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An EPQ model for a deteriorating item with inflation reduced selling price and demand with immediate part payment

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Abstract

This paper presents a continuous EPQ model of deteriorating items with shortages. Also in this paper, an inventory policy for an item is presented with inflation and selling price dependent demand under deterministic and random planning horizons allowing shortages with an immediate part payment to the wholesaler. In this study inventory models under the finite and random planning horizons have been formulated with respect to the retailer's point of view for maximum profit. The GRG method is used to find the optimal solutions and the corresponding maximum profits for the different sets of given numerical data.

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1. Introduction

Normally, the payment for an order is made by the retailer to the supplier immediately just after the receipt of the consignment. Now-a-days, due to the stiff competition in the market, to attract more customers, a credit period is offered by the supplier to the retailer. Moreover, for the speedy movement of capital, a wholesaler tries to maximize his/her market through several means. For this, very often some concessions in terms of raw material cost credit period, etc., are offered to the retailers against immediate full/part payment. To avail these benefits, a retailer is tempted to cash down a part of the payment immediately even making a loan from the money lending source which

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charges interest against this loan. Now the retailer is in dilemma for optimal procurement and also for the amount for immediate part payment. Here an amount, borrowed from the money lending source as a loan with interest, is paid to the wholesaler at the beginning on receipt of goods. In return, the wholesaler/supplier offers a relaxed credit period as permissible delay in payment of rest amount and a reduced unit purchasing price depending on the amount of immediate part payment. Inflation also plays an important role for the optimal order policy and influences the demand of certain products. The effect of inflation and time value of the money cannot be ignored for determining the optimal inventory policy. In the present paper, an inventory control system in which immediate part payment and the delay-payment for the restare allowed by the wholesaler for an item over a finite planning horizon or random planning horizon with selling price and inflation induced demand is considered. In addition, against an immediate part payment (variable) to the wholesaler, there is a provision for (i) borrowing money from a money lending source and (ii) earning some relaxation on credit period from the wholesaler. The models are developed with respect to the retailer for maximum profit. Also in this present paper we have developed a continuous production control inventory model for deteriorating items with shortages in which two different rates of production are available and it is possible that production started at one rate and after some time it may be switched over to another rate .Such a situation is desirable in the sense that by starting at a low rate of production , a large quantum stock of manufactured item at the initial stage is avoided ,leading to reduction in the holding cost . The randomness in the planning horizon has been removed using the chance-constraint technique. Single objective problems incorporating immediate part payment, delay in payment for the rest, and selling price, inflation dependent demand with two production rate are formulated to maximize the profit function with shortages and solved using Generalized Reduced Gradient (GRG)method. The decision variables for these inventory models are the immediate part payment and number of cycles. These models are illustrated with numerical examples. Finally, the sensitivity analyses for the profit function and immediate part payment with respect to some parameters are carried out and the results are presented graphically.

2. Literature Review

First, Goyal [12] introduce the concept of permissible delay in payment in an EOQ model. Since then, lots of literature is available in this area of study. The important papers related to such studies are Chu et al. [6], Chung [7], Jamal et al.[17], Sarker et al. [24] and others. Effect of inflation and time value of money is also well established in inventory problems. The many authors such as Moon and Lee [21], Chen [10], Dey et al. [11], Padmanavan and Vrat[22] and others worked in these area. Jaggi et al. [16] developed an inventory model with shortages, in which units are deteriorating at constant rate and demand rate is increasing exponentially due to inflation over a finite planning horizon using discount cash flow approach. Tripathi et al. [26] developed a cash flow oriented EOQ model under permissible delay in payments for non-deteriorating items and time-dependent demand rate under inflation and time discounting. Huang[15] introduce an EPQ model under two level of trade credit policy. Liao[19] introduce an EPQ model of deteriorating item under permissible delay in payments. Chung ,Cßrdenas- Barr¾n [5] present a simple easy solution procedures to locate the optimal solutions of an inventory model that considers deteriorating items under stock-dependent demand and two-level trade credit. Cßrdenas[1-3] introduce an EOQ model with cash discount offer. Cßrdenas-Barr¾n, Smith and Goyal[4] introduce an EOQ model with planned backorders when the supplier offers a temporary fixed- percentage discount and has specified a minimum quantity of additional units purchase. Widyadana, Cßrdenas- Barr¾n and Wee[28]

proposed an EOQ model of deteriorating with a simplified approach. Cßrdenas- Barr$\frac{3}{4}$n [2] introduce an EOQ model with imperfect quality and quantity discounts. Taleizadeh, Mohammadi, Cßrdenas- Barr$\frac{3}{4}$n and Samimi [27] proposed an EOQ model for perishable product with special sale and shortage. Chen, Cßrdenas- Barr$\frac{3}{4}$n,Teng [9] extend EOQ model under conditionally permissible delay in payment. Jana, Das and Roy [18] introduce a partial backlogging inventory model for deteriorating item under Fuzzy inflation and discounting over random planning horizon. Singh et al. [25] proposed a two warehouse model under inflation with large quantity of purchase orders. Chung and Huang [8] studied ordering policy with permissible delay in payments to show the convexity of total annual variable cost function. Guria et al. [14] proposed a pricing model for petrol/diesel and determined the optimal ordering policy for an existing petrol/diesel retailing station under permissible delay in payment. Several authors like Panda and Maiti[23] investigated the inventory models of this type of item. Maiti et al. [20] introduced the concept of advanced payment for determining the optimal ordering policy under stochastic lead-time and price dependent demand condition. Recently Maiti and Guria [13] introduce an EOQ model with inflation reduced purchasing price, selling price and demand with immediate part payment.
In this paper we introduce an EPQ model for a deteriorating item with inflation reduced selling price and demand with immediate part payment.

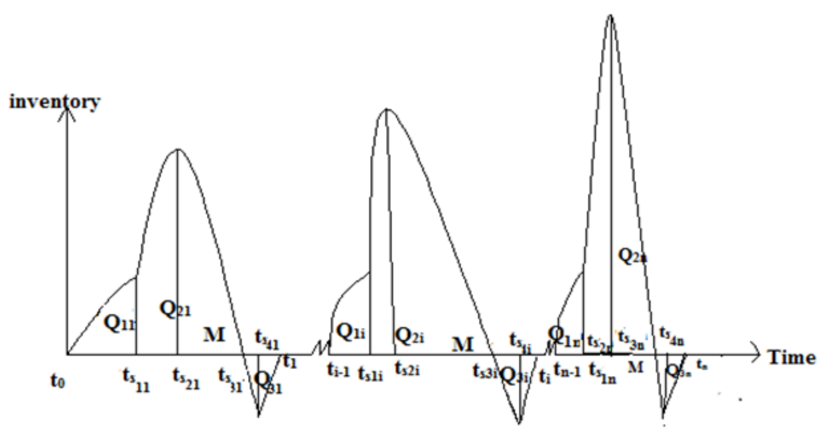

Figure 1. Graphical representation of finite time horizon inventory system

3. Mathematical Model Formulation:

To formulate the mathematical model for the proposed inventory system, the following notations and assumptions are made.

3.1. Notations

(i) $q_i(t)$=inventory level at time t for the ith cycle.
(ii) Q_{1i} = inventory level at time $t = t_{s_{1i}}$ for the ith cycle (t_{i-1}, t_i).

(iii) Q_{2i} = inventory level at time $t = t_{s_{2i}}$ for the ith cycle (t_{i-1}, t_i).
(iv) Q_{3i} = shortage quantity for the ith cycle (t_{i-1}, t_i).
(v) $Q_i = (Q_{1i} + Q_{2i})$ = total inventory for the ith cycle (t_{i-1}, t_i).
(vi) c_3 = set up cost for each cycle.
(vii) A = immediate part payment for each cycle .
(viii) n = number of cycles.
(ix) H= finite planning horizon for the crisp model.
(x) $\hat{H}$ =random planning horizon that follows normal distribution with mean $m_{\hat{H}}$ and standard deviation $\sigma_{\hat{H}}$ for the stochastic model.
(xi) T = length of each cycle.
(xii) M = permissible delay period for each cycle.
(xiii) $M^{'}$= rest period of each cycle after the credit period, i.e $(\frac{H}{n} - M)$.
(xiv) r= the unit raw material cost.
(xv) s_i = selling price per unit quantity for the ith cycle.
(xvi) D_0= original demand at t = 0
(xvii) $D_i(t)$= rate of demand (variable) for the ith cycle.
(xviii) R_i= The unit production cost in the ith cycle.
(xix) $P_1(> D_i(t))$and $P_2(> P_1)$ constant production rates started at time t_{i-1} and at time $t_{s_{1i}}$ respectively for the ith cycle.
(xx) I_e= rate of interest per unit to be earned by the retailer.
(xxi) I_b= rate of interest per unit to be paid by the retailer to money lender against immediate part payment A.
(xxii) c_1 = inventory holding cost per unit quantity per unit time for each cycle.
(xxiii) c_2 = inventory shortage cost per unit quantity per unit time for each cycle.
(xxiv) α = rate of inflation.
(xxv) t_s= time of beginning of shortages which is taken as $t_s = r^{'}\frac{H}{n}$;$0 < r^{'} < 1.0$
(xxvi) β and p_r are two given numbers where $\beta > 0, 0 \leq \beta \leq 1.0$ and $\varepsilon_{\hat{H}}$ is a real number whose standard normal value is p_r.
(xxvii) Z (n,A) is the profit function for the whole period.
(xxviii) t_p= time of beginning the second production P_2 which is taken as $t_p = \frac{r^{'}}{4}\frac{H}{n}$.
(xxix) t_q= time of beginning the demand after the second production $t_q = r^{'}\frac{H}{n}$.
(xxx) t_r= time of beginning the second production P_2 after shortage which is taken as $t_r = r^{''}\frac{H}{n}$;$0 < r^{'} < r^{''} < 1.0$.
(xxxi) $t_{s_{1i}} = t_{i-1} + t_p$, $t_{s_{2i}} = t_{i-1} + t_q$, $t_{s_{3i}} = t_{i-1} + t_s$,$t_{s_{4i}} = t_{i-1} + t_r$.

3.2. Assumption

(i) Lead time is zero.
(ii) Shortages are allowed and fully backlogged.
(iii) Immediate part payment A is made and same for all the cycles and it is paid at the beginning of each cycle. For the 1stcycle, it is borrowed from a money lending source with the condition that it will be paid at the end of business period H with interest at the rate of I_b and for the remaining cycles, the immediate part payment A will be given from the revenue earned up to that time. So, for ith cycle $(i \geq 2)$ the immediate part payment A must be less than the total revenue at t_{i-1}.
(iv) The rest payment for the wholesaler will be done at the end of the credit period M where $t_{i-1} + M \leq t_i$ for each cycle. There is a fixed credit period M_0 and it is assumed that it will be enhanced depending upon the amount of the immediate advanced payment A and is given in the form $M = M_0 + t^{'}A$ where $t^{'} << 1$ and $A < rQ_1$.

(v) The length of each cycle is $T = \frac{H}{n}$ i.e $\frac{H}{n} = t_i - t_{i-1}$ and $t_i = i\frac{H}{n}$, $t_0 = 0$;for i = 1,2,3, . . . , n for crisp model.

(vi) The rates of interest to be earned by the retailer and that to be paid to the money lender by the retailer are same for each cycle and$I_e < I_b$.

(vii) The unit production cost $R_i = \frac{re^{\alpha t_{i-1}}}{A^{\gamma_1}}$, $\alpha > 0$,$\gamma_1 > 0$,r is the unit raw material cost.

(viii) Selling price per unit quantity $s_i = mre^{\alpha t_{i-1}}$ where $m(> 0)$ is the mark-up. Here, the retailer does not share the reduced price obtained due to advance payment with the customers.

(ix) Demand is inflation rate and selling price dependent i.e $D_i = \frac{D_0 e^{\alpha_1 t}}{s_i^{\gamma}}$ where $\alpha_1 = k_2\alpha$; $0 < \gamma < 1.0$; $0 < k_2 < 1.0$ and $t_{i-1} \le t \le t_i$,where D_0 is the original demand.

(x) Holding cost per unit per unit time c_1 is same for all cycles.

(xi) Set up cost c_3 is also same for each cycle.

(xii) Shortage cost per unit per unit time c_2 is same for all cycles.

4. Chance Constraint Method:

Chance constraint programming is one of the techniques of stochastic programming which deals with a situation where some or all parameters of the problem are described by random variables. In this presentation, chance constraint is taken as
Prob$(|nT - \hat{H}| \le \beta) \ge p_r$, where $\hat{H}$ is a random variable .Here p_r is the highest value of the probability with which this chance constraint is satisfied. This can be rewriten as
Prob$(nT - \beta \le \hat{H}) \ge p_r$ and Prob$(\hat{H} - nT \le \beta) \ge p_r$

From the first equality
$Prob(\frac{nT-\beta-m_{\hat{H}}}{\sigma_{\hat{H}}} \le \frac{\hat{H}-m_{\hat{H}}}{\sigma_{\hat{H}}}) \ge p_r$
Now, $\frac{\hat{H}-m_{\hat{H}}}{\sigma_{\hat{H}}}$ represents the standard normal variant with mean 0 and varience 1, i.e
Prob$(nT - \beta \le \hat{H}) \ge p_r = 1 - F(\frac{nT-\beta-m_{\hat{H}}}{\sigma_{\hat{H}}})$
Where F(x) represents the continuous distribution function of standard normal distribution.
Let $\varepsilon_{\hat{H}}$ be the standard normal value such that F$(\varepsilon_{\hat{H}}) = p_r = \frac{1}{\sqrt{2\Pi}}\int_{-\infty}^{\varepsilon_{\hat{H}}} e^{-\frac{t^2}{2}} dt$

Then the statement Prob$(nT - \beta \le \hat{H}) \ge p_r$ is true if and only if
$\frac{nT-\beta-m_{\hat{H}}}{\sigma_{\hat{H}}} \le -\varepsilon_{\hat{H}}$ i.e $nT \le m_{\hat{H}} + \beta - \varepsilon_{\hat{H}}\sigma_{\hat{H}}$
Similarly, the second inequality can be reduced to $m_{\hat{H}} - \beta + \varepsilon_{\hat{H}}\sigma_{\hat{H}} \le nT$.

5. Mathematical Representation of the Model:

5.1 Case-I: Crisp time horizon

Model-IA:

Let $Q_i(t)$ be the on hand inventory for the ith cycle (t_{i-1}, t_i). Fully backlogged shortages are allowed towards at the end of each cycle for a $(1 - r^{'})\frac{H}{n}$ period of time. During that period shortage reaches Q_{3i} and then the production starts with rate P_2. Clearly total ordered quantity for ith cycle is $Q_i = Q_{2i} + Q_{3i}$
The variation of the inventory level $q_i(t)$ with respect to time t due to effect of demand $D_i(t)$, production and deterioration can be described by the following differential equation

$$\frac{dq_i(t)}{dt} + \theta q_i(t) = P_1 - D_i(t) , \qquad t_{i-1} \le t \le t_{s_{1i}} \quad i = 1, 2, 3, 4, ... \tag{1}$$

$\frac{dq_i(t)}{dt}+\theta q_i(t) = P_2 - D_i(t)\ , \qquad t_{s_{1i}} \le t \le t_{s_{2i}} \quad i = 1, 2, 3, 4, ...$ (2)

$\frac{dq_i(t)}{dt}+\theta q_i(t) = -D_i(t)\ , \qquad t_{s_{2i}} \le t \le t_{s_{3i}} \quad i = 1, 2, 3, 4, ...$ (3)

$\frac{dq_i(t)}{dt} = -D_i(t)\ , \qquad t_{s_{3i}} \le t \le t_{s_{4i}} \quad i = 1, 2, 3, 4, ...$ (4)

$\frac{dq_i(t)}{dt} = P_2 - D_i(t)\ , \qquad t_{s_{4i}} \le t \le t_i \quad i = 1, 2, 3, 4, ...$ (5)

With boundary conditions $q_i(t_{i-1}) = 0$,$q_i(t_{s_{1i}}) = Q_{1i}$, $q_i(t_{s_{2i}}) = Q_{2i}$, $q_i(t_{s_{3i}}) = 0$, $q_i(t_{s_{4i}}) = -Q_{3i}, q_i(t_i) = 0$ (6)

The solutions of equations (1),(2),(3),(4),(5)are respectively given below

$q_i(t) = \frac{P_1}{\theta}(1-e^{\theta(t_{i-1}-t)})+\frac{D_0}{s_i^{\gamma}(\alpha_1+\theta)}(e^{(\alpha_1+\theta)t_{i-1}-\theta t}-e^{\alpha_1 t}), \quad t_{i-1} \le t \le t_{s_{1i}}$ i=1,2,3,4,...... (7)

$q_i(t) = \frac{P_2}{\theta}(1 - e^{\theta(t_{i-1}+t_p-t)}) + \frac{D_0}{s_i^{\gamma}(\alpha_1+\theta)}(e^{(\alpha_1+\theta)(t_{i-1}+t_p)-\theta t} - e^{\alpha_1 t}) + Q_{1i}e^{\theta(t_{i-1}+t_p)-\theta t}$,

$t_{s_{1i}} \le t \le t_{s_{2i}}$ i=1,2,3,4,......... (8)

$q_i(t) = -\frac{D_0}{s_i^{\gamma}}\frac{e^{\alpha_1 t}}{(\alpha_1+\theta)} + \frac{D_0}{s_i^{\gamma}(\alpha_1+\theta)}(e^{(\alpha_1+\theta)(t_{i-1}+t_q)-\theta t} - e^{\alpha_1 t}) + Q_{2i}e^{\theta(t_{i-1}+t_p)-\theta t}$,

$t_{s_{2i}} \le t \le t_{s_{3i}}$ i=1,2,3,4,......... (9)

$q_i(t) = \frac{D_0}{s_i^{\gamma}\alpha_1}(e^{\alpha_1(t_{i-1}+t_s)} - e^{\alpha_1 t}), \quad t_{s_{3i}} \le t \le t_{s_{4i}}$ i=1,2,3,4,5,.....

(10)

$q_i(t) = P_2(t-t_i)+\frac{D_0}{s_i^{\gamma}\alpha_1}(e^{\alpha_1 t_i}-e^{\alpha_1 t}), \quad t_{s_{4i}} \le t \le t_i$ i=1,2,3,4,5,..... (11)

The inventory level Q_{1i} is obtained by putting $t = t_{s_{1i}}$in the equation (7) which is as follows

$Q_{1i} = \frac{P_1}{\theta}(1-e^{-\theta t_p})+\frac{D_0}{s_i^{\gamma}(\alpha_1+\theta)}e^{\alpha_1 t_{i-1}}(e^{-\theta t_p}-e^{\alpha_1 t_p}),$ (12)

The inventory level Q_{2i} is obtained by putting $t = t_{s_{2i}}$in the equation (8) which is as follows

$Q_{2i} = \frac{P_2}{\theta}(1-e^{\theta(t_p-t_q)})+\frac{D_0}{s_i^{\gamma}(\alpha_1+\theta)}e^{\alpha_1(t_{i-1}+t_p)}(e^{\theta(t_p-t_q)}-1)+Q_{1i}e^{\theta(t_p-t_q)},$ (13)

The inventory level Q_{3i} is obtained by putting $t = t_{s_{3i}}$in the equation (10) which is as follows

$Q_{3i} = \frac{D_0}{s_i^{\gamma}\alpha_1}e^{\alpha_1 t_{i-1}}(e^{\alpha_1 t_r}-e^{\alpha_1 t_s}),$ (14)

The total produced units during the whole business period is given by

$$\text{TQ}=\sum_{i=1}^{n} Q_i=\sum_{i=1}^{n}(Q_{2i}+Q_{3i}) = \frac{P_2 n}{\theta}(1-e^{(t_p-t_q)})+\frac{D_0(e^{\theta(t_p-t_q)}-1)}{m^{\gamma}r^{\gamma}(\alpha_1+\theta)}\sum_{i=1}^{n} e^{\alpha_1(t_{i-1}+t_p)-\alpha\gamma t_{i-1}}+$$
$$e^{\theta(t_p-t_q)}\frac{P_1 n}{\theta}(1-e^{-\theta t_p})+\frac{D_0(e^{-\theta t_p}-e^{\alpha_1 t_p})}{m^{\gamma}r^{\gamma}(\alpha_1+\theta)}\sum_{i=1}^{n} e^{(\alpha_1 t_{i-1}-\alpha\gamma t_{i-1})}+\frac{D_0}{m^{\gamma}r^{\gamma}\alpha_1}(e^{\alpha_1 t_r}-e^{\alpha_1 t_s})\sum_{i=1}^{n} e^{(\alpha_1 t_{i-1}-\alpha\gamma t_{i-1})}$$

Lemma 1: Prove that $\sum_{i=1}^{n} e^{(\alpha_1 t_{i-1}-\alpha\gamma t_{i-1})} = \frac{(e^{nv}-1)}{(e^{v}-1)}$

Where $\frac{H}{n} = t_{i-1} - t_i$,$v = (\alpha_1 - \alpha\gamma)\frac{H}{n}$

Using **Lemma 1** the total produced units during the whole business period is given by

$$\textbf{TQ}=\frac{P_2 n}{\theta}(1-e^{(t_p-t_q)})+\frac{P_1 n}{\theta}(e^{\theta(t_p-t_q)}-e^{-\theta t_q})+\frac{D_0(e^{\theta(t_p-t_q)}-1)}{m^{\gamma}r^{\gamma}(\alpha_1+\theta)}e^{\alpha_1 t_p}\frac{(e^{nv}-1)}{(e^{v}-1)}+\frac{D_0(e^{-\theta t_p}-e^{\alpha_1 t_p})}{m^{\gamma}r^{\gamma}(\alpha_1+\theta)}\frac{(e^{nv}-1)}{(e^{v}-1)}+\frac{D_0(e^{\alpha_1 t_r}-e^{\alpha_1 t_s})}{m^{\gamma}r^{\gamma}\alpha_1}\frac{(e^{nv}-1)}{(e^{v}-1)} \quad (15)$$

The total revenue earned in the ith cycle (t_{i-1}, t_i) and during the whole business (0,H) are respectively given by

$$T_{s_i} = \int_{t_{i-1}}^{t_i} s_i D_i dt = \frac{(mr)^{(1-\gamma)}e^{\alpha(1-\gamma)t_{i-1}}D_0}{\alpha_1}(e^{\alpha_1 t_i} - e^{\alpha_1 t_{i-1}})$$

$$TS = \sum_{i=1}^{n} T_{s_i} = \sum_{i=1}^{n}\frac{(mr)^{(1-\gamma)}e^{\alpha(1-\gamma)t_{i-1}}D_0}{\alpha_1}(e^{\alpha_1 t_i} - e^{\alpha_1 t_{i-1}})$$

Lemma 2: Prove that $\sum_{i=1}^{n} e^{\alpha_1 t_i+\alpha(1-\gamma)t_{i-1}} = e^{-x}\frac{e^{w}(e^{nw}-1)}{(e^{w}-1)}$

Where $\frac{H}{n} = t_{i-1} - t_i$,$x = \alpha(1-\gamma)\frac{H}{n}$,$w = [\alpha_1 + \alpha(1-\gamma)]\frac{H}{n}$

Lemma 3: Prove that $\sum_{i=1}^{n} e^{\alpha_1 t_{i-1}+\alpha(1-\gamma)t_{i-1}} = \frac{(e^{nw}-1)}{(e^{w}-1)}$

Where $\frac{H}{n} = t_{i-1} - t_i$,$w = [\alpha_1 + \alpha(1-\gamma)]\frac{H}{n}$

Therefore **TS**=$\frac{(rm)^{1-\gamma}}{\alpha_1} D_0 \frac{e^{w}(e^{nw}-1)}{(e^{w}-1)}(e^{-x}+e^{-w})$ (Using lemma 2 & 3) (16)

The holding cost for ith cycle (t_{i-1}, t_i) and during the whole business period (0,H) are respectively obtained as

$HC_i = c_1 \int_{t_{i-1}}^{t_{s_{3i}}} q_i(t)dt = c_1\{\frac{P_1}{\theta}(\theta t_p+e^{-\theta t_p}+1)+\frac{P_2}{\theta}(\theta(t_p-t_q)+e^{\theta(t_p-t_q)}-1)+\frac{D_0}{(mr)^{\gamma}(\alpha_1+\theta)}\frac{1}{\alpha_1\theta}e^{(\alpha_1 t_{i-1}-\alpha\gamma t_{i-1})}$ $[(\alpha_1+\theta) - \alpha_1 e^{-\theta t_p} - \theta e^{\alpha_1 t_s} + \alpha_1 e^{\alpha_1 t_p}(1-e^{\theta(t_p-t_q)}) + \alpha_1 e^{\alpha_1 t_q}(1-e^{\theta(t_q-t_s)})] - \frac{Q_{1i}}{\theta}(1-e^{\theta(t_p-t_q)}) + \frac{Q_{2i}}{\theta}(1-e^{\theta(t_q-t_s)})\}$

THC =$\sum_{i=1}^{n} HC_i = c_1\{n\frac{P_1}{\theta}(\theta t_p+e^{-\theta t_p}+1)+n\frac{P_2}{\theta}(\theta(t_p-t_q)+e^{\theta(t_p-t_q)}-1)+\frac{D_0}{(mr)^{\gamma}(\alpha_1+\theta)}\frac{1}{\alpha_1\theta}[(\alpha_1+\theta)-\alpha_1 e^{-\theta t_p}-\theta e^{\alpha_1 t_s}+\alpha_1 e^{\alpha_1 t_p}(1-e^{\theta(t_p-t_q)})+\alpha_1 e^{\alpha_1 t_q}(1-e^{\theta(t_q-t_s)})]\sum_{i=1}^{n} e^{(\alpha_1 t_{i-1}-\alpha\gamma t_{i-1})} - \frac{1}{\theta}(1-e^{\theta(t_p-t_q)})\sum_{i=1}^{n} Q_{1i} + \frac{1}{\theta}(1-e^{\theta(t_q-t_s)}\sum_{i=1}^{n} Q_{2i}\}$

$=c_1\{n\frac{P_1}{\theta}(\theta t_p + e^{-\theta t_p} + 1) + n\frac{P_2}{\theta}(\theta(t_p - t_q) + e^{\theta(t_p-t_q)} - 1) + \frac{D_0}{(mr)^{\gamma}(\alpha_1+\theta)}\frac{1}{\alpha_1\theta}[(\alpha_1 + \theta) - \alpha_1 e^{-\theta t_p} - \theta e^{\alpha_1 t_s} + \alpha_1 e^{\alpha_1 t_p}(1-e^{\theta(t_p-t_q)}) + \alpha_1 e^{\alpha_1 t_q}(1-e^{\theta(t_q-t_s)})]\frac{(e^{nv}-1)}{(e^{v}-1)} + \frac{nP_1}{\theta^2}(1-e^{\theta(t_p-t_q)})(1-e^{-\theta t_p}) - \frac{D_0}{(rm)^{\gamma}(\alpha_1+\theta)}\frac{1}{\theta}(1-e^{\theta(t_p-t_q)}(e^{-\theta t_p}-e^{\alpha_1 t_q})\frac{(e^{nv}-1)}{(e^{v}-1)} + \frac{P_2 n}{\theta^2}(1-e^{\theta(t_q-t_s)})(1-e^{\theta(t_p-t_q)}) + \frac{P_1 n}{\theta^2}(1-e^{\theta(t_q-t_s)})(e^{\theta(t_p-t_q)}+e^{-\theta t_q}) + \frac{D_0}{(rm)^{\gamma}(\alpha_1+\theta)}\frac{1}{\theta}(1-e^{\theta(t_q-t_s)})(e^{\theta(t_p-t_q)}-1)e^{\alpha_1 t_p}\frac{(e^{nv}-1)}{(e^{v}-1)} + \frac{D_0}{(rm)^{\gamma}(\alpha_1+\theta)}\frac{1}{\theta}(1-e^{\theta(t_q-t_s)})(e^{-\theta t_p}-e^{\alpha_1 t_p})\frac{(e^{nv}-1)}{(e^{v}-1)}\}$ (17)

The shortage cost for the i-th cycle (t_{i-1}, t_i) and during the whole business period (0,H) are respectively obtained as

$SHC_i = c_2[\int_{t_{s_{3i}}}^{t_{s_{4i}}} -q_i(t)dt + \int_{t_{s_{4i}}}^{t_i} -q_i(t)dt]$

$=-c_2[\frac{D_0}{s_i^{\gamma}\alpha_1^2}e^{\alpha_1 t_{i-1}}(\alpha_1 t_r e^{\alpha_1 t_s}-\alpha_1 t_s e^{\alpha_1 t_s}+e^{\alpha_1 t_s})+\frac{D_0}{s_i^{\gamma}\alpha_1^2}e^{\alpha_1 t_i}(\alpha_1\frac{H}{n}-\alpha t_r-1)-\frac{P_2}{2}(\frac{H}{n}-t_r)^2]$

TSHC $=-c_2[\frac{D_0}{(mr)^{\gamma}\alpha_1^2}(\alpha_1 t_r e^{\alpha_1 t_s} - \alpha_1 t_s e^{\alpha_1 t_s} + e^{\alpha_1 t_s})\frac{(e^{nv}-1)}{(e^{v}-1)} + \frac{D_0}{(mr)^{\gamma}\alpha_1^2}(\alpha_1\frac{H}{n} - \alpha t_r - 1)e^{v}\frac{(e^{nv}-1)}{(e^{v}-1)} - \frac{P_2}{2}(\frac{H}{n}-t_r)^2 n]$ (18)

The raw material cost of all units purchased for the i-th cycle (t_{i-1}, t_i) and during the whole business period (0,H) are respectively obtained as

$Rc_i = rQ_i$

TRC =$r\sum_{i=1}^{n} Q_i = r[\frac{P_2 n}{\theta}(1-e^{(t_p-t_q)})+\frac{P_1 n}{\theta}(e^{(t_p-t_q)}-e^{-\theta t_q})+\frac{D_0(e^{\theta(t_p-t_q)-1})}{(mr)^{\gamma}(\alpha_1+\theta)}e^{\alpha_1 t_p}\frac{(e^{nv}-1)}{(e^{v}-1)}+\frac{D_0(e^{-\theta t_p-e^{\alpha_1 t_p}})}{(mr)^{\gamma}(\alpha_1+\theta)}\frac{(e^{nv}-1)}{(e^{v}-1)}+\frac{D_0}{(mr)^{\gamma}\alpha_1}(e^{\alpha_1 t_r}-e^{\alpha_1 t_s})\frac{(e^{nv}-1)}{(e^{v}-1)}]$

(19)

In the finite time horizon, total interest can be earned in two ways:

(i) interest is earned from the revenue due to the continuous sale during the whole time horizon (i.e., TIECS) and

(ii)interest is earned from the part of the revenue remaining at the end of each cycle after paying due amount of this cycle and the advance (immediate part payment) for the next cycle from the total sale revenue for the present cycle. This is followed for all cycles except the first one (i.e., IET_n).

Therefore, for the first case, the interest earned ($IECS_i$) by continuous sale during the interval $(t_{i-1}, t_i + t_s)$ is given by

$IECS_i = I_e[\int_{t_{i-1}}^{t_{i-1}+M} s_i D_i(t_{i-1}+M-t)dt + \int_{t_{i-1}+M}^{t_{i-1}+t_s} s_i D_i(t_{i-1}+t_s-t)dt]$
$= I_e[(t_{i-1}+M)\int_{t_{i-1}}^{t_{i-1}+M} s_i D_i dt + (t_{i-1}+t_s)\int_{t_{i-1}+M}^{t_{i-1}+t_s} s_i D_i dt - \int_{t_{i-1}}^{t_{i-1}+t_s} s_i D_i t dt]$
$= I_e \frac{(mr)^{1-\gamma}D_0}{\alpha_1}[(M - r'\frac{H}{n})e^{\alpha_1 M} - M + \frac{1}{\alpha}(e^{\alpha_1 r'\frac{H}{n}} - 1)e^{\{\alpha_1+\alpha(1-\gamma)\}t_{i-1}}$
Therefore, the total interest (TIECS) earned by continuous sale during the whole business period (0,H) is given by
TIECS $= I_e \frac{(mr)^{1-\gamma}D_0}{\alpha_1}[(M - r'\frac{H}{n})e^{\alpha_1 M} - M + \frac{1}{\alpha}(e^{\alpha_1 r'\frac{H}{n}} - 1)\frac{(e^{nw}-1)}{(e^w-1)}]$
(20)
Similarly, for the second case, the interest earned (IET_n) is given by
$IET_n = I_e[M'\{\int_{t_0}^{t_0+M} s_1 D_1 dt + \int_{t_1}^{t_1+M} s_1 D_1 dt + \int_{t_2}^{t_2+M} s_2 D_2 dt + ... + \int_{t_{n-1}}^{t_{n-1}+M} s_1 D_1 dt - (Q_1R_1 + Q_2R_2 + ... + Q_nR_n) + nA\} + M\{(Ts_1 - Q_1P_1) + (Ts_1 + Ts_2 - Q_1P_1 - Q_2P_2) + ... + (Ts_1 + Ts_2 + Ts_3 + Ts_4 + Ts_5 + ... + Ts_{n-1} - Q_1P_1 - Q_2P_2 - Q_3P_3 - Q_4P_4 - .... - Q_{n-1}P_{n-1})\} + M'\{(Ts_1 - Q_1R_1) + (Ts_1 + Ts_2 - Q_1R_1 - Q_2R_2) + ... + (Ts_1 + Ts_2 + Ts_3 + Ts_4 + Ts_5 + ... + Ts_{n-1} - Q_1P_1 - Q_2P_2 - Q_3P_3 - Q_4P_4 - .... - Q_{n-1}P_{n-1})\}]$
$= I_e[M'\{\sum_{j=1}^{n}\int_{t_{j-1}}^{t_{j-1}+M} s_j D_j dt - \sum_{j=1}^{n} Q_jP_j + nA\} + (M + M')\{\sum_{j=1}^{n-1}(n-j)(Ts_j - Q_jR_j)\}]$
Now $\sum_{j=1}^{n}\int_{t_{j-1}}^{t_{j-1}+M} s_j D_j dt = \frac{(mr)^{1-\gamma}D_0(e^{\alpha_1 M}-1)}{\alpha_1}\sum_{j=1}^{n} e^{\alpha_1 t_{i-1}+\alpha(1-\gamma)t_{i-1}}$
$= \frac{(mr)^{1-\gamma}D_0(e^{\alpha_1 M}-1)}{\alpha_1}\frac{(e^{nw}-1)}{(e^w-1)}$ (using Lemma 3)
$\sum_{j=1}^{n-1}(n-j)(Ts_j - Q_jR_j) = \sum_{j=1}^{n-1}(n-j)(s_jQ_j - Q_jR_j) = \sum_{j=1}^{n-1}(n-j)(s_j - R_j)Q_j$
$= (mr - \frac{r}{A^{\gamma_1}})[\{\frac{P_2}{\theta}(1 - e^{\theta(t_p-t_q)}) + \frac{P_1}{\theta}(e^{\theta(t_p-t_q)} - e^{\theta t_q})\sum_{j=1}^{n-1}(n-j)e^{\alpha t_{j-1}} + \frac{D_0}{(mr)^{\gamma}(\alpha_1+\theta)}(e^{\theta t_q} - e^{\alpha_1 t_p})\sum_{j=1}^{n-1}(n-j)e^{\{\alpha_1+(1-\gamma\alpha)\}t_{j-1}} + \frac{D_0}{(rm)^{\gamma}\alpha_1}\sum_{j=1}^{n-1}(n-j)e^{\{\alpha_1+(1-\gamma\alpha)\}t_{j-1}}\}]$

Lemma 4: Prove that $\sum_{j=1}^{n} je^{jx} = \frac{ne^{(n+2)x} - (n+1)e^{(n+1)x} + e^x}{(e^x-1)^2}$
Now $\sum_{j=1}^{n-1}(n-j)e^{\alpha t_{j-1}} = \sum_{j=1}^{n-1}(n-j)e^{\alpha(j-1)\frac{H}{n}} = e^{-\alpha\frac{H}{n}}\sum_{j=1}^{n-1}(n-j)e^{\alpha j\frac{H}{n}}$
$= e^{-x}\sum_{j=1}^{n-1}(n-j)e^{jx}$, where $x = \alpha\frac{H}{n}$
$= e^{-x}[n\sum_{j=1}^{n-1}e^{jx} - \sum_{j=1}^{n-1}je^{jx}]$
$= e^{-x}[n\frac{e^x(e^{(n-1)x}-1)}{(e^x-1)} - \frac{((n-1)e^{(n+1)x} - ne^{nx} + e^x)}{(e^x-1)^2}]$
$= [n\frac{(e^{(n-1)x}-1)}{(e^x-1)} - \frac{((n-1)e^{nx} - ne^{(n-1)x} + 1)}{(e^x-1)^2}]$ (using Lemma 4)
Lemma 5: Prove that $\sum_{j=1}^{n-1}(n-j)e^{\{\alpha_1+(1-\gamma)\alpha\}t_{j-1}} = [n\frac{(e^{(n-1)w}-1)}{(e^w-1)} - \frac{((n-1)e^{nw} - ne^{(n-1)w} + 1)}{(e^w-1)^2}]$
Also $\sum_{j=1}^{n} Q_jR_j = \frac{r}{A^{\gamma_1}}[\{\frac{P_2}{\theta}(1 - e^{\theta(t_p-t_q)}) + \frac{P_1}{\theta}(e^{\theta(t_p-t_q)} - e^{-\theta t_q})\}\sum_{j=1}^{n} e^{\alpha t_{j-1}} + \frac{D_0}{(rm)^{\gamma}(\alpha_1+\theta)}(e^{-\theta t_q} - e^{\alpha_1 t_p})\sum_{j=1}^{n} e^{\{\alpha_1+(1-\gamma)\alpha\}t_{j-1}} + \frac{D_0}{(rm)^{\gamma}\alpha_1}(e^{\alpha_1 t_r} - e^{\alpha_1 t_s})\sum_{j=1}^{n} e^{\{\alpha_1+(1-\gamma)\alpha\}t_{j-1}}]$

Now, $\sum_{j=1}^{n} e^{\alpha t_{j-1}} = \sum_{j=1}^{n} e^{\alpha (j-1)\frac{H}{n}} = e^{-\alpha \frac{H}{n}} \sum_{j=1}^{n} e^{\alpha j \frac{H}{n}} = e^{-x} \sum_{j=1}^{n} e^{jx} = e^{-x} \frac{e^{x}(e^{nx}-1)}{(e^{x}-1)}$
$= \frac{(e^{nx}-1)}{(e^{x}-1)}$
And $\sum_{j=1}^{n} e^{\alpha_1 t_{j-1}+\alpha(1-\gamma)t_{i-1}} = \frac{(e^{nw}-1)}{(e^{w}-1)}$
Therefore $\sum_{j=1}^{n} Q_j R_j = \frac{r}{A^{\gamma_1}}[\{\frac{P_2}{\theta}(1-e^{\theta(t_p-t_q)})+\frac{P_1}{\theta}(e^{\theta(t_p-t_q)}-e^{-\theta t_q})\}\frac{(e^{nx}-1)}{(e^{x}-1)}+\frac{D_0}{(rm)^{\gamma}(\alpha_1+\theta)}$
$(e^{-\theta t_q}-e^{\alpha_1 t_p})\frac{(e^{nw}-1)}{(e^{w}-1)}+\frac{D_0}{(rm)^{\gamma}\alpha_1}(e^{\alpha_1 t_r}-e^{\alpha_1 t_s})\frac{(e^{nw}-1)}{(e^{w}-1)}]$
Using **Lemma 5** IET_n is given by
$IET_n = I_e\{M'[\frac{(mr)^{1-\gamma}D_0}{\alpha_1}(e^{\alpha_1 M}-1)\frac{(e^{nw}-1)}{(e^{w}-1)} - \frac{r}{A^{\gamma_1}}(\{\frac{P_2}{\theta}(1-e^{\theta(t_p-t_q)})+\frac{P_1}{\theta}(e^{\theta(t_p-t_q)} -$
$e^{-\theta t_q})\}\frac{(e^{nx}-1)}{(e^{x}-1)}+\frac{D_0}{(mr)^{\gamma}(\alpha_1+\theta)}(e^{-\theta t_q}-e^{\alpha_1 t_p})\frac{(e^{nw}-1)}{(e^{w}-1)}+\frac{D_0}{(mr)^{\gamma}\alpha_1}(e^{\alpha_1 t_r}-e^{\alpha_1 t_s})\frac{(e^{nw}-1)}{(e^{w}-1)})+$
$nA]+(M+M')[(mr-\frac{r}{A^{\gamma_1}})(\{\frac{P_2}{\theta}(1-e^{\theta(t_p-t_q)})+\frac{P_1}{\theta}(e^{\theta(t_p-t_q)}-e^{-\theta t_q})\}(n\frac{(e^{(n-1)x}-1)}{(e^{x}-1)} -$
$\frac{((n-1)e^{nx}-ne^{(n-1)x}+1)}{(e^{x}-1)^2})+\frac{D_0}{(mr)^{\gamma}(\alpha_1+\theta)}(e^{-\theta t_q}-e^{\alpha_1 t_p})$
$(n\frac{(e^{(n-1)w}-1)}{(e^{w}-1)}-\frac{((n-1)e^{nw}-ne^{(n-1)w}+)}{(e^{w}-1)^2})))]\}$ (21)
The interest paid at the end of business period (0,H) is given by
TIP $= I_b$ AH
(22)
Total set-up cost at the end of the business period (0,H) is given by
$TC_3 = nc_3$ (23)
Hence the total profit function for the retailer is given over the whole business period is given by
$Z(n,A) = TS + TIECS + IET_n - TRC - THC - TSHC - TC_3 - TIP$ (24)
Here the optimization problem for this model is
Max Z(n,A)
(25)
Subject to the constrains
Assumption (iii) & (iv)
(26)

5.2 Case -II: Random time horizon

Chance Constraint for random time horizon

Here, the time horizon $\hat{H}$ is considered as random but the total time horizon is partitioned into n cycles with length T for each cycle. Therefore, following Section 4, corresponding chance constraint for the whole business period is given by $Prob(|(nT-\hat{H})| \leq \beta) \geq p_r$ which is reduced to $m_{\hat{H}} - \beta + \varepsilon_{\hat{H}}\sigma_{\hat{H}} \leq nT \leq m_{\hat{H}} + \beta - \varepsilon_{\hat{H}}\sigma_{\hat{H}}$
(27)

where $p_r = \frac{1}{\sqrt{2\Pi}} \int\limits_{-\infty}^{\varepsilon_{\hat{H}}} e^{-\frac{t^2}{2}} dt$ is a cumulative probability $Prob[t \leq \epsilon]$ available in standard table for different values of $\varepsilon_{\hat{H}}$.

Model-IIA: model with shortages

Considering $H = m_{\hat{H}}$ in Model-IA, the model with fully backlogged shortages in random time horizon is formulated. Hence, total profit function for the retailer over the whole business period is given by the Eq.(24).
Here the optimization problem for this model is
Max Z(n,A)
(28)
Subject to the constrains (26) & (27) (29)

6. Numerical Experiments

For illustration, we use following data for **Model-IA:**
D_0=15 units/year,$r = 2.5, r^{'} = 0.8, r^{''} = 0.9, t^{'} = 0.001, P_1 = 530, P_2 = 550$ $\gamma = 0.08, \gamma_1 = 0.05, \alpha = 0.1, k_2 = 0.7, \theta = 0.3, m = 1.3, H = 12, c_3 = \$6/order$, $m_0 = 0.2, I_e = \$1.7/dollar/year$, $I_b = \$0.8/dollar/year, c_2 = \$4/unit/year, c_1 = \$3/unit/year$
Considering $\beta = 0.5, p_r = 0.6554, \varepsilon_{\hat{H}} = 0.4, m_{\hat{H}} = 11.1, \sigma_{\hat{H}} = 0.6$, together with the above numerical parameters the results of **Model-IIA** are derived.

Table 1:-Profit of different shortage period for the crisp Inventory model(i.e effect of $r^{'}$ and $r^{''}$on profit in Model-IA)

$r^{'}$	$r^{''}$	Z	A	n	M	THC	$TSHC$	TRC
0.55	0.85	34493.43	2759.615	3	2.959615	27282.22	2916.143	3869.383
0.56	0.84	34392.44	2766.780	3	2.966780	27657.29	2659.746	3912.416
0.57	0.83	34304.64	2773.887	3	2.973887	28028.99	2393.959	3955.110
0.58	0.82	34229.99	2780.938	3	2.980938	28397.35	2118.790	3997.467
0.59	0.81	34168.42	2787.932	3	2.987932	28762.41	1834.244	4039.488
0.6	0.8	34119.88	2794.872	3	2.994872	29124.22	1540.325	4081.176
0.61	0.79	34084.33	2801.756	3	3.001756	29482.82	1237.040	4122.533
0.62	0.78	34061.71	2808.585	3	3.008585	29838.24	924.3951	4163.559

Table 2:-Variation of profit with different $\varepsilon_{\hat{H}}$&$\sigma_{\hat{H}}$ parameters in random time horizon for stochastic inventory model (i.e effect of $\varepsilon_{\hat{H}}$&$\sigma_{\hat{H}}$ on profit -Model IIA)

$\varepsilon_{\hat{H}}$	$\sigma_{\hat{H}}$	Z	A	n	nT	M	T
	0.6	24333.74	2800.193	3	11.42	3.000193	3.806667
	0.7	24333.74	2792.192	3	11.39	2.992192	3.796667
0.3	0.8	24333.74	2784.202	3	11.36	2.984202	3.786667
	0.9	24333.74	2776.223	3	11.33	2.976223	3.776667
	1.0	24333.74	22768.256	3	11.30	2.968256	3.766667
	0.6	24248.77	2784.202	3	11.36	2.984202	3.786667
	0.7	24193.03	2773.566	3	11.32	2.973566	3.773333
0.4	0.8	24138.02	2762.951	3	11.28	3.962951	3.760000
	0.9	24083.73	2752.355	3	11.24	2.952355	3.746667
	1.0	24030.17	2741.779	3	11.20	2.941779	3.733333
	0.6	24165.43	2768.256	3	11.30	2.968256	3.766667
	0.7	24097.23	2755.002	3	11.25	2.955002	3.750000
0.5	0.8	24030.17	2741.779	3	11.20	2.941779	3.733333
	0.9	23964.23	2728.587	3	11.15	2.928587	30716667
	1.0	23899.42	2715.425	3	11.10	2.915425	3.700000

Table 3:-Variation of profit with different values of Interest earned in random time horizon for stochastic inventory model (i.e effect of I_e and $I_b$on profit-Model-II A)

I_e	I_b	Z	A	n	M
	0.9	21143.74	2682.486	3	2.882486
	0.8	24248.77	2784.202	3	2.984202
1.7	0.7	27469.66	2886.476	3	3.086476
	0.6	30807.04	2989.263	3	3.189263
	0.9	25991.75	2733.271	3	2.933271
	0.8	29151.41	2829.591	3	3.029591
1.8	0.7	32420.76	2926.390	3	3.126390
	0.6	35800.34	3023.631	3	3.223631
	0.9	30867.12	2778.834	3	2.978834
	0.8	34075.79	2870.293	3	3.070293
1.9	0.7	37388.59	2962.166	3	3.162166
	0.6	40805.97	3054.425	3	3.254425

7. Results and Discussions:

Optimal solutions for Model-IA is given in Table 1 for above set of input parameters. In Table 1, with different shortage periods in finite time horizon crisp inventory model (i.e., effect of $r^{'}$ & $r^{''}$on profit function of Model-IA), values of maximum profit are presented. It is noted from Table 1 and fig 3, that as the period of shortages decreases (i.e., $r^{'}$ increases), the total profit decreases as expected. It is also noted that with the increase in $r^{'}$ optimal values of immediate part payment also increases in Model-IA. Further increase in $r^{'}$ also increases the optimal profit and delay period.

Table 2 presents the values of profit function with different $\varepsilon_{\hat{H}}$ & $\sigma_{\hat{H}}$ in random time horizon (i.e., effects of $\varepsilon_{\hat{H}}$ & $\sigma_{\hat{H}}$ on profit: Model-IIA). From Table 2 and fig 1, it is revealed that as the total time horizon is nearer to 12.0 (the crisp value of H), profit becomes maximum. Here, as the dispersion about the mean value of $\hat{H}(\sigma_{\hat{H}})$ is small, time horizon is nearer to 12.0 and hence the profit is maximum. Thus with the increase

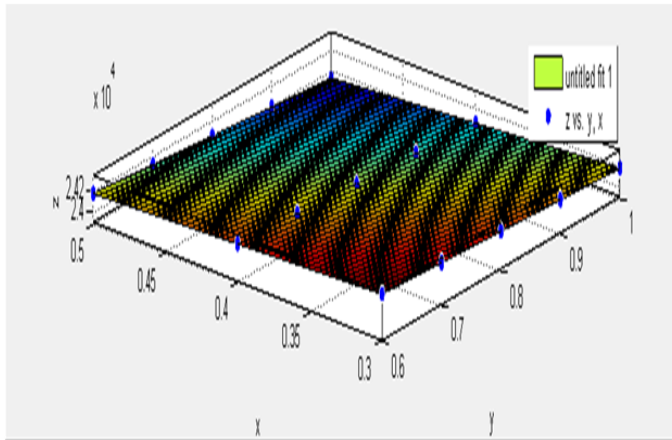

Figure 2. Effect of $\varepsilon_{\hat{H}}$ & $\sigma_{\hat{H}}$ on profit Z

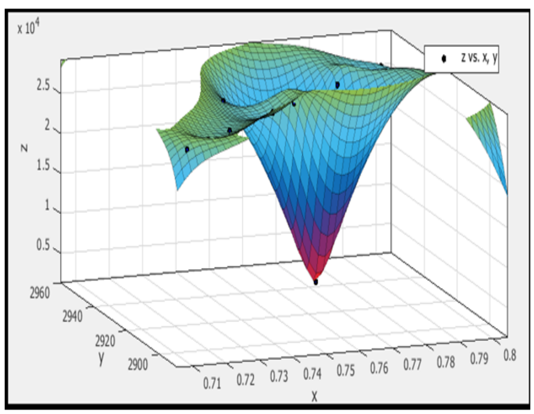

Figure 3. Effect of $r^{'}$ on profit Z and A(Model-IA)

in dispersion, profit decreases. Same trend is observed with the variation of $\varepsilon_{\hat{H}}$ .Smaller $\varepsilon_{\hat{H}}$ indicates that the chance constraint (27) is strictly satisfied and with the decrease of $\varepsilon_{\hat{H}}$ profit increases. Therefore, all these trends are as per the formulation of the model. Table 3 presents the values of profit function with different values of Interest earned in random time horizon (i.e., effect of I_e on profit function of Model-IIA). It is noted from Table 3 that as rate of interest earned increases the total profit increases as expected. It is also noted that with the increase in I_e optimal values of immediate part payment also increases in Model-IIA. Further decrease of I_b (interest payable) increases the optimal profit immediate part payment and delay period.

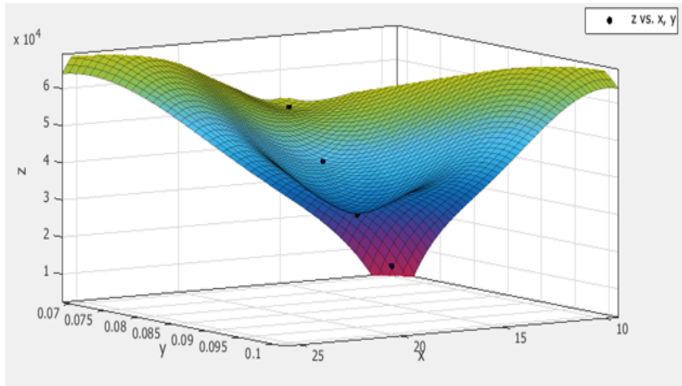

Figure 4. Effect of D_0, gama on profit Z (Model-IA)

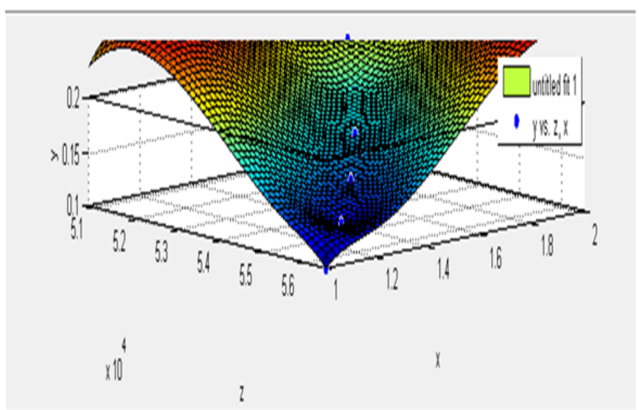

Figure 5. Effect of c_1,$r^{'}$ on profit Z (Model-IIA)

8. Sensitivity analyses:

The effects of change in the demand parameter (γ) on the immediate part payment (A) and the profit (Z) are studied and presented in Fig. 9. From this graph, it is observed that, when the value of γ increases, the profit and the immediate part payment decrease with same number of replenishment and almost same value of credit period, but the effect of changes in γ is significant, as with the increase in γ demand decreases and hence the optimum profit and advance payment decrease.

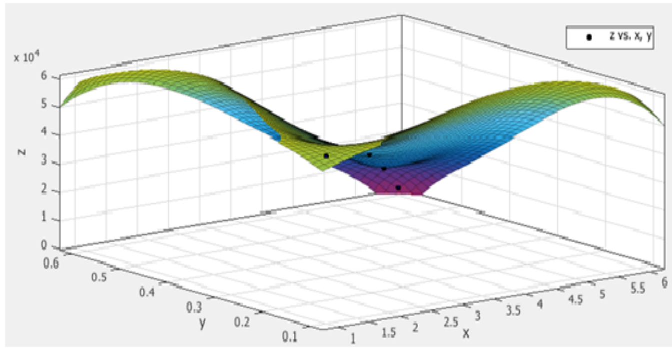

Figure 6. Effect of c_1,r' on profit Z (Model-IA)

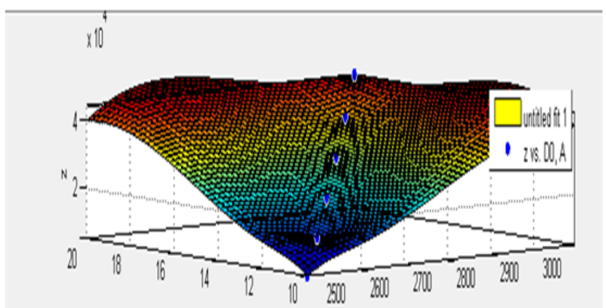

Figure 7. Effect of D_0 on profit Z and immediate part payment A(Model-IIA)

The effects of initial demand D_0 on the immediate part payment (A) and the profit (Z) are studied and presented in Fig. 7. From this graph, it is observed that, when the value of D_0 increases, the profit and the immediate part payment increase with same number of replenishment and almost same value of credit period, but the effect of changes in D_0 is significant, as with the increase of demand the profit also increase.
The effects of change of holding cost and setup cost on the profit (Z) are studied and presented in Fig. 8. From this graph, it is observed that, with the increase of holding

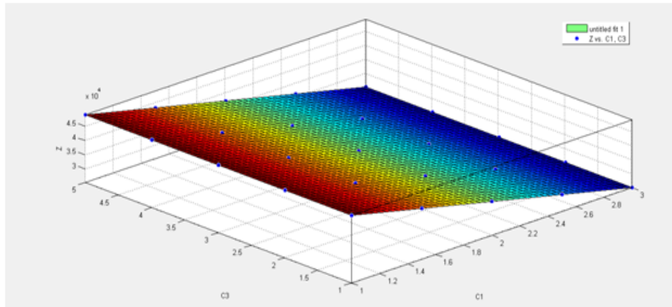

Figure 8. Effect of c_1,c_3 on profit Z (Model-IA)

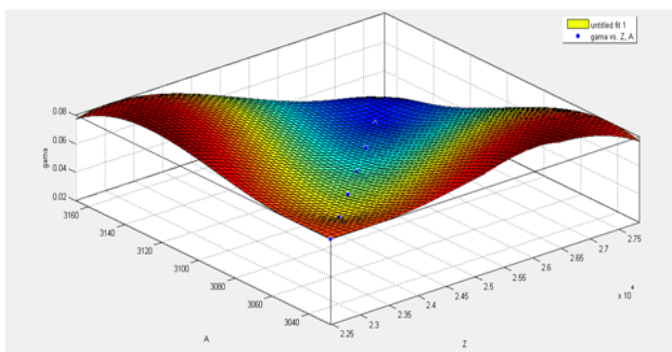

Figure 9. Effect of γ on profit Z and immediate part payment A (Model-IA)

cost and setup cost the total profit increase.

9. Conclusions:

In this paper we introduce the concept of immediate part payment in EPQ model in both deterministic and random planning horizon under inflation with allowing shortages.

This study presents deterministic EPQ model for inflation rate and selling price dependent demand under a situation in which the supplier offers a trade credit period to his retailer to settle down the account for purchased quantities and reduced unit production cost against an immediate part payment paid on the receipt of the raw material by the retailer who, in this situation, makes loan from the money lending source. Numerical results for all models with immediate part payment show that immediate part payment plays an important role for the retailer to earn more profit. The intuitive reason is that, when the immediate part payment increases then credit period also increases. In this situation the retailer earn more by taking loan from money lending source. Hence this analysis answers to the retailer's dilemma how much to make for immediate part payment to enjoy the wholesaler's concessions for maximum profit in spite of the fact that more part payment means more loan and more interest paid.

The above model can be extended in various ways. We should extend this model for two trade credit; taking partial trade credit; taking demand, selling price as a fuzzy number etc.

A. Appendix:

Lemma 1: Prove that $\sum_{i=1}^{n} e^{(\alpha_1 t_{i-1} - \alpha\gamma t_{i-1})} = \frac{(e^{nv}-1)}{(e^{v}-1)}$

Where $\frac{H}{n} = t_{i-1} - t_i$,$v = (\alpha_1 - \alpha\gamma)\frac{H}{n}$

Proof: $\sum_{i=1}^{n} e^{(\alpha_1 t_{i-1} - \alpha\gamma t_{i-1})} = \sum_{i=1}^{n} e^{(\alpha_1 - \alpha\gamma) t_{i-1}}$

$=\sum_{i=1}^{n} e^{(\alpha_1 - \alpha\gamma)(i-1)\frac{H}{n}}$

$=\sum_{i=1}^{n} e^{(\alpha_1 - \alpha\gamma)i\frac{H}{n}} e^{-(\alpha_1 - \alpha\gamma)\frac{H}{n}}$

$$=\sum_{i=1}^{n} e^{-v} e^{iv} = e^{-v}\sum_{i=1}^{n} e^{iv} = e^{-v}\frac{e^{v}(e^{nv}-1)}{(e^{v}-1)} = \frac{(e^{nv}-1)}{(e^{v}-1)} \tag{A.1}$$

Lemma 2: Prove that $\sum_{i=1}^{n} e^{\alpha_1 t_i + \alpha(1-\gamma)t_{i-1}} = e^{-x}\frac{e^{w}(e^{nw}-1)}{(e^{w}-1)}$

Where $\frac{H}{n} = t_{i-1} - t_i$,$x = \alpha(1-\gamma)\frac{H}{n}$,$w = [\alpha_1 + \alpha(1-\gamma)]\frac{H}{n}$

Proof: $\sum_{i=1}^{n} e^{\alpha_1 t_i + \alpha(1-\gamma)t_{i-1}} = \sum_{i=1}^{n} e^{\alpha_1 i\frac{H}{n} + \alpha(1-\gamma)(i-1)\frac{H}{n}}$

$=\sum_{i=1}^{n} e^{-\alpha(1-\gamma)\frac{H}{n}} e^{i[\alpha_1 + \alpha(1-\gamma)\frac{H}{n}]}$

$=e^{-\alpha(1-\gamma)\frac{H}{n}} \sum_{i=1}^{n} e^{i[\alpha_1 + \alpha(1-\gamma)\frac{H}{n}]}$

$$=e^{-x}\sum_{i=1}^{n} e^{iw} = e^{-x}\frac{e^{w}(e^{nw}-1)}{(e^{w}-1)} \tag{A.2}$$

Lemma 3: Prove that $\sum_{i=1}^{n} e^{\alpha_1 t_{i-1} + \alpha(1-\gamma)t_{i-1}} = \frac{(e^{nw}-1)}{(e^{w}-1)}$

Where $\frac{H}{n} = t_{i-1} - t_i$,$w = [\alpha_1 + \alpha(1-\gamma)]\frac{H}{n}$

Proof: $\sum_{i=1}^{n} e^{\alpha_1 t_{i-1} + \alpha(1-\gamma)t_{i-1}} = \sum_{i=1}^{n} e^{(i-1)[\alpha_1 + \alpha(1-\gamma)]\frac{H}{n}} = \sum_{i=1}^{n} e^{-[\alpha_1 + \alpha(1-\gamma)]\frac{H}{n}} e^{i[\alpha_1 + \alpha(1-\gamma)]\frac{H}{n}}$

$$=e^{-[\alpha_1 + \alpha(1-\gamma)]\frac{H}{n}} \sum_{i=1}^{n} e^{i[\alpha_1 + \alpha(1-\gamma)]\frac{H}{n}} = e^{-w}\frac{e^{w}(e^{nw}-1)}{(e^{w}-1)} = \frac{(e^{nw}-1)}{(e^{w}-1)} \tag{A.3}$$

Lemma 4: Prove that $\sum_{j=1}^{n} j e^{jx} = \frac{ne^{(n+2)x} - (n+1)e^{(n+1)x} + e^{x}}{(e^{x}-1)^2}$

$$\text{Proof: We have } \sum_{j=1}^{n} e^{jx} = \frac{e^{x}(e^{nx}-1)}{(e^{x}-1)} \tag{A.4}$$

Differentiating both sides of (A.4) with respect to x

$$\sum_{j=1}^{n} je^{jx} = \frac{(e^x-1)\{e^x(e^{nx}-1)+e^x ne^{nx}\}-e^x(e^{nx}-1)e^x}{(e^x-1)^2} = \frac{ne^{(n+2)x}-(n+1)e^{(n+1)x}+e^x}{(e^x-1)^2} \quad \text{(A.5)}$$

Lemma 5: Prove that $\sum_{j=1}^{n-1}(n-j)e^{\{\alpha_1+(1-\gamma)\alpha\}t_{j-1}} = [n\frac{(e^{(n-1)w}-1)}{(e^w-1)} - \frac{((n-1)e^{nw}-ne^{(n-1)w}+1)}{(e^w-1)^2}]$

Proof: $\sum_{j=1}^{n-1}(n-j)e^{\{\alpha_1+(1-\gamma)\alpha\}t_{j-1}} = \sum_{j=1}^{n-1}(n-j)e^{\{\alpha_1+(1-\gamma)\alpha\}(j-1)\frac{H}{n}}$

$=e^{-\{\alpha_1+(1-\gamma)\alpha\}\frac{H}{n}} \sum_{j=1}^{n-1}(n-j)e^{\{\alpha_1+(1-\gamma)\alpha\}j\frac{H}{n}}$

$=e^{-w}\sum_{j=1}^{n-1}(n-j)e^{jw}$,

$=e^{-w}[n\sum_{j=1}^{n-1}e^{jw} - \sum_{j=1}^{n-1}je^{jw}]$

$=e^{-w}[n\frac{e^w(e^{(n-1)w}-1)}{(e^w-1)} - \frac{((n-1)e^{(n+1)w}-ne^{nw}+e^w)}{(e^w-1)^2}]$

$=[n\frac{(e^{(n-1)w}-1)}{(e^x-1)} - \frac{((n-1)e^{nw}-ne^{(n-1)w}+1)}{(e^w-1)^2}]$ (Using Lemma 4) (A.6)

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