

## Modelling and Stability of a Single-Shaft Combined Cycle Power Plant

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### Abstract

The subject of this paper is the development of a dynamic model for a single-shaft combined cycle plant and the analysis of its response to electrical load and frequency transients. In particular the stability of the frequency control, as well as the implications of overheat control are investigated. The model is developed in the Simulink environment of Matlab as part of an educational and research simulation package for autonomous and interconnected systems.

*Keywords: Combined cycle plant, gas turbines, steam turbines, frequency control, modelling, simulation*

### 1. Introduction

During the last decades there has been continuous development of combined cycle power plants due to their increased efficiency and their low emissions. The dynamic response of such power plants to load and frequency transients is rather problematic, since the compressor and the fuel supply system are both attached to the shaft of the unit. Thus rotor speed and frequency have a direct effect on air and fuel supply, which introduces a negative effect on system stability (CIGRE, 2003).

In addition, combined cycle power plants function on the temperature limits (above a relatively low power level) so as to achieve the best efficiency in the steam generator (CIGRE, 2003). This fact raises further issues relative to the response of combined cycle power plants (CCPP) during frequency drops or variations at load power. Temperature should be maintained (apart from the first seconds of the disturbance) below certain limits for the protection of the plant.

This paper is based on the modelling proposed in Kakimoto and Baba (2003) and Rowen (1983), while the model developed is integrated into an educational and research simulation package developed in the Electrical Energy Systems Lab of NTUA (Vournas et al., 2004). Other similar models are presented in Kunitomi et al. (2003), Lalor et al. (2005), Zhang and So (2000), which, however, are slightly different. For instance, in Lalor and O'Malley (2003) the structure of the steam turbine is more detailed, while in this paper we use a simplified

steam turbine model, assuming that the power generation depends exclusively on the heat recovery from the gas turbine. On the other hand, our paper examines more closely the limits of airflow control and extends the results of Kakimoto and Baba (2003) by:

- including a supervisory control of the combustion temperature,
- presenting a stability analysis of control loops through linearization and eigenvalues,
- proposing a method to stabilize the plant response to frequency transients during full load operation.

The paper is structured as follows: In Section 2 the plant model, the control loops, and the steady-state operating regions are presented. Section 3 demonstrates the necessity for the introduction of speed control. Section 4 refers to the stability of control loops and presents the eigenvalues of the linearized model, as well as a stable response to a frequency drop. In Section 5 a case of unstable response for the same disturbance under full load operation is presented and a solution is proposed, in order to stabilize the response of the plant by increasing airflow gate opening limits. The final section forms the conclusions of the paper.

### 2. Combined Cycle Plant Model

#### 2.1 Steam-Gas Turbine

The combined cycle power plant model shown in Fig. 1 consists of the power generation units and the control branches. The thermodynamic part giving the available thermal power to the gas turbine and the steam turbine is

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modelled by algebraic equations, corresponding to the adiabatic compression and expansion, as well as to the heat exchange in the recovery boiler. These equations correspond to the block 'Algebraic equations of energy transform' in Fig.1. These algebraic equations are presented below (Spalding and Cole, 1973):

From the adiabatic compression equation the following relation holds, where  $x$  is the ratio of input-output temperatures for isentropic compression:

$$x = \frac{t_{d,is}}{t_i} = (P_r)^{\frac{\gamma-1}{\gamma}} \quad (1)$$

In (1)  $P_r$  is the actual compressor ratio. For nominal airflow ( $W=1$ pu), this is equal to the nominal ratio  $P_{r0}$ . When airflow is different from nominal ( $W \neq 1$ ), the actual compression ratio is

$$P_r = P_{r0}W \quad (2)$$

and therefore:

$$x = (P_{r0}W)^{\frac{\gamma-1}{\gamma}} \quad (3)$$

From the definition of compressor efficiency:

$$\eta_c = \frac{t_{d,is} - t_i}{t_d - t_i} \quad (4)$$

from which, based on the definition of  $x$  in (1)

$$t_d = t_i \left(1 + \frac{x-1}{\eta_c}\right) \quad (5)$$

The gas turbine inlet temperature depends on the fuel to air ratio (assuming that air is always in excess). The temperature rises with the fuel injection  $W_f$  and decreases with airflow  $W$ .

From the energy balance equation in the combustion chamber, the following normalized equation results:

$$W \frac{t_f - t_d}{t_{f0} - t_{d0}} = W_f \quad (6)$$

or:

$$t_f = t_d + (t_{f0} - t_{d0}) \frac{W_f}{W} \quad (7)$$

Similar to (4), the gas turbine efficiency is given by

$$\eta_t = \frac{t_f - t_e}{t_f - t_{e,is}} \quad (8)$$

For the adiabatic expansion, noting from (3) that the right hand side is the same as in the compression (the mass that enters the

compressor is the same with the one in the output of the gas turbine) we have:

$$x = \frac{t_f}{t_{e,is}} \quad (9)$$

from which we obtain for the actual exhaust temperature similarly to (5):

$$t_e = t_f \left[1 - \left(1 - \frac{1}{x}\right)\eta_t\right] \quad (10)$$

The power produced by the gas turbine is proportional to temperature difference ( $t_f - t_e$ ), and the mechanical power consumed in the compressor is proportional to ( $t_d - t_i$ ). Both are also proportional to airflow  $W$  (we assume that the mixture of air and gas is almost equal to airflow). Therefore the net power converted to mechanical is:

$$E_g = K_0[(t_f - t_e) - (t_d - t_i)]W \quad (11)$$

The thermal power absorbed by the heat exchanger of the recovery boiler is proportional to airflow and exhaust temperature.

$$E_s = K_1 t_e W \quad (12)$$

The control branches and the transfer functions are shown in Fig. 1. Note that in the control loop temperature variables are replaced by normalized variables  $T_f, T_e$  defined as:

$$T_f = \frac{t_f - 273}{t_{f0}} \quad (13)$$

$$T_e = \frac{t_e - 273}{t_{e0}} \quad (14)$$

Note that  $t_f$  and  $t_e$  are in Kelvin degrees while  $t_{f0}$  and  $t_{e0}$  are in Celsius. Thus for normalized conditions  $T_f = T_e = 1$  (pu).

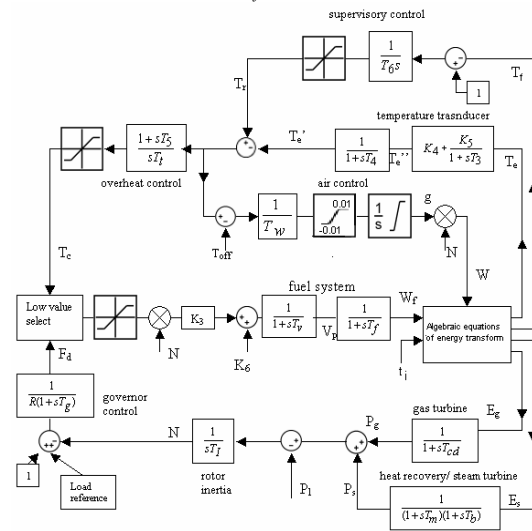


Figure 1. Single-shaft combined cycle model

TABLE I. MODEL PARAMETERS

	Parameter	Value
$t_{i0}$	Ambient temperature (K)	303
$t_{d0}$	Nominal compressors discharge temperature (C)	390
$t_{f0}$	Nominal gas turbine inlet temperature (C)	1085
$t_{e0}$	Nominal exhaust temperature (C)	532
$P_{r0}$	Nominal compressor pressure ratio	11.5
$\gamma$	Ratio of specific heat ( $C_p/C_v$ )	1.4
$\eta_c$	Compressor efficiency	0.85
$\eta_t$	Turbine efficiency	0.85
$K_0$	Gas turbine output coefficient (1/K)	0.00303
$K_1$	Steam turbine output coefficient (1/K)	0.000428
R	Speed governor regulation	0.04
$T_g$	Governor time constant (s)	0.05
$K_4$	Gain of radiation shield (instantaneous)	0.8
$K_5$	Gain of radiation shield	0.2
$T_3$	Time constant of radiation shield (s)	15
$T_4$	Time constant of thermocouple (s)	2.5
$T_5$	Time constant of temperature control (overheat) (s)	3.3
$T_t$	Temperature control (overheat) integration rate (s)	0.4699
$T_{c \max}$	Temperature control upper limit	1.1
$T_{c \min}$	Temperature control lower limit	0
$F_{d \max}$	Fuel control upper limit	1.5
$F_{d \min}$	Fuel control lower limit	0
$K_3$	Ratio of fuel adjustment	0.77
$K_6$	Fuel valve lower limit	0.23
$T_v$	Valve positioner time constant (s)	0.05
$T_F$	Fuel system time constant (s)	0.4
$T_6$	Time constant of $T_f$ control (s)	60
$g_{\max}$	Air valve upper limit	1.001
$g_{\min}$	Air valve lower limit	0.73
$T_w$	Time constant of air control (s)	0.4699
$T_{cd}$	Gas turbine time constant (s)	0.2
$T_m$	Steam turbine time constant (s)	5
$T_b$	Heat recovery boiler time constant (s)	20
$T_I$	Turbine rotor inertia constant (s)	18.5
$T_{\text{off}}$	Temperature offset	0.01

Also note that the airflow depends on the product of gate opening and rotor speed:

$$W = g \cdot N \quad (15)$$

## 2.2 Control Loops

Two control loops are introduced so that the combined cycle unit functions properly. The first one is the frequency control loop, which includes the speed governor. The second one is the overheat control loop.

*Speed control:* The first loop involves the speed governor, which detects frequency deviation from the nominal value and determines the fuel demand signal ( $F_d$ ) so as to balance the difference between generation and load. Autonomous operation is assumed, so power imbalances will cause electrical frequency deviations as shown in the rotor inertia block of Fig. 1.

*Temperature control:* The second loop is the temperature control and consists of two branches. The normal temperature control branch acts through the air supply control. When the temperature of the exhaust gases exceeds its reference value ( $T_r$ ), this controller acts on the air valves to increase the airflow, so as to decrease exhaust gas temperature (air control loop in Fig. 1). In certain situations, however, this normal temperature control is not enough to maintain safe temperatures. Thus, in cases of a severe overheat, the fuel control signal is reduced through a low-value-select function (LVS) that determines the actual fuel flow into the combustion chamber.

Reference temperature ( $T_r$ ) is the parameter defined by the supervisory control for the exhaust gas temperature ( $T_e$ ). This control branch reacts by decreasing  $T_r$  when gas turbine inlet temperature ( $T_f$ ) exceeds its nominal value.

*Low value select:* Inputs to the LVS are the fuel demand signal determined by the speed governor and an overheat control variable, which decreases from an initial ceiling value when the exhaust temperature exceeds its reference. During the operation of the unit, only one of the control branches is active, the one whose control variable has the lowest value.

## 2.3 Operating regions

As can be easily observed, there are eight independent variables in the algebraic equations (3), (5), (7) and (10)-(12). Thus, in order to solve the system during dynamic simulation it is necessary to define fuel flow and airflow, as shown in the corresponding block of Fig. 1.

In steady state and for initialization purposes, one required parameter is the plant output  $P$  (in percent of rated), which is equal to the sum of the normalized generation of the two turbines, i.e.:

$$E_g + E_s = P \quad (16)$$

However, it is necessary to define one more variable in order to determine the operating point of the system in steady state. Depending upon which variable is defined we can distinguish three operating regions for the combined cycle plant.

In operating region I the exhaust temperature is lower than the nominal value (light loading of the plant). This is because the exhaust temperature is lower than reference temperature, so the input of the air control branch is negative in steady state and the airflow gate opening obtains its minimum value.

As plant production increases (with airflow fixed at its minimum value), it is obvious that the exhaust temperature will continue to rise as the fuel flow rises (Fig. 2). When  $T_e$  reaches the nominal value, the plant enters region II. The exhaust temperature tends to exceed the reference value and thus air control is activated and the exhaust temperature is fixed at its nominal level. In other words operating region II is the one where the temperature control is active to keep the exhaust temperature  $T_e$  at its nominal value, while the combustion temperature  $T_f$  is below its rated value (intermediate loading). In this area,  $T_e$  is equal to nominal.

In the operating region III (heavy load),  $T_f$  tends to exceed its nominal value. In this case the supervisory combustion temperature control designed in this paper is activated to change the exhaust temperature reference so as to control  $T_f$  to its nominal value. So the supervisory control sets  $T_f$  at its nominal value and decreases the exhaust temperature below nominal, by adjusting the reference temperature.

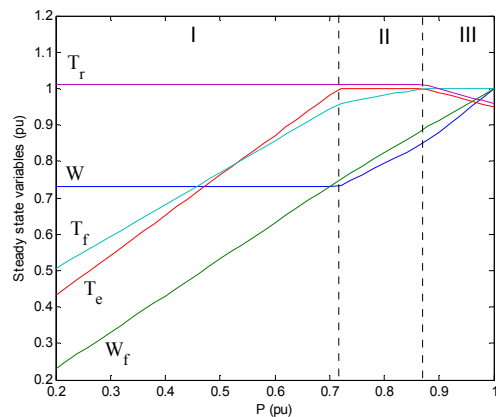


Figure 2. Operating regions in steady state

In Fig. 2 the three operating regions described above are plotted in steady state as a function of power output and some critical variables. The data used for the combined cycle plant are shown in Table I (Kakimoto and Baba, 2003; Rowen, 1983).

### 3. Frequency Control Requirement

In single shaft combined cycle power plants, the fuel control system as well as the fans, which provide air to the compressor, are attached to the generator shaft, so their performance is directly linked to rotor speed. This affects the stability of the system as will be demonstrated in this section.

Without speed (frequency) control, a decrease in frequency will result in a decrease in air and fuel flow ( $W$  and  $W_f$ ), as the shaft of the plant will reduce its speed. The decrease of these two parameters will cause a decrease in power generation and consequently the frequency will continue to drop. So even if only a small transient disturbance is applied, it will eventually lead the uncontrolled unit out of service, as at some point the underfrequency protection of the plant will be activated. The response of frequency to such a temporary load increase without speed control is shown in Fig. 3 together with the disturbance in electrical load.

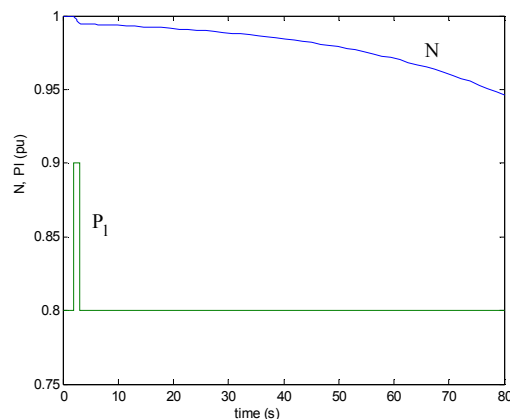


Figure 3. Frequency response to a load disturbance without control loops

Therefore, it becomes obvious that speed regulation is absolutely necessary for the stability of the combined cycle plant.

### 4. Stability of Control Loops

The low-value-select (LVS) function acts like a switch that activates one of the two control loops (frequency or overheat) by selecting the lower value of the two control variables ( $T_c$  or  $F_d$ ). Since, as discussed above, the speed control is vital for the stability of the system, its temporary interruption by the LVS during overheat conditions is critical.

The calculation of linearized system eigenvalues is done separately for the case where the speed control, or the temperature control, branch is active. The calculated eigenvalues are presented in Table II. This table shows that when the speed loop is active the

system is stable, whereas with the LVS switched to overheat control, the system becomes unstable with a positive eigenvalue corresponding to shaft speed. Thus, a necessary condition for the system to achieve a steady state after a disturbance is that the LVS reactivates the speed control soon enough. The other eigenvalues of Table II demonstrate a satisfactory behavior (relatively fast and without undamped oscillations) of the combined cycle plant.

TABLE II. EIGENVALUES OF THE MODEL (s<sup>-1</sup>)

Speed control		Temperature control	
Eigenvalue	Dominant variable	Eigenvalue	
-20.85	V <sub>p</sub>	-20.231	
-18.996	F <sub>d</sub>	-20	
-6.037	P <sub>g</sub>	-4.9764	
-0.781 ± 1.064j	N	T <sub>e</sub> '	-1.1644
	W <sub>f</sub>	W <sub>f</sub>	± 1.87368j
-0.2168	T <sub>e</sub> '	N	+0.042793
± 0.8587j	g	g	-0.38359
-0.16097	P <sub>s</sub>	-0.196	
-0.08819	Q	-0.0598146	
-0.0804	T <sub>e</sub> ''	-0.084078	
-4.86644	T <sub>r</sub>	-4.86644	

This type of control (switching between stable and unstable systems) is typical of “sliding mode” control systems (Utkin et al., 1999). It should be noted that a switching system could become temporarily unstable and still maintain its overall stability if the switching device (LVS in our model) performs properly.

At this point it is necessary to note that the airflow control branch, which performs the normal temperature control, has a major influence on the overall stability of the system. When this branch is active, it is possible to increase the power generation without overheating, as a proportional increase of fuel and air will maintain the combustion temperature constant according to (7). Thus, with proper airflow control, even if the system passes temporarily to the unstable operation, it can eventually end up in a stable steady state.

During a disturbance (e.g. decrease of frequency or increase of the load), due to the high gain and the small time constant of the speed control loop, a quick increase of the fuel demand signal is observed. Thus, soon after the disturbance, the LVS function activates the temperature control to avoid overheating. When the air control functions properly, it allows the increase of the power produced with constant temperature. In this way the frequency error

decreases, resulting in the decrease of fuel demand F<sub>d</sub> and the system returns to the stable operation of frequency control and ends up in steady state. If, on the other hand, the generation is not increased, the system does not return to the stable loop of speed control and the plant will go out of service.

The normal response of the plant for an instantaneous 3% frequency drop is shown in Figs. 4-6 for an initial operating point corresponding to 85% of nominal power output. The frequency drop through the speed governor control immediately results in an increase of F<sub>d</sub> causing the value of this parameter to exceed the ceiling of 110%, while temperatures increase and overheat control is activated. This limits the fuel flow and power production, as shown in the figures.

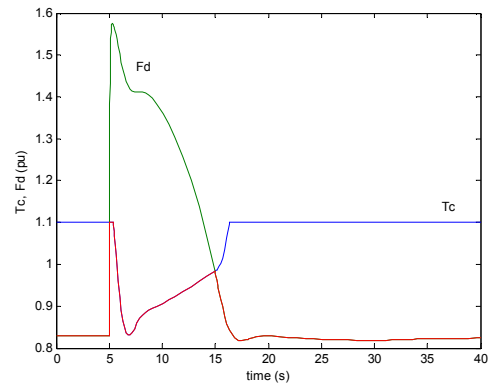


Figure 4. P=0.85, Inputs of LVS

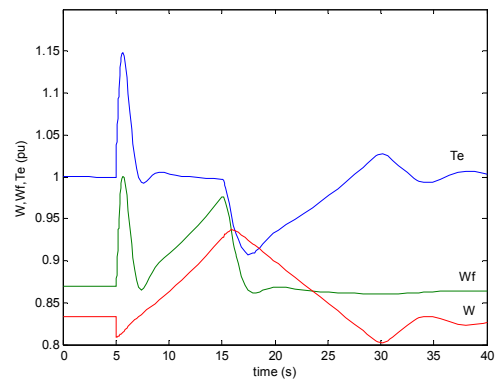


Figure 5. P=0.85, T<sub>e</sub>, W<sub>f</sub>, W responses

However, after the first post-disturbance period of two seconds, the airflow continues to increase and determines the response of the model. Although the unstable branch is activated for 10 seconds, the proper functioning of air control allows an increase in fuel consumption and thus the frequency error and the fuel demand signal decrease again, so that eventually speed control is activated again, making the response stable.

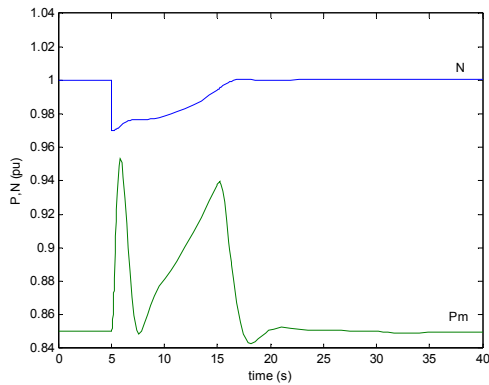


Figure 6.  $P=0.85$ , power and speed response

### 5. Air Flow Gate Opening Limits

The same disturbance is fatal if applied to the full load operating point, as seen in Figs. 7-9. In this case the plant cannot recover because the maximum value of airflow gate opening  $g$  is set to 100.1%, so during the disturbance it is not possible to increase the airflow, as in the previous example. Power generation is thus reduced to avoid overheating (fuel flow is decreasing) and the frequency experiences a continuous drop.

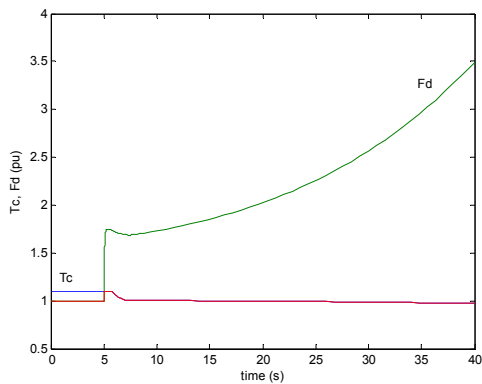


Figure 7.  $P=1$ , Inputs of LVS

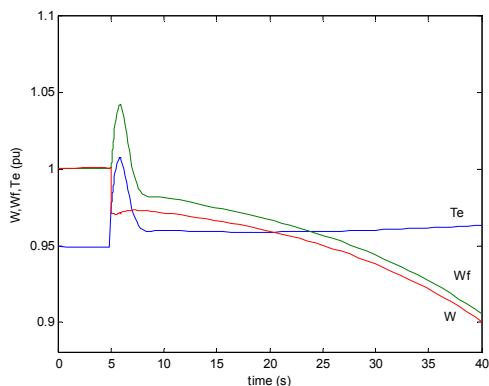


Figure 8.  $P=1$ ,  $T_e$ ,  $W_f$ ,  $W$  responses

Therefore, the crucial factor for the stability of the plant is the air control capability. In the example of this section the cause of the instability was the fact that there were no margins of increasing the airflow.

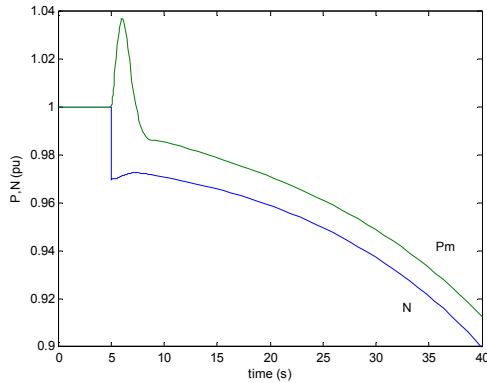


Figure 9.  $P_m=1$ , power and speed response

Increasing the maximum opening of the air valve beyond the nominal levels, the plant will improve its response to frequency disturbances. For example, Figs. 10-12 show the response of the model for the previous 3% frequency drop disturbance if the maximum opening of the air valve is increased by 10%. With dashed lines in Figs. 11-12, the responses of the previous example (with limited airflow) are shown.

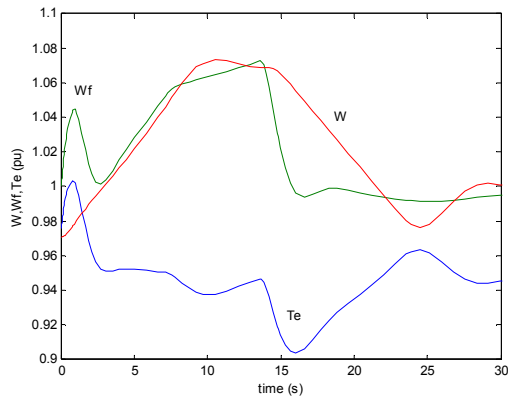


Figure 10.  $P=1$ ,  $g_{max}=1.1$ ,  $T_e$ ,  $W_f$ ,  $W$  response

As can be observed, the model has the same response during the first two seconds because air control response is slow anyway. Two seconds after the disturbance, when overheat control limits the fuel flow, the responses become distinct, since now the air control is allowed to open the air valves so that fuel flow can increase without overheating.

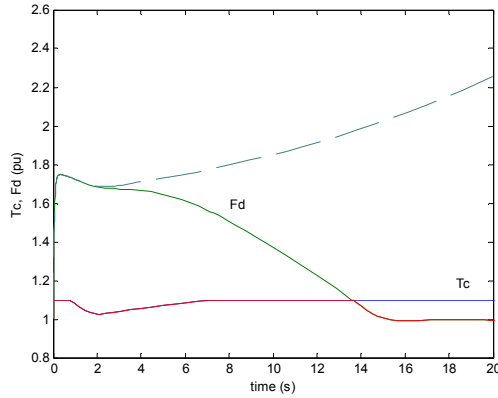


Figure 11.  $P=1$ ,  $g_{max}=1.1$ , Inputs of LVS

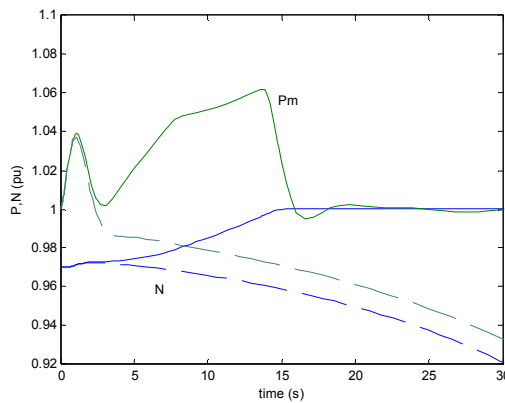


Figure 12.  $P=1$ ,  $g_{max}=1.1$ , power and speed

## 6. Conclusions

Within this paper we analyzed the model and the stability of a single shaft combined cycle plant, as well as its control loops. Furthermore we examined the behaviour of such a plant in frequency disturbances. The obtained results can be summarized as follows:

- A speed control loop is necessary for the stability of the plant, as the frequency feedback in fuel flow and airflow render the plant very sensitive to disturbances.

- Low value select function plays a major role in the response of the model. In case of overheat it enables the temperature control, which limits the fuel supply. Otherwise it enables the speed control loop, which is essential for system stability.

- Without frequency control, the plant is unstable, so in order to end up in steady state after a disturbance, a necessary condition is for the LVS to switch back to speed control.

- Air control contributes significantly to the stability of the plant, as it can help increase power generation, so as to correct frequency errors without creating overheat.

- Since the model response is relatively slow, some blocks with small time constants could be ignored in order to reduce the order of the model and simplify calculations.

Following the above discussion, in order to allow reliable and stable operation of the plant during full load, it was proposed to expand the airflow gate opening limits at least by 10% above that corresponding to the nominal full load air flow.

## Nomenclature<sup>1</sup>

$P$	generated power
$P_l$	load power
$P_g$	gas turbine generated power
$P_s$	steam turbine generated power
$E_g$	thermal power converted by the gas turbine
$E_s$	thermal power input of the heat recovery boiler
$g$	airflow gate opening
$t_e$	exhaust temperature
$T_e'$	measured exhaust temperature (pu)
$T_e''$	measured temperature at radiation shield (pu)
$t_f$	gas turbine inlet temperature
$t_d$	compressor discharge temperature
$T_r$	reference temperature (pu)
$t_i$	ambient temperature
$T_c$	overheat control signal
$t_{e0}$	nominal exhaust temperature (C)
$t_{f0}$	nominal gas turbine inlet temperature (C)
$F_d$	fuel demand signal
$K_0$	gas turbine output coefficient (1/K)
$K_1$	steam turbine output coefficient (1/K)
$P_r$	compressor pressure ratio
$N$	rotor speed (frequency)
$W$	airflow
$W_f$	fuel flow
$V_p$	fuel valve position
$Q$	heat flow from heat recovery boiler to steam
$R$	speed governor regulation
$T_g$	governor time constant (s)
$K_4$	gain of radiation shield (instantaneous)
$K_5$	gain of radiation shield

<sup>1</sup> Temperatures are in Kelvin degrees and other variables in per unit with respect to rated operating point, unless otherwise specified.

$T_3$	time constant of radiation shield (s)
$T_4$	time constant of thermocouple (s)
$T_5$	time constant of temperature control (overheat) (s)
$T_t$	overheat control integration rate (s)
$K_3$	ratio of fuel adjustment
$K_6$	fuel valve lower limit
$T_v$	valve positioner time constant (s)
$T_F$	fuel system time constant (s)
$T_6$	time constant of supervisory control (s)
$T_w$	time constant of air control (s)
$T_{cd}$	gas turbine time constant (s)
$T_m$	steam turbine time constant (s)
$T_b$	heat recovery boiler time constant (s)
$T_I$	turbine rotor inertia constant (s)
$T_{off}$	temperature offset

#### Greek letters

$\gamma$	specific heat ratio ( $c_p/c_v$ )
$\eta_c$	compressor efficiency
$\eta_t$	turbine efficiency

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