001). In this paper, the following two approximations will be used for GR entropy fluxes

$$J_{GR} = \tfrac{4}{3}\sigma\varepsilon T^3 \left[1 - \tfrac{45}{4\pi^4}(2.336 - 0.260\varepsilon)\ln(\varepsilon)\right] \quad (8)$$

which is within 0.33% of the numerical integration results for the emissivity range 0.005 to 1.000, and

$$J_{GR} = \tfrac{4}{3}\sigma\varepsilon T^3 \left[1 - \tfrac{45}{4\pi^4}(2.292 - 0.150\varepsilon)\ln(\varepsilon)\right] \quad (9)$$

which is within 0.03% of the numerical integration results for emissivities greater than 0.2.

3. The Entropy Flux of Blackbody Radiation Emission

For the emission of blackbody radiation from a surface, it is clear that the heat conduction entropy expression is not applicable. However, for ease of calculation and comparison, a coefficient n may be introduced in the entropy flux expression such that

$$J_{Emi} = n\frac{H_{Emi}}{T} \qquad (10)$$

For BR emission this coefficient is simply equal to 4/3. For non-blackbody radiation (NBR) emission this coefficient is greater than that of BR emission (Wright et al., 2001). This has been proven from stability arguments and from plotting non-dimensionalized forms of Planck's spectral energy and entropy radiance. The fact that $n > 4/3$ for NBR emission may seem surprising since BR emission is characterized with maximum entropy emission for all radiation with the same emission temperature. It also may appear to violate the second law of thermodynamics in that BR is the equilibrium state of all radiation with the same energy. That is, an enclosed TR system will naturally equilibrate to a system of BR with the same energy level.

However, as a alternative, concise proof, Planck's formula's can be utilized to show that BR

has the minimum entropy-to-energy ratio of all TR with the same emission temperature, and thus represents the minimum value of the coefficient n and represents the minimum departure from the heat conduction entropy flux expression. This can be done simply from considering the derivatives of the spectral entropy radiance (L_ν) with respect to the spectral energy radiance (K_ν). After some mathematical manipulation using (1) and (2) it can be shown that the first derivative becomes

$$\frac{dL_\nu}{dK_\nu} = \frac{k}{h\nu}\ln\left(\frac{2h\nu^3}{c^2 K_\nu} + 1\right). \qquad (11)$$

and is always positive, proving that the entropy radiance L_ν always increases with increasing energy K_ν. The second derivative can be expressed as

$$\frac{d^2 L_\nu}{dK_\nu^2} = \frac{k}{h\nu}\left(\frac{-2h\nu^3/c^2 K_\nu}{K_\nu + 2h\nu^3/c^2}\right) \qquad (12)$$

This second derivative is clearly negative for all values of K_ν. This shows that even though the slope remains positive, it is always decreasing with increasing K_ν. Thus, the monochromatic entropy radiance (L_ν) is a monotonically increasing function of the spectral energy radiance (K_ν). As a result, the monochromatic entropy-to-energy ratio L_ν/K_ν, the slope of the line from the origin to a location on the L_ν versus K_ν plot, decreases as K_ν increases. That is, the higher the energy at a particular frequency, the lower the entropy-to-energy ratio. BR emission has the highest spectral energy radiance K_ν at all frequencies, and therefore the lowest entropy-to-energy ratio at all frequencies and the lowest entropy-to-energy ratio of all TR with the same emission temperature.

Figure 1 shows a graybody radiation (GR) emission spectrum at an emission temperature T, BR emission spectrum at the same emission temperature (T), and BR emission spectrum at the same energy level as the graybody radiation. Three observations regarding TR, all consistent with Planck's blackbody formulas, are illustrated in *Figure 1*,

1) BR emission has the highest energy and entropy emission of all TRs with the same emission temperature,

2) BR has the highest entropy of all TRs with the same energy radiance Enclosed isolated TR systems will tend to equilibrate to BR with the same emission energy level, and,

3) BR has the lowest ratio of entropy-to-energy, and the minimum value (4/3) of the

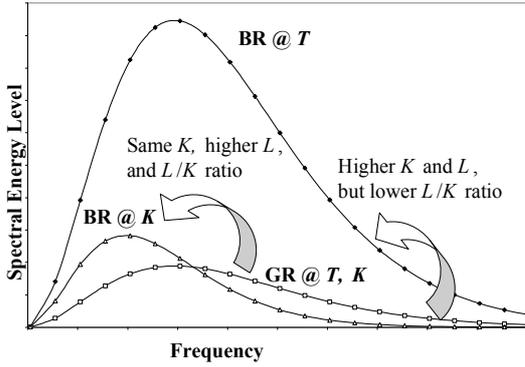coefficient n, for all TR emissions with the same emission temperature.

Figure 1. Qualitative entropy and energy considerations for BR and NBR emissions

Also, a BR emission spectrum, with emission temperature T, encloses all TR spectrums with a lower emission temperature, as well as all NBR spectrums with the same emission temperature. Combining this with the fact that the entropy-to-energy ratio of BR increases with decreasing emission temperature (proportional to the inverse of temperature), one may deduce a fourth point,

4) BR emission has the lowest ratio of entropy-to-energy for all TRs with the same or lower emission temperature, that is, all TR enclosed under its energy spectrum.

4. The Net Entropy Flux with Blackbody Radiation Transfer

The focus of the present analysis is to consider the net entropy flux by radiative heat transfer. That is, to consider the combination of emitted, incident and reflected fluxes commonly seen in practice rather than just emission alone. All matter emits radiation, and consequently, most calculations of net transfer at a surface involve incident and reflected fluxes from surrounding materials. One difficulty is that the entropy flux of reflected and emitted radiation cannot be calculated independently. This is due to the fact that their frequency spectrums overlap and the radiation travels in the same hemisphere of directions away from the surface.

For the determination of the net entropy flux from a surface for ease of calculation and comparison one may express the net entropy flux such that

$$J_{Net} = n\frac{H_{Net}}{T} \qquad (13)$$

where T is the emission temperature of the surface and the coefficient n is unique to the surface character, its temperature and the incident radiation flux. In this section, we will consider a blackbody surface with temperature T and with incident isotropic BR with emission temperature T_S, as depicted in *Figure 2*. For simplicity and clarity, the only form of heat transfer from the surface considered is radiation transfer, so the pressure of the fluid or gas adjacent to the surface may be seen as zero or effectively zero.
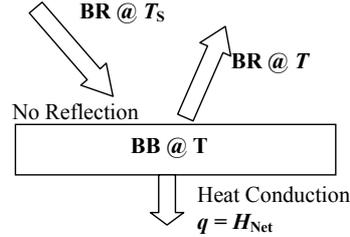

Figure 2. net radiation transfer from a BB surface with incident BR

The illustration in *Figure 2* depicts heat transfer by conduction from the BB absorber, indicating that the incident BR flux has a higher emission temperature than the BB surface, $T_S > T$. However, this analysis will also consider cases where $T > T_S$ and heat is supplied to the absorber by heat conduction.

For this scenario the entropy coefficient n can be expressed, after some algebraic manipulation as

$$n = \frac{J_{Net}}{(H_{Net}/T)} = \frac{4\theta(1-\theta^3)}{3(1-\theta^4)} \qquad (14)$$

where θ is the temperature ratio T/T_S. *Figure 3* shows the plot of the entropy coefficient n against the temperature ratio θ.

Figure 3 provides a graphical tool that can be used to determine the entropy coefficient n given the temperatures T and T_S. For θ greater than unity ($T > T_S$), it can be seen in *Figure 3* that the entropy flux coefficient (n) asymptotically approaches the value of 4/3 that occurs for BR emission. This is due to the fact that there is no reflected radiation and at higher surface temperatures the emitted flux dominates the net energy and entropy flux calculations. At θ equal to unity ($T = T_S$) there is no error, $n = 1$, simply because there is no net energy transfer through the surface ($q = 0$).

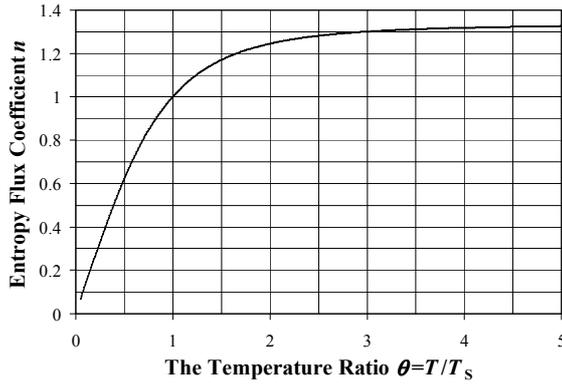*Figure 3. The entropy coefficient n versus the temperature ratio θ for BR (T_S) incident on a BR surface (T)*

For θ less than unity ($T < T_S$), *Figure 3* shows that the coefficient n actually approaches the value of zero. This can be explained by considering a temperature very near absolute zero. In this case there is very little radiation emitted from the surface, while the incoming BR at T_S carries an arbitrary amount of entropy towards the surface. However, the absorption of the incoming BR by the absorber with temperature near absolute zero results in comparatively large entropy production. This is because the entropy flux of heat conduction near absolute zero is much greater than the entropy carried by BR at the same energy flux. In the limit of T approaching zero, the incoming entropy flux by BR is inconsequential and results in a value of n approaching zero, since n is the ratio of the net entropy flux by radiation to that by heat conduction with the same energy flux.

It is evident from *Figure 3* that the entropy flux coefficient n can take on values from zero to a maximum value of 4/3. This means that the incorrect application of the entropy flux expression for heat conduction can either underestimate the actual net entropy flux by radiation for $T > T_S$, or overestimate it for $T < T_S$.

5. The Net Entropy Flux for a Graybody Surface

In this section a more complex scenario is considered where reflected radiative fluxes are also involved. Incident BR (T_S) is considered on a graybody surface (ε,T). The graybody model is often implemented in practice and represents a fairly good approximation for many scenarios in practice. The specification of incident BR also has reasonable practical application because, even if a surface is not surrounded by a blackbody, the incident radiation from a relatively large enclosure with non-blackbody character is accurately approximated as BR due to multiple reflections and geometry considerations.

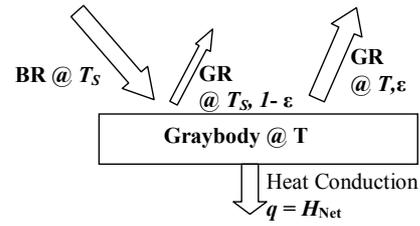The radiative fluxes involved in this analysis are depicted in *Figure 4*.

Figure 4. Net radiation transfer from a graybody surface with incident BR

The entropy flux of the reflected BR and the emitted GR are not independent because their energy spectrums overlap, as depicted in *Figure 5*.

In *Figure 5* a number of example GRs reflected spectrums with different emission temperatures T_S, along with an example emitted GR spectrum with emission temperature T. The *Figure* shows that, in general, there is no scenario where the reflected and emitted spectrums do not significantly overlap. When θ is greater than unity, overlap of these spectrums may not be complete. For θ less than unity, there is a 100% overlap of the energy spectrums.
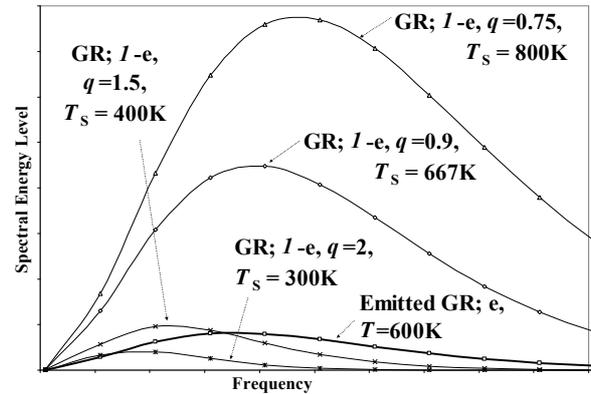

Figure 5. Overlap of an emitted graybody spectrum and graybody reflected spectrums with different emission temperatures

The net energy flux is given by

$$H_{Net} = \varepsilon\sigma\left(T_S^4 - T^4\right) \qquad (15)$$

The net entropy flux is

$$J_{Net} = \tfrac{4}{3}\sigma T_S^3 - \tfrac{4}{3}\sigma T^3 \int_0^\infty \tfrac{45}{4\pi^4} x^2 [(1+f)\ln(1+f)$$

$$- f \ln f]dx \quad (16)$$

where

$$f = \frac{1-\varepsilon}{e^{x_s}-1} + \frac{\varepsilon}{e^x-1} \qquad (17)$$

and where $x = h\upsilon/kT$ and $x_S = h\upsilon/kT_S = x\theta$. The entropy coefficient n for the net entropy flux can be expressed as

$$n = \frac{\frac{4}{3}\theta\left\{1 - \frac{45}{4\pi^4}\theta^3 \int_0^\infty x^2[(1+f)\ln(1+f) - f\ln f]dx\right\}}{\varepsilon(1-\theta^4)}$$

$$(18)$$

The entropy coefficient (18) is plotted in *Figure 6* for the emissivities 0.01, 0.05 and in increments of 0.05 up to 1.00. *Figures 6, 7* and *8* provide graphical tools that can be used to determine the entropy coefficient n given the temperature's ratio $\theta = T/T_S$, and the emissivity.

It is clear from *Figure 6* that the entropy coefficient has a broad range of values depending on the value of the emissivity and the temperature ratio θ. For high temperature ratios θ as in *Figure 7*, the entropy flux coefficient (n) asymptotically approaches a fixed value. This fixed value is higher for lower emissivities, and can be greater than 2.5 times the value given by the heat conduction expression q/T.
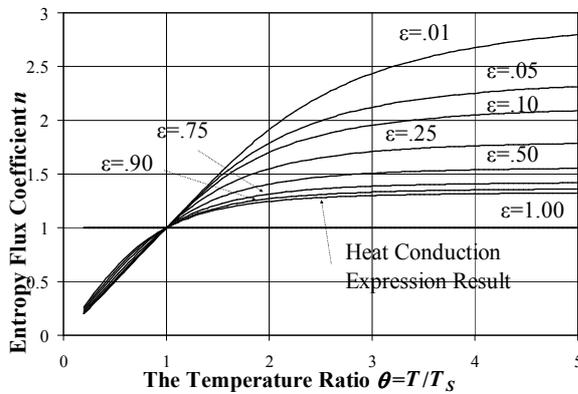
Figure 6. The entropy coefficient n versus the temperature ratio θ for BR (T_S) incident on a GR surface (ε,T); overview

For $\theta > 1$, lower emissivity is always associated with higher values of the entropy coefficient n. It may also be noted that for $\theta < 1$ ($T < T_S$), as in *Figure 8*, the entropy coefficient n is approximately equal to the temperature ratio ($n \approx \theta$) for low emissivities.
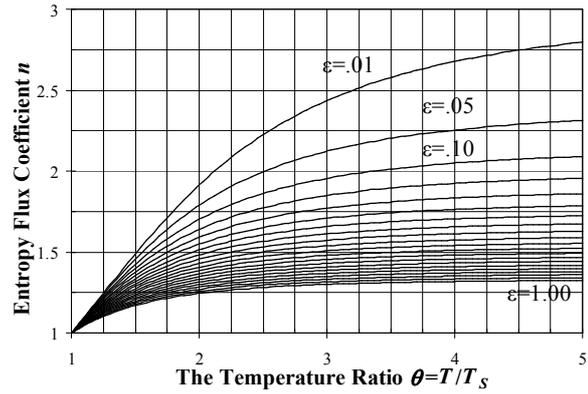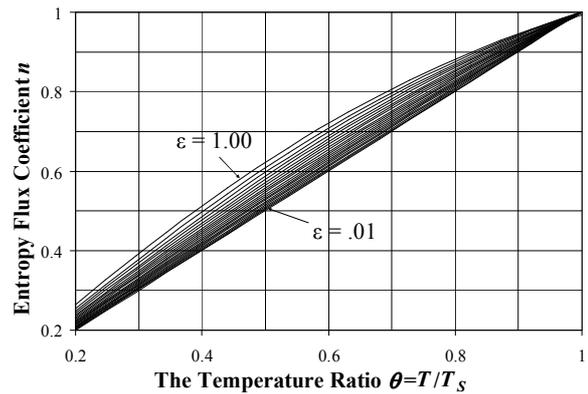

Figure 7. The entropy coefficient n versus the temperature ratio θ for BR (T_S) incident on a GR surface (ε,T); θ > 1

The approximations for the entropy of GR (8) and (9) cannot be used for the reflected and emitted entropy fluxes since they cannot be calculated independently. To obtain an approximate analytical expression, the entropy of the combined emitted and reflected spectrums may be approximated as that of BR with the same energy flux. This is done by calculating an effective BR temperature (T_{eff}) from the energy equality, and then using this effective temperature in the BR entropy expression

$$H_{\substack{\text{Ref,}\\\text{Emi}}} = \sigma\left\{\varepsilon T^4 + (1-\varepsilon)T_S^4\right\} \equiv \sigma T_{\text{Eff}}^4 \qquad (19)$$

and

$$J_{\substack{\text{Ref,}\\\text{Emi}}} \approx \tfrac{4}{3}\sigma T_{\text{Eff}}^3 = \tfrac{4}{3}\sigma\left\{\varepsilon T^4 + (1-\varepsilon)T_S^4\right\}^{3/4} \qquad (20)$$

Figure 8. The entropy coefficient n versus the temperature ratio θ for BR (T_S) incident on a GR surface (ε,T); θ < 1

With the approximation (20), and after some algebraic manipulation, the entropy coefficient n can be expressed as

$$n = \frac{J_{Net}}{(H_{Net}/T)} = \frac{\frac{4}{3}\theta\left\{1 - \left[\varepsilon\theta^4 + (1-\varepsilon)\right]^{3/4}\right\}}{\varepsilon(1-\theta^4)} \quad (21)$$

The error in the approximation (20) increases with increasing temperature ratio θ and decreasing emissivity. The absolute error in the entropy coefficient n, given by (21), is within 0.06 for $0 < \theta \le 2.5$ and $\varepsilon \ge 0.20$.

6. The Net Entropy Flux for Incident Graybody Radiation on a Blackbody Surface

In this final section, the analysis will consider another common scenario of TR transfer, non-blackbody radiation (NBR) incident on a blackbody surface. The incident NBR will be modeled as graybody radiation (GR). The radiative fluxes involved in this analysis are depicted in *Figure 9*.
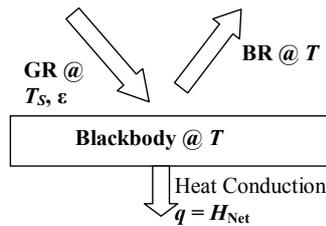
Figure 9. net radiation transfer for incident GR on a blackbody surface

The net energy flux is simply

$$H_{Net} = \sigma\left(\varepsilon T_S^4 - T^4\right) \quad (22)$$

The net entropy flux, using the approximation (8) for the incident flux, is

$$J_{Net} = \varepsilon\left(\frac{4}{3}\sigma T_S^3\right)\left[1 - \frac{45}{4\pi^4}(2.336 - 0.260\varepsilon)\ln\varepsilon\right] - \frac{4}{3}\sigma T^3 \quad (23)$$

and the entropy coefficient n can be expressed, after some algebraic manipulation, as

$$n = \frac{\frac{4}{3}\theta\left\{\varepsilon\left[1 - \frac{45}{4\pi^4}(2.336 - 0.260\varepsilon)\ln\varepsilon\right] - \theta^3\right\}}{\varepsilon - \theta^4} \quad (24)$$

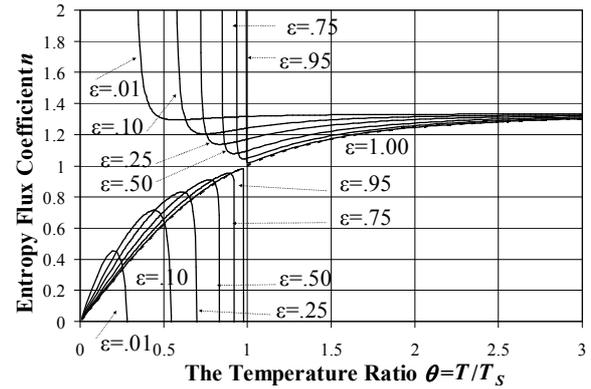*Figure 10* depicts the entropy coefficient n for incident GR on a BB surface.

Figure 10. The entropy coefficient n versus the temperature ratio θ for GR (ε, T_S) incident on a blackbody surface (T)

Unlike the entropy coefficient plots in the last two scenarios, plots for $\varepsilon < 1$ do not pass through the point were $n = 1$ and $\theta = 1$ ($T = T_S$). That is, n is greater than unity ($n > 1$) when T and T_S are equal. There is a positive amount of energy transferred away from the surface by radiation, and the net entropy carried away by radiation is greater than that given by q/T. For high θ, incident GR becomes insignificant, and the entropy coefficient for net transfer approaches the value for BR emission. Note that the plot for emissivity of unity in *Figure 10* represents the case of BR at T_S incident on a BB surface at temperature T (as in *Figure 3*).

In the region $\theta < 1$ ($T < T_S$), for each emissivity, the entropy coefficient approaches positive and negative infinity at the location $\theta_{q=0} = \varepsilon^{1/4}$, where the subscript '$q = 0$ 'on θ signifies that the net heat flux by radiation is zero at this temperature ratio. Approaching this location from higher values of θ, results in the limit of positive infinity ($+\infty$), and approaching from lower values of θ, results in the limit of negative infinity ($-\infty$). Regardless of approach direction, the heat flux through the surface approaches zero. The entropy transfer associated with this amount of heat conduction in the absorber is non-zero but small relative to the net entropy flux by radiation. However, the net transfer of entropy away from the surface by radiation is independent of the approach direction. This net entropy transfer by radiation represents the increase in entropy for GR conversion by a BB surface into BR with the same energy flux.

The percent entropy increase for the conversion of GR into BR with the same energy flux, using (8), is given by

$$\frac{J_{GR} - J_{BR}}{J_{GR}} = 1 - \varepsilon^{1/4}\left[1 - \frac{45}{4\pi^4}(2.336 - 0.260\varepsilon)\ln\varepsilon\right]$$

(25)

For $\varepsilon = 0.1$ as an example, the percent increase in entropy is 9.22%. That is, whatever the value of the incoming entropy of the GR flux, the outgoing entropy by BR at the same energy level is 9.22% higher. The entropy coefficient is simply the ratio of the entropy increase to the ratio of energy flow to absorber temperature. Thus, the sign of the entropy coefficient only depends on the sign of the net energy transfer as $\theta_{q=0}$ is approached, and may be negative or positive. The negative sign occurs when the net energy flux by radiation is in the opposite direction of the net entropy flux.

7. Conclusions

Blackbody radiation emission has the minimum entropy-to-energy ratio for all radiation with the same emission temperature. That is, the misuse of the heat conduction entropy flux expression for radiative heat transfer results in greater error for non-blackbody radiation (NBR) emission than for BR emission. In practice, analysis of radiative transfer includes incident, reflected and emitted fluxes. This net entropy flux can be put in the form $n(q/T)$, where n is a coefficient unique to the radiative fluxes involved. This allows the entropy flux to be easily calculated given the energy flux and the surface temperature of the absorbing material. This entropy coefficient can vary greatly, taking on values less than unity and values greater than unity. This means that misuse of the heat conduction entropy flux expression can vary from overestimating to underestimating the net radiative entropy flux. In the case of incident GR on a BB surface, it can take on large positive values as well as large negative values. Graphical tools and simplified approximate expressions are presented that allow the entropy coefficient n to be quickly determined in certain general scenarios of radiative transfer encountered in practice.

Nomenclature

TABLE 1. TR ENERGY AND ENTROPY NOMENCLATURE

	Energy		Entropy	
	Symbol	Units	Symbol	Units
Flow Rate	$\dot{E}$	W	$\dot{S}$	W/K
Irradiance	H	W/m^2	J	W/K m^2
Radiance	K	W/m^{2}sr	L	W/K m^{2}sr
Spectral Radiance	K_ν	J/m^{2}sr	L_ν	J/Km^{2}sr

c	Speed of light = $(2.9979)10^8$ m/s
ε	Emissivity for gray radiation (GR)
k	Boltzmann's constant = $(1.38)10^{-23}$ J/K
h	Planck's constant = $(6.626)10^{-34}$ J s
n	Entropy coefficient in Equation (10) and (13)
q	Heat flux (W/m^2)
T	Material emission temperature or temperature of a gaseous system (K)
T_S	Emission temperature of Source Radiation (K)
x	Non-dimensional group, $h\nu/kT$
x_S	Non-dimensional group, $h\nu/kT_S$

Greek

θ	Temperature ratio T/T_S
ν	Frequency (s^{-1})
σ	Stefan-Boltzmann constant = $(5.67)10^{-8}$W/m^{2}K^4
π	Mathematical constant, 3.14159…

Acronyms

BB	Blackbody
BR	Blackbody radiation
DBR	Diluted blackbody radiation
GR	Graybody radiation
NBR	Non-blackbody radiation

References

A. Bejan, 1987, "Unification of Three Different Theories Concerning the Ideal Conversion of Enclosed Radiation", *ASME Journal of Solar Energy Engineering*, Vol. 109, pp. 46-51.

A. Bejan, 2006, *Advanced Engineering Thermodynamics*, 3rd edition, John Wiley and Sons, New Jersey.

G. P. Beretta and E. P. Gyftopoulos, 1990, "Electromagnetic Radiation: A Carrier of Energy and Entropy, *Fundamentals of Thermodynamics and Exergy Analysis*, ASME Advanced Energy Systems Division, Winter Annual Meeting, Vol. 19.

Y. A. Cengel and M. A. Boles, 2006, *Thermodynamics: An Engineering Approach*, 5th edition, Wiley, New York.

A. De Vos and H. Pauwels, 1986, Discussion, *ASME Journal of Solar Energy Engineering*, Vol. 108, pp. 80-84.

R. A. Gaggioli, 1998, "Available Energy and Exergy", *Int. J. Applied Thermodynamics*, Vol. 1, pp. 1-8.

A. F. Haught, 1984, "Physics Considerations of Solar Energy Conversion", *ASME Journal of Solar Energy Engineering*, Vol. 106, pp. 3-15.

M. J. Moran and H. N. Shapiro, 2004, *Fundamentals of Engineering Thermodynamics*, edition, Wiley, USA.

P. T. Landsberg and G. Tonge, 1979, "Thermodynamics of the Conversion of Diluted Radiation", *J. Phys. A: Math. Gen.*, Vol. 12, Issue 4, pp. 551-562.

P. Landsberg and G.Tonge, 1980, "Thermodynamic Energy Conversion Efficiencies", *J. Appl. Phys.*, Vol. 51, Issue 7, pp. R1-R20.

P. T. Landsberg, 1984, "Non-Equilibrium Concepts in Solar Energy Conversion", *Proceedings of the NATO Advanced Study Institute on Energy Transfer Processes in Condensed Matter*, June 16-30, 1983, Sicily, Italy, edited by B. Di Bartolo, Plenum Press, New York, pp. 537-592.

L. H. Liu and S. X. Chu, 2006, "On the Entropy Generation Formula of Radiation Heat Transfer", *Transactions of the ASME*, Vol. 128.

R. Petela, 1964, "Exergy of Heat Radiation", *ASME Journal of Heat Transfer*, Vol. 86, pp. 187-192.

M. Planck, 1914, *The Theory of Heat Radiation*, translation by Morton Mausius, Dover Publications, NY.

M. Santillan, G. Ares de Parga, and F. Angulo-Brown, 1998, "Black-body radiation and the maximum entropy production regime", *Eur. J. Phys.*, Vol. 19.

G. L. Stephens and D. M. Obrien, 1993, "Entropy and Climate. I: ERBE Observations of the Entropy Production of the Earth", *Quarterly Journal of the Royal Meteorological Society,* Vol. 119, pp. 121-152.

S. E. Wright, D. S. Scott, M. A. Rosen, and J. B. Haddow, 2001, "On the Entropy of Radiative Heat Transfer in Engineering Thermodynamics", *Int. J. Eng. Sci.*, Vol. 39, pp. 1691-1706.

S. E. Wright, M. A. Rosen, D. S. Scott and J. B. Haddow, 2002, "The Exergy Flux of Radiative Heat Transfer for the Special Case of Blackbody Radiation", *Exergy, An International Journal.*