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Thermodynamic Optimization of GSHPS Heat Exchangers*

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Abstract

In this paper, a new method for determining the optimized dimensions of a ground source heat pump system (GSHPS) heat exchanger is presented. Using the GSHPS is one of the ways for utilization of infinite, clean and renewable energies in the environment. In recent years, due to limitation of physical space for installing the heat exchangers and avoiding the environmental effects on heat exchanger operation, vertical GSHP systems are used more than the other ones. Determination of optimum heat exchanger size is one of the most important parameters in the optimization of the heat exchanger design. In this study, optimum length and diameter for the heat exchanger is determined for different mass flows by using the second law of thermodynamics. The optimal length and diameter minimize entropy generation and therefore result in increased efficiency of the heat pump.

Keywords: GSHP, ground source heat pump, heat exchanger, entropy minimization, thermodynamical design.

1. Introduction

The natural environments contain unlimited resources of energy at low exergy levels. This energy is very cheap or even free. Hence the interest in its utilization increases with the increasing costs or inconvenience of obtaining the highly exergetic energy. The utilization of these resources is possible by using heat pumps. The principles of operation of heat pumps are similar to refrigeration equipment. One of the most interesting research fields in the study of heat exchangers is to find methods to increase the heat pump's efficiency. Most heat pumps are used for the heating or cooling of residential buildings. In these cases, air and soil are usually the only available resources and utilization of underground or surface water is usually impossible. There are three loops in each GSHP:

a) The first loop, with regard to its application, is an air/water or air/air loop which transfers heat from the warmer to the colder area.

b) The second loop is the refrigerant cycle.

c) The third loop is the ground heat exchanger which transfers heat from the ground/heat exchanger to the heat exchanger/ground.

The GSHPS is used more often than air types, because the ground source or sink temperature is more stable than the air temperature over time (Claesson and Eskilson, 1987).

In the GSHP, heat absorption is done by circulating the working fluid in the heat exchanger. This working fluid can be pure water, a mixture of water and anti-freeze, or brine that usually circulates in the high density polyethylene (HDPE) pipes installed vertically in boreholes (VGSHP) or horizontally in grooves (HGSHP).

HGSHPs are not commonly used in houses because these systems need more space for installation than the VGSHP and temperature variations have greater effects on them. As shown in Figure 1, in VGSHP systems, the heat exchanger usually consists of two or more boreholes (with depths between 20 and 90 meters). There is a u-tube in each borehole and the boreholes are filled with grout.

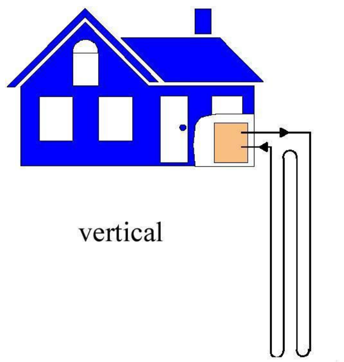

Figure 1. A vertical ground source heat pump

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In recent years, many analytic and numeric methods for the determination of u-tube sizes have been presented (Chiasson, 1999). The purpose of a thermodynamic design is to achieve a working system. A goal of the design is high efficiency; minimization of entropy production is a way to achieve this. For the determination of the u-tube optimum sizes, the total generated entropy should be minimized in the heat exchanger because, for a given heating load, there is a direct relationship between entropy generated and the required power input.

In this paper, the temperature distribution along the u-tube is analytically determined, and then with regard to the second law of thermodynamics, an equation is presented for the calculation of the generated entropy in the heat exchanger. By using this equation, the optimum Reynolds number and therefore the optimum length and diameter of the heat exchanger is determined. Finally an example application of this method is presented along with results and related figures.

2. Entropy generation through internal flows

The irreversibility of convective heat transfer is due to two effects (Bejan, 1988): a) heat transfer across a finite (non-zero) temperature difference, b) fluid friction.

Considering the flow passage through arbitrary cross section A and the wetted perimeter P, bulk properties of the stream m are T, h, s and p when heat is transferred to the stream at a rate q' [W/m], across a finite temperature difference. Focusing on a slice of thickness dx as a system, the rate of entropy generation is found with an entropy balance:

$$d\dot{S}_{gen} = \dot{m}ds - \frac{qdx}{T + \Delta T} \tag{1}$$

In order to illustrate the dependence of $\dot{S}_{gen}$ on the Stanton number and friction factor information, consider the case where the heat transfer rate per unit length q' and the mass flow rate m are specified; combining definition with formula yields:

$$\dot{S}'_{gen} = \frac{q^2}{4T^2 \dot{m} C_P} \cdot \frac{D}{St} + \frac{2\dot{m}^3}{\rho^2 T} \cdot \frac{f}{DA^2} \tag{2}$$

where

$$St = \frac{h}{\rho V C_P}$$

Under the present assumptions, Eq. (2) has two degrees of freedom, the wetted perimeter P and the cross-sectional area A or any other couple of independent parameters such as (Re_D, D). In a round tube of diameter D, the rate of entropy generation per unit length Eq. (2), assumes the form:

$$\dot{S}'_{gen} = \frac{q^2}{\pi k T^2 Nu\left((\mathrm{Re})_D, \mathrm{Pr}\right)} + \frac{32\dot{m}^3}{\pi^2 \rho^2 T} \cdot \frac{f\left((\mathrm{Re})_D\right)}{D^5} \tag{3}$$

Note that Eq. (3) depends on only one geometric parameter [D or $(\mathrm{Re})_D$]. As the tube diameter increases, $(\mathrm{Re})_D$ decreases; the interesting effect on S'_{gen} is that while the heat transfer contribution increases, the fluid friction term decreases. This is one example in which a variation of one geometric parameter always has opposite effects on the two terms of S_{gen}. Consequently, it is possible to determine the optimum tube diameter, or $(\mathrm{Re})_D$, which leads to minimum irreversibility. If the pipe flow is turbulent and fully developed, the Nusselt number and friction factor are given by the well known correlations:

$$Nu = 0.023[(\mathrm{Re})_D]^{0.8}(\mathrm{Pr})^{0.4}$$

$$f = 0.046\,[(\mathrm{Re})_D]^{-0.2}$$

Combining this formula with Eq. (3) yields (Bejan, 1994):

$$\dot{S}'_{gen} = \left(\frac{43.48q^2}{\pi k T^2\, \mathrm{Pr}^{0.4}}\right)\mathrm{Re}^{-0.8} + \left(1.44\times10^{-3}\right)\left(\frac{\pi^3\mu^5}{\rho^2 T\dot{m}^2}\right)\mathrm{Re}^{4.8} \tag{4}$$

3. Determination of entropy generated in the U-tube

For determination of entropy generated in the u-tube heat exchanger, we shall first determine the temperature distribution along the u-tube. Considering Eq. (4), as is illustrated in Figure 2 and shown in Eq. (5), the heat exchanger consists of two heat fluxes which are absorbed by working fluid in the heat exchanger: dq_{con} due to heat transfer from ground and dq_{int} due to heat transfer between the warmer and colder branches of u-tube.

$$dQ = dq_{con} + dq_{int} \tag{5}$$

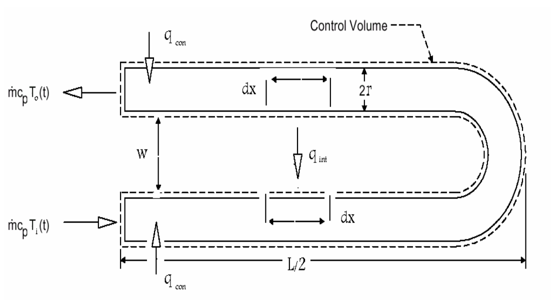

Figure 2. Heat fluxes in the u-tube heat exchanger

4. Heat flux due to heat transfer from ground

The ground temperature changes at the zones around the heat exchanger (Rybach, and Sanner, 2000):

First, when the heat pump starts its operation, the ground temperature decreases and after time, the ground temperature reaches a new stable temperature which is 1 to 2 Kelvin lower than the original temperature.

Second, the ground temperature changes decrease, as the distance from the heat exchanger increases. At distances greater than 5 to 10 meters from the heat exchanger, the temperature changes are less than 1 Kelvin.

To determine the heat flux due to heat transfer from the ground, we can assume the heat flux is absorbed by the heat exchanger radially at steady state heat conduction. We can divide the area around the heat exchanger into three coaxial cylinders whose radiuses are r_1 (u-tube radius), r_2 (borehole radius) and r_3 (the position where the ground temperature is not affected by the heat exchanger).

The heat transfer coefficient can be found with Eq.. (6). With a known heat transfer coefficient, the heat flux can be found with $dq_{con}=h_t\,(T_\infty - T)$ (Holman, 1997).

$$h_t = \frac{1}{\sum_{i=1}^{3} \frac{r_1}{k_i} \ln \frac{r_i}{r_{i-1}}} \qquad (6)$$

5. Heat Transfer between Two Branches of a U-Tube

In a u-tube heat exchanger, heat transfer occurs due to temperature differences between the two branches of the u-tube. The u-tube heat exchanger can be modeled as two very long cylinders (which are parallel at a distance equal to W) in an infinite media (ground). With reference to Figure 2, the heat flux can be determined.

$$q_{\text{int}} = SK\left[T(x) - T(L-x)\right] \qquad (7)$$

where:

$$S = \frac{2\pi L}{\cosh^{-1}\left(\frac{2W^2}{r^2}\right)} \qquad (8)$$

6. Temperature Distribution along U-Tube

By writing the energy equation in a differential element of the u-tube of length dx (Figure 3), we have:

$$dQ = -\dot{m}\,dh \qquad (9)$$

where:

$$dh = c_P dT$$

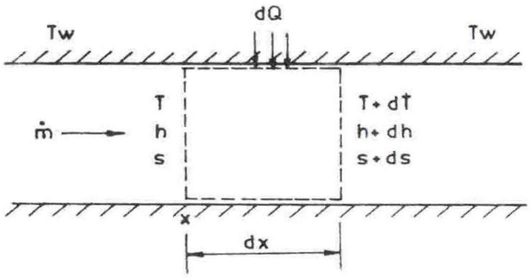

Figure 3. Differential element of the u-tube

The absorbed heat flux (dQ) consists of two fluxes, (dq_{con} due to heat absorbed from ground and dq_{int} due to heat transferred between the two branches of the u-tube), so by using the previously mentioned material, we can determine dq_{con}:

$$dq_{con} = h_t\, 2\pi r dx\ \theta \qquad (10)$$

By substitution, Eqs. (7), (8) and (10) into Eq. (5) and solving the resulting non-homogeneous differential equation, the temperature distribution along the u-tube is found (Mukherjee, 1987):

$$\theta = (\theta_i - b)\exp(-az) + b \qquad (11)$$

where:

$$z = xr$$

$$a = \frac{h_t 2\pi}{\dot{m} c_P}$$

$$b = \frac{-KQ}{2 h_t \dot{m} c_P \cosh^{-1}(\frac{2W^2}{r^2})}$$

7. Entropy Minimization in the Heat Exchanger

By using the temperature distribution given by Eq. (11) and substituting it into Eq (4), the generated entropy in the heat exchanger can be written in terms of the Reynolds number. By differentiating this with respect to the Reynolds number and equating it to zero, the optimum Reynolds number can be determined:

$$\text{Re}_{opt} = (2.626)\left(\frac{M}{N}\right)^{0.131} (\text{Pr})^{-0.053}\, B_\circ^{\,0.263} \qquad (12)$$

where:

$$B_\circ = \frac{h_t \dot{m}^{\frac{3}{2}} \rho}{k^{\frac{1}{2}} \mu^{\frac{7}{2}}}$$

$$N = \int_0^n \frac{dz}{T}$$

$$M = \int_0^n \frac{\Delta T}{T^2} dz$$

$$dz = rdx$$

$$n = rL$$

By using Re_{opt}, optimum length and diameter for the u-tube can be determined.

8. The generated entropy due to the heat transfer between the two branches of the U-tube

Heat transfer between the two branches of the u-tube increases the entropy in the heat exchanger (as shown in Fig. 2). Generated entropy in a slice of thickness dx can be written:

$$\dot{S}'_{int} = \frac{q_{int}}{T(x)} - \frac{q_{int}}{T(L-x)} = q_{int}\left[\frac{1}{T(x)} - \frac{1}{T(L-x)}\right] \quad (13)$$

where:

$\dot{S}'_{int}$ is the generated entropy due to heat transfer between the two u-tube branches.

By adding this equation to the above mentioned equations and differentiating with respect to the Reynolds number, Eq. (12) can be modified to Eq. (14) with an additional term A_3 as follows, when considering the heat transfer between the two u-tube branches:

$$A_1 \, Re_{opt}^{-2.8} - A_2 \, Re_{opt}^{4.8} + A_3 = 0 \quad (14)$$

where:

$$A_1 = \frac{626.112 \, M h_t^2 \dot{m}}{\mu k \, Pr^{0.4}}$$

$$A_2 = \left(4.176 \times 10^{-3}\right)\left(\frac{\pi^4 \mu^6 N}{\rho^2 \dot{m}^2}\right)$$

$$A_3 = \frac{\pi^2 \mu k Q^2 n}{4 \dot{m}^3 C_P^2}\left(\frac{1}{T_1 T_2}\right)\left(\frac{1}{\cosh^{-1}\left(\frac{2W^2}{r^2}\right)}\right)$$

Note that if heat transfer between the two branches of the u-tube is not considered and the A_3 term deleted, Eq. (14) will change to the simpler form of Eq. (12).

9. Results

As an example, consider a u-tube with a one-inch diameter as a heat exchanger installed in a borehole, and assume the surrounding area is filled with grout and sand. If we divide the area around the u-tube into three zones, where the geometric and thermophysical characteristics of these zones are as described in Table I:

TABLE I. GEOMETRIC AND THERMOPHYSIC CHARACTERS OF THE ZONES AROUND THE U-TUBE HEAT EXCHANGER

material	heat conductivity [W/m K]	zone radius [m]
u-tube pipe (HDPE)	0.33	0.0301
grout	1.8	0.1
soil	2.5	5

The distance between two branches of u-tube is W=0.114 m. With the information presented in Table 1, the total heat transfer coefficient can be obtained:

$$h_t = 14.43 \quad W/m^2K \quad (15)$$

The ground and fluid entrance temperatures are $T_\infty = 293$ K and $T_i = 283$ K, respectively. With water as a working fluid in the u-tube, and assuming the water physical properties do not change with temperature, the thermophysical properties are as follows:

$$C_P = 4200 \quad J/kgK$$

$$\rho = 999.8 \quad kg/m^3$$

$$\mu = 70 \times 10^{-5} \quad pas$$

$$Pr = 10$$

Considering the u-tube as a control volume and applying the first law of thermodynamics as well as Equation (11), n can be determined:

$$n = \left(\frac{-1}{a}\right) \ln\left[\frac{Q}{\dot{m} c_P (\theta_i - b)} + 1\right] \quad (16)$$

By using n and Equation (12), the optimum Reynolds number and then the optimum u-tube length and diameter can be found for different mass flow and heat loads. In order to do this, two integrals, M and N, are calculated numerically. The results are presented as graphs in Figures 4 and 5. Eq. (14) is a nonlinear equation, therefore Re_{opt} is found through numeric methods. The optimum u-tube length and diameter for different mass flows and heat loads are shown in Figures 4 and 5 with dashed lines.

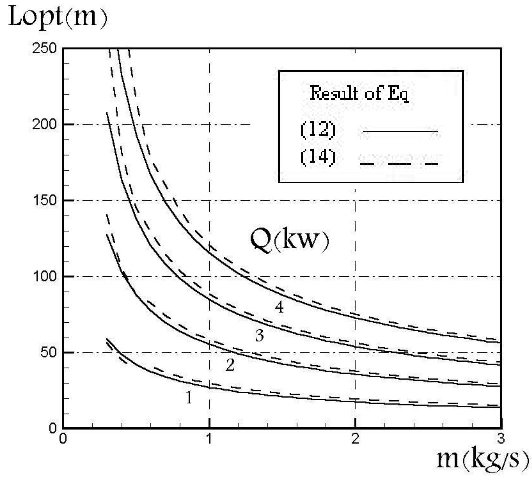

Figure 4. Optimum length vs. mass flows for different heating loads

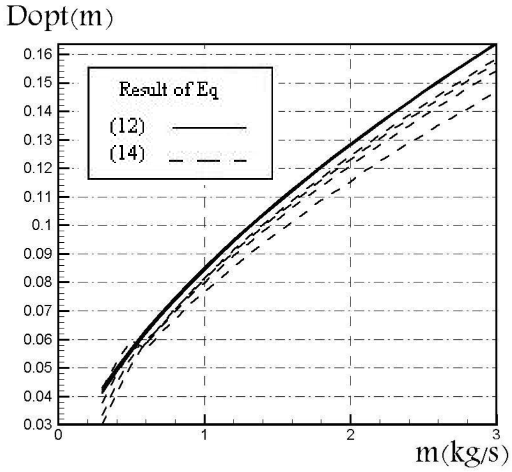

Figure 5. Optimum diameter vs. mass flows for different heating loads

10. Conclusion

By using the method presented in this paper, and therefore minimizing the entropy generated in the heat exchanger, optimum u-tube length and diameter can be determined. Thus less exergy is destroyed, less power is required for a given duty, and the heat pump efficiency increases. In this method, heat exchanger optimum sizes can be determined; if we have circulation pump characteristics, then the proper mass flow is selected for it using the graphs presented.

Figure 4 shows that the optimum u-tube length decreases when the mass flow increases or when the heating load increases. Figure 5 shows that the optimum u-tube diameter increases when the mass flow increases. However, increased heating loads have no significant effect. The dashed lines in Figures 4 and 5 compare the difference between the solutions using Eq. (14) (considering the effects of u-tube branches on each other) and those using Eq. (12). Considering the dashed lines in Figures 4 and 5, it can be seen the solution of the non-linear Eq. (14) can be avoided. The difference between the two solutions is negligible. In recent years, many different analytic and numeric methods have been presented for designing a heat pump's u-tube heat exchanger, and there is much software based on them. The method presented in this paper is for the optimization of the heat exchanger sizes with regard to the second law of thermodynamics. This method can also be used with available software and incorporates the second law of thermodynamics into the design of GSHP heat exchangers.

Nomenclature

A	surface area [m^2]
a	constant parameter
b	constant parameter
B_0	duty parameter
C_p	heat capacity [J/kgK]
D	hydraulic diameter
f	friction factor
h	enthalpy of working fluid [kJ/kg]
h_t	total heat transfer coefficient [W/m^2K]
K, k_i	thermal conductivity [W/mK]
L	length of u-tube [m]
M	duty parameter
m	mass flow rate [kg/s]
N	duty parameter
n	duty parameter
Nu	Nusselt number
Pr	Prandtl number
Q	total heat transfer rate [W/m]
q	local heat transfer rate [W/m]
r	radius of u-tube [m]
Re	Reynolds number
S	shape factor
S_{gen}	rate of entropy generation [W/mK]
St	Stanton number
T	temperature [K]
W	distance between u-tube branches [m]
z	- r x
ρ	density [kg/m^3]
μ	viscosity [kg/sm]
θ	$T - T_\infty$

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