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Entropic Skins Geometry Applied to Dynamics of Turbulent Reactive Fronts

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Abstract

The determination of the velocity U_T of a turbulent propagating interface resulting from the interaction between a turbulent intensity U' and a reactive front with a laminar velocity U_L is still a fundamental and open problem. In this paper, we propose a new approach to deal with this phenomenon by introducing a geometrical structure called *entropic skins geometry* based on scale entropy and its dynamics in scale-space. In the specific case (called *parabolic scaling*) of equipartition of « scale-evolutivity » through scale space, we derive a law for turbulent velocity. This law has a second-order logarithmic form which is verified by experimental data.

Keywords: Turbulent reactive fronts, turbulent combustion, scale-dependent fractals, entropic skins geometry, parabolic scaling, Yakhot's law, scale-entropy diffusion equation, scale diffusivity.

1. Introduction

Our aim is to propose a geometrical approach to study the dynamics of turbulent reactive fronts by using the concept of scale-entropy and a general scale-entropy diffusion equation. This equation is established thanks to the introduction of a new quantity in physics called *scale diffusivity* which characterizes how information concerning a multi-scale structure propagates through scale-space and how it unifies the system giving it a sort of "structural viscosity" in scale-space. This description derives from a more general framework called *entropic skins geometry* introduced to deal with the phenomenon of intermittency in turbulence and turbulent reactive systems.

In this paper, our approach is applied to the interaction of a turbulent flow and a reactive front which can lead to turbulent flames in the field of combustion and more generally to aqueous autocatalytic reaction fronts obtained by some specific chemical reaction. The choice of studying aqueous autocatalytic reaction fronts is due to the fact that, at high turbulence intensities, turbulent combustion generates secondary phenomena such as heat losses which have extinction effects on the flame: it is then difficult to study the interaction itself. A way to avoid this is to work with fronts produced by an aqueous autocatalytic chemical front evolving in a turbulent flow. In this case, large U'/U_L values can be reached without any problem of extinction due to heat losses. Nevertheless, our main motivation concerns the field of turbulent combustion; that is why we will frequently refer to this specific domain.

After a recall of the phenomenon of turbulent combustion, we will present our geometrical approach and then we will show how it can be applied to the field of turbulent reactive fronts. Our long-term aim within this work is to establish links between this geometrical approach and the classical quantities of non-equilibrium thermodynamics. We also think that such an approach can help to visualize some traditional thermodynamical quantities as it is already suggested here through a study on Yakhot's law (elaborated by using the renormalization group theory) proposing a geometrical interpretation to this law in the context of the entropic skins approach.

2. Turbulence-reactive Front Interaction: Multi-scale Structure and the Turbulent Velocity

The scientific study of turbulent combustion started with Mallard and Le Chatelier (1883) at the end of the nineteenth century (a work realized at Ecole des Mines de Saint-Etienne). The fundamental problem in turbulent combustion is

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to characterize how scales of turbulence influence the flame structure (front wrinkling, front thickening, quenching) and how this latter, by heat production or hydrodynamical instabilities, influences the characteristics of turbulence. Thanks to the development of laser visualization techniques, the multi-scale structure of turbulent flames can be revealed. This led to the application of fractal theory to the field of turbulence but no real success was attained in terms of the description of turbulent flames dynamics. A more complex multi-scale geometry is needed. The main objective of this paper is to propose a multi-scale geometry to treat these questions.

Figure 1a represents an example of turbulent flame obtained by laser tomography (Queiros-Conde, 1996). The turbulent velocity U_T results from the wrinkling process of the whole scale range of the turbulence cascade which can be defined by its turbulent intensity U'. The chemical part is defined by the thickness of the front δ and the laminar velocity U_L. The flame surface is increased by the effect of wrinkling and leads to a turbulent surface S_T which has a multi-scale structure. The main effect of turbulence is to wrinkle the flame front and, if the flow contains scales smaller than the flame thickness, to thicken the front. In the "flamelet" regime (Borghi and Destriau, 1995), the problem is simplified by the fact that chemical time and flame thickness are much smaller than the time and spatial scales characterizing the turbulence cascade. Our study concerns this case since our main aim is to understand turbulence-combustion interaction. There is a direct relation between velocities and surfaces through the relation $U_T/U_L=S_T/S_L$ where S_L is the projected flame surface perpendicular to the mean direction given by U_T and which also corresponds to the flame surface obtained without turbulence. The ratio $\Sigma=S_T/S_L$ is called the roughness of the front.

In order to explain simply how the turbulence-flame interaction can be considered and analyzed in scale-space, let us consider a part of a flame in a domain defined by the integral scale of the flow noted l_0. Let us note l_c the smallest scale displayed by the front. This means that the turbulent velocity results from the action on the front of scale range $[l_c;l_0]$ which leads to a total roughness noted $\Sigma_{c,0}$; for this reason, we can note the turbulent velocity $U_T=U_{c,0}$ with $U_{c,0}/U_L=\Sigma_{c,0}$. On this initial front, one can define a smaller front (*Figure 1b*) contained this time in a box of size l_i. The physical problem should be exactly the same if we consider now the scale range $[l_c;l_i]$ which leads to the roughness noted $\Sigma_{c,i}$. The turbulent velocity has simply decreased since the front is less rough but we can also introduce a velocity indicated as $U_{c,i}$ with $U_{c,i}/U_L=\Sigma_{c,i}$. Again, it is possible to define on this front a new front (*Figure 1c*) contained in a box of size l_j (pay attention: for convenience $l_j<l_i$). It gives a velocity $U_{c,i}.<U_{c,j}$ with $U_{c,j}/U_L=\Sigma_{c,j}$. Locally, for small windows of size l_c, the front has a laminar velocity $U_{c,c}=U_L$. The front of *Figure 1a* is a true photography (it means that the photography has been taken at the real size corresponding to the combustion chamber) obtained for $U'/U_L=2.12$. But *Figures. 1b* and *1c* are only rescaled parts of the initial front. However, they could be « real » in the sense that these fronts could be obtained for smaller U'/U_L values.

A turbulent reactive front thus displays two remarkable properties. (i) A piece of turbulent front (such as in *Figures 1b* and *1c* extracted from the original one in *Figure 1a*) leads to a turbulent front which could be obtained in some different experimental configurations using a smaller U'/U_L value. (ii) Everywhere on the front, the velocity is U_L but, owing to the multi-scale structure which allows the mixture to burn faster, the front has a turbulent velocity (with $U_T>>U_L$). Hence, we have a remarkable system to study the link between multi-scale geometry and dynamics here.

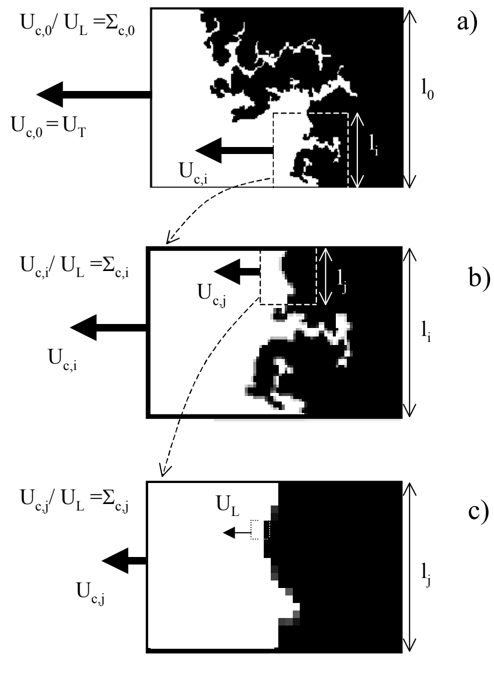

Figure 1: Scale-dependent behavior of turbulent velocity and roughness. a) $U'/U_L=2.12$. Equivalence ratio $\phi=1.05$, dilution percentage $\delta=15\%$; b). Part of front of Fig. 1a rescaled; c) Part of front of Fig. b rescaled. The fronts b) and c) are extracted from one "real" front obtained experimentally. They could correspond to real fronts obtained for smaller values of U'/U_L (scale covariance).

If the aim is to study the turbulence-front interaction, we must also be aware of the influence of the front on the flow. In fact, the front with a propagation velocity U_T imposes a velocity gradient in the downstream part of the fluid whose order of magnitude is precisely the velocity U_T of the front (exactly as if the front were a moving piston pushing fresh gases). This generates velocity fluctuations of order U_T. Two sorts of fluid velocity fluctuations must thus be considered: the ones coming from the turbulence itself generated by external means (a grid for example, intensity quantified by U') and the ones resulting from a background flow generated by the feedback of the front propagation on the downstream fluid simply by the creation of a velocity gradient due to the propagation itself. So, the turbulence quantified by U' is superposed to this background flow generated by the front propagation on the flow (see *Figure. 2*). To characterize this property, we propose to introduce two kinds of kinetic energies. Let us consider a cube of fluid with the size of an integral scale l_0 (*Figure 2*).

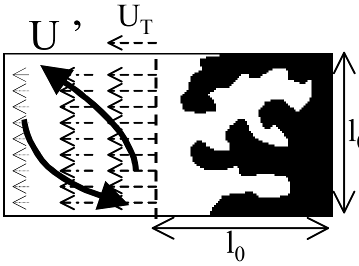

Figure 2: Sketch to explain $E_{BF}(l_0)=(1/2)\rho l_0^d U_T^2$ and $E_{Turb}(l_0)=(1/2)\ \rho l_0^d U'^2$. Background fluid velocities are symbolized by dashed arrows.

The front (like a piston) creates downstream a velocity gradient equal to U_T over a distance l_0. This generates turbulent fluctuations. It means that one can define a kinetic energy associated to these velocity fluctuations $E_{BF}(l_0)=(1/2)\rho l_0^d U_T^2$ where l_0^d (d=3) is the volume associated to an integral scale. To these background velocity fluctuations, the fluctuations due to turbulence itself (injected by external means) are superposed. If we assume that the turbulent intensity U' is distributed homogeneously in this cube, we can define a turbulent kinetic energy injected to the system by $E_{Turb}(l_0)=(1/2)\rho l_0^d U'^2$. We expect that the ratio $E_{Turb}(l_0)/E_{BF}(l_0)$ (which is equal to $U'^2/U_T{}^2$) will intervene in the study of the flow-front interaction.

3. Turbulent Velocity Laws Obtained by Renormalization Group Theory and Scale Covariance

The fundamental problem in turbulent reactive fronts is to express their turbulent velocity, namely the ratio U_T/U_L as a function of the characteristic ratio U'/U_L. Although a large number of expressions or laws (empirical, phenomenological or theoretical) have been proposed, the problem is not entirely resolved. However, three approaches, based on rigorous theoretical frameworks, deserve to be quoted. (i) The first one is the work by Clavin and Williams (1979) which, for small U'/U_L values, used a perturbation method and obtained $U_T/U_L=1+(U'/U_L)^2$. (ii) The second important law $U_T/U_L=\exp\left[(U'/U_T)^2\right]$ has been obtained by Yakhot (1988) using the renormalization group theory. It gives an implicit form which remains difficult to grasp physically. The Clavin-Williams' law is recovered for small U'/U_L values. This law seems to be adequate to describe experimental measurements (Ronney et al., 1992, 1995). (iii) The third theoretical approach (Queiros-Conde, 1996; Pocheau and Queiros-Conde, 1996) assumes a property of scale covariance for the law, i.e. it imposes the fact that the explicit form of the law must be independent of the scale-range over which it is expressed. It leads to a quadratic form $\left(U_T/U_L\right)^2=1+c\left(U'/U_L\right)^2$ where c is an order-one constant depending on the equivalence-ratio; it also gives the Clavin-Williams' law for small U'/U_L. In the context of this paper, we will mainly focus on Yakhot's law for a geometrical interpretation proposed here.

4. Scale Entropy and Diffusion Equation in Scale-Space: the Scale-Space as a Dimension

The entropic skins theory (ESG hereafter) (Queiros-Conde, 2000, 2001, 2003) is a geometrical framework which describes the phenomenon of intermittency in fully developed turbulence. It also appeared adequate to describe multi-scale features of turbulent interfaces (Queiros-Conde, 2003). ESG introduces a hierarchy of multi-scale sets (skins) having different space-filling properties (quantified by *scale-entropy*), all these skins being linked to each other by an entropic flux through scale space. Concerning the specific multi-scale structure of turbulent interfaces, scale analyses showed that turbulent interfaces are not really fractal and that the measured fractal dimension is scale-dependent, which is paradoxical. Let us recall the classical box-counting method used to realize a scale analysis. The interface is covered by a grid of mesh size l_i, then one counts the number of meshes $N(l_i)$ touched by the interface. By varying the mesh size, if a power law $N(l_i)\sim l_i^{-D_f}$ can be evidenced over a sufficiently extended scale range, then the interface is said to be fractal with a fractal dimension D_f. However, measurements showed that this is not the case and that fractality can only be an ideal limit

(Queiros-Conde, 1996, 2003). This ideal limit is obtained for U'/U_L numbers or large scales. The values $D_f=7/3$ is consistent with Kolmogorov cascade without intermittency. With intermittency, it has been shown that $D_f=1+2\gamma$ with $\gamma=((1+3/\sqrt{8})^{1/3}+(1-3/\sqrt{8})^{1/3})^3\approx 0.68$ i.e. $D_f=2.36$ (Queiros-Conde, 2000). But this is only a limit case and, generally, fractal dimension is scale-dependent and a « local fractal dimension » must be defined.

In ESG, scale invariance is assumed only locally in scale space: For 2 scales l_i and l_{i+1} (with $l_{i+1}<l_i$) close enough in scale space, one can define a local fractal dimension Δ_i implying that, for scales l with $l_{i+1}\leq l\leq l_i$, one has $N(l)\sim l^{-\Delta i}$ where $N(l)$ is the number of balls of size l necessary to cover the structure. Let us consider a multi-scale system having scales ranging from an inner cut-off length l_c to an outer cut-off length l_0. To these scales l_c and l_0 correspond two local fractal dimensions Δ_c and Δ_0 with $\Delta_c<\Delta_0$. Δ_c as the dimension of the most localized structure in the system (called the *crest*) and Δ_0 as the dimension of the most extended structure (called the *bulk*, the dimension Δ_0 usually corresponds to the fractal dimension obtained in the limit of large scales or high turbulent intensities: $\Delta_0=D_f$). In the case of a turbulent flow, l_c is simply the Kolmogorov scale (scale of dissipation) and l_0 is the integral scale (scale of energy injection). At each scale l_i, one can define V_i as the volume occupied by the system at the scale l_i: It can be written $V_i=N_{i,0}l_i^d$ where $N_{i,0}$ is the number of balls of size l_i needed to cover the system embedded in a volume of characteristic scale l_0 (d is the embedding dimension of space: here d=3; to simplify we do not consider the coefficient $\pi/4$ in the volume). Since $N_{0,0}=1$, we have $V_0=l_0^d$. We then introduce the scale entropy $S_{i,0}$ at scale l_i defined by $S_{i,0}=\ln(V_0/V_i)$. Denoting $x=\ln(l_i/l_0)$ (called *ln-scale*), $S_x=S_{i,0}$, we introduce the scale-entropy flux $\phi_x=dS_x/dx$. It can easily be shown that $\phi_x=\Delta_x-d$ where Δ_x is the local fractal dimension considered at scale l_i defined by $x=\ln(l_i/l_0)$.

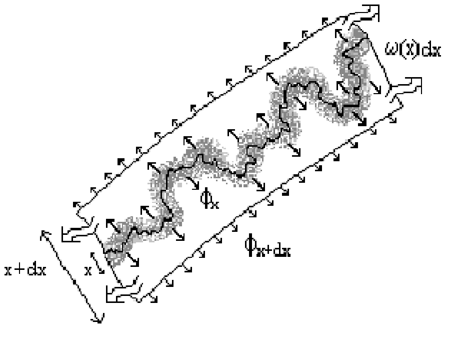

Figure 3. Sketch describing the scale-entropy balance argument.

By an argument of scale-entropy flux balance (see *Figure 3*), it is obtained $d^2S_x/dx^2-\omega(x)=0$ where $\omega(x)$ as a sink of scale-entropy flux in dx; $\omega(x)$ could also be a source but here we only consider the case of a sink. If we assume the simple case of a uniform scale-entropy flux sink ($\omega(x)=\beta$ with $\beta\geq 0$, $\beta=0$ gives back the fractal case) then it can easily be obtained $S_x=(\beta/2)x^2+(\Delta_0-d)x$ where $\beta=(\Delta_0-\Delta_c)/\ln(l_0/l_c)$. This case has been called *parabolic scaling*. This purely geometrical assumption probably has a close link with Tondeur and Kvaalen (1987)'s assumption of equipartition entropy production. Recently, a link between this parabolic scaling and constructal optimization has been evidenced (Queiros-Conde et al., 2007). The constructal optimization corresponds to a uniform distribution of scale entropy sink in scale space (i.e. parabolic scaling). Parabolic scaling leads to remarkable relations namely

$$N_{i,0}=(l_0/l_i)^{(\Delta_0+\Delta_i)/2} \quad (1)$$

and

$$\Delta_i=d+\beta\ln(l_i/l_0)=\Delta_c+\beta\ln(l_i/l_c). \quad (2)$$

Let us introduce the quantity $\overline{\Delta}_{c,0}=(\Delta_0+\Delta_c)/2$ as the mean dimension between bulk and crest; we thus have

$$N_{c,0}=(l_0/l_c)^{(\Delta_0+\Delta_c)/2}=(l_0/l_c)^{\overline{\Delta}_{c,0}}. \quad (3)$$

This leads to a total scale-entropy of a multi-scale system following parabolic scaling noted $S^P_{c,0}$ equal to $S^P_{c,0}=\ln\left[(l_0/l_c)^{d-\overline{\Delta}_{c,0}}\right]$. Let us also remark that $\beta=S^P_{c,0}/\left[\ln(l_0/l_c)\right]^2$; this quantity thus represents a sort of surfacic scale-entropy density in which the spatial-coordinate would be scale-logarithm. Moreover, let us remark that the maximum scale-entropy is $S^{max}_{c,0}=\ln\left[(l_0/l_c)^{d-\Delta_c}\right]$: it represents the scale-entropy of a fractal system for the scale range $[l_c;l_0]$ which would have the crest-dimension Δ_c as fractal dimension.

At this level, in order to simplify our following presentation and to make clear the concepts we defined, it is convenient to define some adequate terminology and quantities. The absolute value of scale entropy flux $\phi_x=dS_x/dx=\Delta_x-d$ represents in fact, at ln-scale x, an evolutive capacity for the system (the minus sign is due to our convention for scale definition). Its maximal value $|\varphi_c|=|\Delta_c-d|$ is obtained for the inner cut-off scale and its minimal value $|\varphi_0|=|\Delta_0-d|$ is given for the integral scale. For this reason, we will specify the scale-entropy flux ϕ_x in a more simple way as a *scale evolutive flux* for the ln-scale x. The scale-entropy flux sink $\omega(x)$ is thus a gradient of evolutive flux in scale-space; it quantifies how the evolutive flux is itself evolving through scale-space; it can be seen as a *scale evolutive capacit* ; we will call it a *scale-evolutivity* for the ln-scale x.

Hence, the parabolic scaling case ($\omega(x)=\beta$) corresponds to a constant scale-evolutivity through scale-space and to an evolutive flux which decreases (in absolute value) from inner cut-off scale to integral scale. For this case, let us introduce a « parabolic volume». The total volume (one can call it an « Euclidean volume ») for a ball of size l_0 is $V_0=l_0^d$. The parabolic volume corresponds to the exact volume occupied by the multi-scale front following a parabolic scaling; it can be written $V_p=N_{c,0}l_c{}^d=V_0(l_0/l_c)^{\bar{\Delta}_{c,0}-d}$. It represents the volume occupied by the really active part (dissipative part) of the front contained in its Euclidean volume. The ideal fractal case would give a « fractal volume » defined by $V_F=N^F_{c,0}\ l_c{}^d=V_0(l_0/l_c)^{\Delta_0-d}$ where $N^F_{c,0}$ is the number of balls needed to cover a fractal having a dimension Δ_0 in the scale range $[l_c;l_0]$. We have the relation $V_P/V_F=(l_0/l_c)^{\bar{\Delta}_{c,0}-\Delta_0}$. We also define a global scale entropy based on a fractal behavior: $S^F_{c,0}=\ln\left[(l_0/l_c)^{d-\Delta_0}\right]$. At this point, we define the deviation $\sigma^{P/F}_{c,0}$ of scale-entropy in the parabolic case by taking the ideal fractal case as a reference: $\sigma^{P/F}_{c,0}=S^P_{c,0}-S^F_{c,0}$. It can be shown immediately that

$$\sigma^{P/F}_{c,0}=\ln\left[(l_0/l_c)^{\Delta_0-\bar{\Delta}_{c,0}}\right]=\ln(V_F/V_P)\,. \quad (4)$$

This quantity which takes the ideal fractal case as a reference quantifies a sort of *scale-entropy production*. The fractal behavior becomes here a reference for parabolic behavior which describes the multi-scale geometry of turbulent flames much better. Let us remark that, for the case of turbulent reactive fronts in the flamelet regime, if we take $\Delta_0=7/3$ (corresponding to Kolmogorov cascade without intermittency) then, since $\Delta_c=2$, we have $\Delta_0-\bar{\Delta}_{c,0}=1/6$ and finally

$$\sigma^{P/F}_{c,0}=(1/6)\ln(l_0/l_c) \quad (5)$$

Using the maximum scale entropy $S^{max}_{c,0}=\ln\left[(l_0/l_c)^{d-\Delta_c}\right]$, we define *a scale structure efficiency* by

$$\eta=\frac{S^P_{c,0}-S^F_{c,0}}{S^{max}_{c,0}}=\frac{\sigma^{P/F}_{c,0}}{S^{max}_{c,0}}\,. \quad (6)$$

It is easily shown that

$$\eta=\frac{\Delta_0-\bar{\Delta}_{c,0}}{d-\Delta_c}=\left(\frac{1}{2}\right)\frac{\Delta_0-\Delta_c}{d-\Delta_c}\,. \quad (7)$$

For a pure fractal system (fractal over the scale-range $[l_c;l_0]$), this efficiency is null. The fractal system does not produce scale entropy: it does not display any capacity to make structure. For turbulent reactive fronts (since $\Delta_0=7/3$ and $\Delta_c=2$), we have $\eta=1/6$. In a more general way (systems different from the one studied in this paper but having a multi-scale behavior described by ESG), it is interesting to deal with two specific cases given by (i) bulk-dimension having its maximum value (i.e. the embedding dimension i.e. $\Delta_0=d=3$) and (ii) the crest-dimension having the smallest possible value (i.e. $\Delta_c=0$). In fact, the first case gives, whatever the value of Δ_c, the maximum value of scale structure efficiency: $\eta^{max}=1/2$. The second case leads to $\eta_{lim}=(1/2)(\Delta_0/d)$. Assuming a system displaying a turbulent structure for the bulk ($\Delta_0=7/3$) and a point-like one for the crest ($\Delta_c=0$), we determine $\eta_{lim}=7/18$. Other cases can be considered; nevertheless, the maximum value of scale structure efficiency cannot be higher than ½ whatever the crest-dimension is.

We emphasize the fact that the previous derivation corresponds to a specific case « parabolic scaling » for which there is uniform scale-entropy flux sink through scale-space. Let us now come back to the general case. The equation $d^2S_x/dx^2-\omega(x)=0$ can be generalized to time behavior for non-stationary systems. It has been shown that scale entropy is piloted by a diffusion equation where the spatial coordinate is the scale logarithm. It gives the fundamental and new equation

$$\frac{\partial^2 S_{x,t}}{\partial x^2}-\omega(x,t)=\frac{1}{\chi}\frac{\partial S_{x,t}}{\partial t} \quad (8)$$

where the quantity χ (a new quantity in physics) is called 'scale diffusivity'. Scale diffusivity defines the capacity for the system to propagate perburtations through scale-space and gives access to a scale dynamics since it becomes possible to calculate information transfer in scale-space. The scale diffusion equation leads to a breakthrough in the way to consider multi-scale systems since it gives a tool to study scale dynamics analytically and to calculate some temporal quantities which so far have remained non-accessible. We now apply this formalism to turbulent reactive fronts, namely turbulent flames.

5. Determination of the Inner Cut-off Scale in Turbulent Combustion

Let us come back to the definition of the inner cut-off scale. It is linked to the Kolmogorov scale of the flow defined by $l_k=(v^3/\varepsilon)^{1/4}$ where ε is the rate of energy dissipation and v the kinematic viscosity (with $\varepsilon=U'^3/l_0=u'_k{}^3/l_k$, u'_k being the characteristic velocity associated to the Kolmogorov scale). This is without chemical reaction. Let us consider the specific case of turbulent combustion. In the case of premixed turbulent combustion, due to the density change at the front between fresh and burnt gases, the interesting viscosity is not the viscosity of fresh fluid but the viscosity of burnt gases and we can determine a Kolmogorov scale $l_{k,BG}$ concerning only burnt gases. We propose to take this scale as an inner cut-off scale. Let us determine the evolution of this scale with the ratio U'/U_L. For this, we consider that the viscosity of burnt gases can be formulated as $l_{k,BG}=U_L\delta$ where δ is a characteristic thickness of the front; for a turbulent flame, it would be the thermal thickness. The Reynolds number can then be written as $Re=(U'/U_L)(l_0/\delta)$: this way of expressing the Reynolds number is not classical, its objective is to decompose the Reynolds number and express it using the characteristic ratio U'/U_L which is the main control variable of experiments in turbulent combustion. The existence of a Kolmogorov cascade implies $l_0/l_c=Re^{3/4}$. We thus can write $l_c/\delta=(l_0/\delta)^{1/4}(U'/U_L)^{-3/4}$. Experimentally, it has been found that a multiplicative factor 2 appears giving $l_c\approx 2l_{k,BG}$ (Queiros-Conde,1996). Multiplicative factor 2 can be explained by the following simple qualitative argument: the turnover time corresponding to $l_{k,BG}$ is $t_{k,BG}=l_{k,BG}/U_L$. This time $t_{k,BG}$ being the minimum turnover-time of the flow means that the corresponding vortex just does a turn before dissipating. To be able to wrinkle the front, it must live at least $2t_{k,BG}$ which means $l_c\approx 2\, l_{k,BG}$. We thus finally propose the expression $l_c/\delta=2(l_0/\delta)^{1/4}(U'/U_L)^{-3/4}$.

This order of magnitude and the exponent –3/4 are experimentally well verified (Queiros-Conde, 1996). Let us remark in this context that the exponent –3 in the Gibson scale $l_G=l_0(U'/U_L)^{-3}$ introduced by Peters (1986) has no experimental validation; moreover, it can be shown that, in the context of multi-scale approach, it leads to some paradoxical conclusions (Queiros-Conde, 1996). A Gibson scale is derived assuming that, to wrinkle a flame, a vortex must have a characteristic velocity such as $u'_k\geq U_L$. Such a condition does not take into account any constraints on the relative scales of the two interacting objects (front and vortex).

Another simple derivation (without the multiplicative coefficient 2) of formula $l_c/\delta=(l_0/\delta)^{1/4}(U'/U_L)^{-3/4}$ is possible. Apart from the reactive time δ/U_L characterizing combustion and the turnover time $t_k=l_k/u'_k$ characterizing a vortex, to really characterize the interaction vortex-flame, we have to introduce times which are characteristic of the entanglement of velocities and scales. We define $t_k(f)=l_k/u'_k$ as the time needed by a vortex of size l_k and thus of velocity u'_k to cross the thermal thickness δ of the front. In the same way, we define $t_f(k)=l_k/U_L$ as the time needed by the flame to cross the vortex. Let us examine the inequality $t_k(f)>t_f(k)$: in this case the flame crosses the vortex so fast that this latter has no time to wrinkle the flame. The wrinkling condition is thus $t_k(f)\leq t_f(k)$. The identity $t_k(f)=t_f(k)$ defines the cut-off scale (without the multiplicative coefficient 2).

6. Velocity of Turbulent Reactive Fronts Determined by Entropic Skins Geometry

Let us consider a turbulent reactive front as a multi-scale system having scales ranked from an inner cut-off length l_c to an outer cut-off length l_0. The total roughness of the front which results from all the scale range $[l_c; l_0]$ can be expressed by $\Sigma_{c,0}=(N_{c,0}l_c^2)/l_0^2$ where $N_{c,0}$ is the number of balls of size l_c (minimum resolution) needed to cover the front contained in a ball of size l_0. We can define a roughness at any scale l_i by $\Sigma_{c,i}=(N_{c,i}l_c^2)/l_i^2$ where $N_{c,i}$ is the number of balls of size l_c needed to cover the front contained in a ball of size l_i. We can also vary the resolution and define the roughness $\Sigma_{i,0}=(N_{i,0}l_i^2)/l_0^2$ where $N_{i,0}$ is the number of balls of size l_i needed to cover the front contained in a ball of size l_0. Multiplicativity of roughness implies $\Sigma_{c,i}\Sigma_{i,0}=\Sigma_{c,0}$ for any scale l_i. We can thus introduce for any scale range $[l_j;l_i]$ a roughness $\Sigma_{j,i}=(N_{j,i}l_j^2)/l_i^2$ with $\Sigma_{j,i}=U_{c,i}/U_{c,j}$.

We assume that turbulent reactive fronts belong to the case of parabolic scaling (i.e. following $d^2S_x/dx^2-\beta=0$). In the case of a constant scale evolutivity β in scale-space, it leads to the simple and general relation $\ln\Sigma_{c,i}=(\beta/2)[\ln(l_i/l_c)]^2$. The global roughness due to the effect of scale range $[l_c;l_0]$ is thus $\ln\Sigma_{c,0}=(\beta/2)[\ln(l_0/l_c)]^2$. By using Eq. (3) and the relation $\Sigma_{c,0}=N_{c,0}\, l_c^2/\, l_0^2$, it can be written $\Sigma_{c,0}=(l_0/l_c)^{(\Delta_c+\Delta_0-d-1)/2}$. It is interesting to notice at this stage that since $\Delta_c=2$, we have $d+1=2\Delta_c$ and then $\Delta_c+\Delta_0-(d+1)=\Delta_0-\Delta_c$.

It follows that the scale-entropy production $\sigma_{c,0}^{P/F} = S_{c,0}^{P} - S_{c,0}^{F}$ can be written

$$\sigma_{c,0}^{P/F} = \ln \Sigma_{c,0} \,. \qquad (9)$$

Since $\Sigma_{c,0}=U_T/U_L$, it implies $\ln(U_T/U_L)=(\beta/2)[\ln(l_0/l_c)]^2$. It has been shown (Queiros-Conde, 2003) that the parameter β is not varying with U'/U_L: experimentally, by measuring how local fractal dimension varies with scale-logarithm, it gives β=0.177. Using $l_0/l_c=Re^{3/4}$ and $Re=(U'/U_L)(l_0/\delta)$, the following relation is easily derived

$$\ln(U_T/U_L)=(\alpha/2)\left[\ln(U'/U_L)\right]^2+\mu\ln(U'/U_L)+(\mu/2)^2 \quad (10)$$

where $\alpha=(9/16)\beta$ and $\mu=\alpha\ln(l_0/\delta)$.

Using the experimental result β=0.177 measured (Queiros-Conde, 2003), we thus should obtain for the parameter α a value close to $\alpha_{th}=0.099$. In this law, the variations of l_0/δ are considered to display a slight contribution compared to variations of the main variable U'/U_L.

To test the experimental validity of the previous relation, a large range of U'/U_L values is necessary. Let us recall that the experimental problem of propagating flames at high turbulence is that it generates annex phenomena such as heat losses which have extinction effects on the flame, and this represents an obstacle for the study of the interaction itself. A way to avoid such a case is to work with fronts produced by an aqueous autocatalytic chemical front evolving in a turbulent flow. Large U'/U_L values can then be reached. The previous law can be compared to experimental results obtained by Ronney and his team (1992) and to those achieved by Shy et al. (1995) on fronts in the general case of aqueous autocatalytic reaction fronts for several turbulent flow configurations.

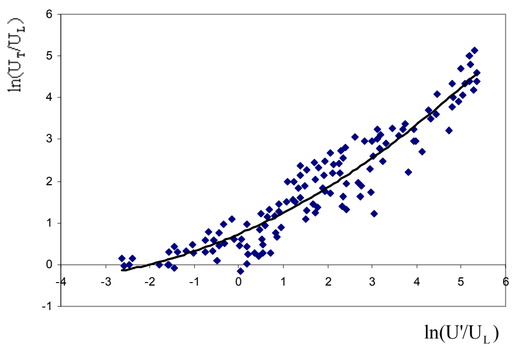

Fig. 4 : Experimental measurements $\ln(U_T/U_L)$ vs $\ln(U'/U_L)$ from Ronney et al. (1995). Second-order polynomial fit gives:

$$\ln(U_T/U_L)=0.048\left[\ln(U'/U_L)\right]^2+0.46\ln(U'/U_L)+0.73$$

The second-order polynomial interpolation presented in Figure 4 implies α=0.096 in very good agreement with the expected value α_{th}. The experimental data cover several turbulent flow configurations (and its dispersion is due to this fact) because the data obtained for one single flow are too limited. However, an improvement of our approach would be to take into account this point (we considered here that it represents a second-order effect). A possible way to proceed is to introduce scales of interaction larger than the integral scale and which would be system-dependent i.e. linked to the external large-scale boundaries of the flow. The problem is that, for these scales, the Kolmogorov cascade is not valid anymore and the corresponding turbulent intensity cannot be calculated easily. Another way would be to know more precisely the integral scale l_0 and the bulk dimension Δ_0 corresponding to each flow configuration in order to calculate $\beta=(\Delta_0-\Delta_c)/\ln(l_0/l_c)$ and then $\alpha=(9/16)\beta$ to have the good parameter in Equation (10). In fact, we assumed here that this parameter is a constant since we did not distinguish between flow configurations. A more specific study on these configurations would be needed.

7. Geometrical Interpretation of Yakhot's Law by Using Scale-entropy Production

Yakhot's law $U_T/U_L=\exp\left[(U'/U_T)^2\right]$ (Yakhot, 1988) has been established by using the renormalization group theory. Thanks to its theoretical derivation and a good experimental validation (Ronney et al., 1992, 1995), it has received great attention. In the context of our geometrical framework and by using scale-entropy production, this law can be interpreted in a simple way. To do so, we assume the validity of Yakhot's law in order to derive all its implications in the context of our geometrical framework. We thus write $\ln(U_T/U_L)=U'^2/U_T^2$. Since $\Sigma_{c,0}=U_T/U_L$ and $\sigma_{c,0}^{P/F}=\ln\Sigma_{c,0}$, we thus can write $\sigma_{c,0}^{P/F} = U'^2/U_T^2$ i.e., by using turbulent kinetic energy and background kinetic energy, $\sigma_{c,0}^{P/F}=E_{Turb}(l_0)/E_{BF}(l_0)$. This implies that, since $\sigma_{c,0}^{P/F}=\ln(V_F/V_P)$, $\exp[-E_{Turb}(l_0)/E_{BF}(l_0)]=V_P/V_F$. Let us remark that the scale structure efficiency can be written $\eta=\left[E_{Turb}(l_0)/S_{c,0}^{max}\right]/E_{BF}(l_0)$.

We thus see that the scale-entropy production $\sigma_{c,0}^{P/F}$ corresponds to the ratio of two energies. Let us now make an analogy and look for an equivalent Boltzmann factor exp(-E/kT) giving, in statistical physics, the probability p(E) to have the energy E (for a system having a temperature T). It is easy to see that the energy $E_{BF}(l_0)$ (background velocity fluctuations) would correspond to kT and $E_{Turb}(l_0)$ to the energy E of the system. We then have p(E)=exp[-

$\sigma_{c,0}^{P/F}$]=V_P/V_F. So there is, assuming Yakhot's law, a direct and remarkable link between the probability p(E) to have an energy E and the volume fraction V_P/V_F characterizing the multi-scale extension of a parabolic front relatively to an ideal fractal one.

We thus conclude that Yakhot's law expresses the fundamental link between statistics and multi-scale geometry through the relation $[E_{Turb}(l_0)]=V_P/V_F$.

8. Conclusion

In conclusion, entropic skins geometry appears to be an adequate multi-scale geometry to describe turbulent reactive fronts and namely turbulent flames. We determined an expression for the inner cut-off scale, i.e. the smallest scale of the front. Then, based on this geometrical framework, we proposed a new law for turbulent velocity that we verified with experimental measurements. This led us to introduce the concept of scale-entropy production which is directly linked to front roughness. Scale-entropy production offers the possibility of a comparison between the real multi-scale behavior (with dissipation and thus entropy production) and the ideal fractal one (which does not dissipate). We finally showed that Yakhot's law can acquire a simple geometrical interpretation. Our approach is based so far on geometrical arguments but recent developments show that it could be linked to thermodynamical arguments resulting from specific optimization methods in thermodynamics (Feidt, 1996). We also would like to investigate the possible connections between our approach and the constructal theory (Bejan, 2000). We hope to develop these aspects in our future work.

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Nomenclature

U'	turbulent intensity
U_L	laminar front velocity
U_T	turbulent front velocity
S_T	turbulent front surface
S_L	projected front surface
l_i	scale
l_c	inner cut-off scale
l_0	outer cut-off scale (integral scale)
l_k	Kolmogorov scale
u'_k	Kolmogorov velocity
t_k	Kolmogorov time
$l_{k,BG}$	Kolmogorov scale (in burnt gases)
x	scale-logarithm: $x=\ln(l_i/l_0)$
$U_{c,i}$	turbulent velocity due to $[l_c;l_i]$
$E_{BF}(l_0)$	background (indirect) kinetic energy
$E_{Turb}(l_0)$	direct kinetic energy
$N(l_i)$	number of covering balls at scale l_i
V_i	volume at scale l_i: $V_i=N_{i,0}l_i^d$
D_f	fractal dimension
d	embedding dimension (d=3)
S_x	scale-entropy
V_0	volume at integral scale
V_F	fractal volume
V_P	parabolic volume
$N_{c,0}^F$	covering balls in $[l_c;l_0]$, fractal case
$N_{c,0}^P$	covering balls in $[l_c;l_0]$, parabolic case
$S_{c,0}^F$	fractal scale-entropy
$S_{c,0}^P$	parabolic scale-entropy

Greek Letters

ε	rate of energy dissipation
Σ	front roughness
$\Sigma_{c,i}$	roughness due to scale range $[l_c;l_i]$
δ	thermal front thickness
γ	intermittency factor
Δ_c	crest fractal dimension
Δ_0	bulk fractal dimension
$\overline{\Delta}_{c,0}$	mean fractal dimension
Δ_x	local fractal dimension
ϕ_x	scale-entropy flux
$\omega(x)$, β	scale evolutivity
η	scale structure efficiency
χ	scale diffusivity
$\sigma_{c,0}^{P/F}$	scale-entropy production: $\sigma_{c,0}^{P/F}=S_{c,0}^P-S_{c,0}^F$

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