# First Forbidden Beta Transitions of $1 / 2^{+} \leftrightarrow 1 / 2^{-2}$ States for $\Delta \mathbf{J}=\mathbf{0}$ In Spherical Odd Mass Nuclei 

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#### Abstract

In this study, the first forbidden beta transitions of $\frac{1}{2}^{+} \leftrightarrow \frac{1}{2}^{-}$states for $\Delta \mathrm{J}=0$ have been investigated in spherical odd mass nuclei. The pn-QRPA model is used with a schematic separable interaction to calculate first forbidden transitions by considering the Woods-Saxon potential basis in the Chepurnov parameterization. The transition probabilities in this model have been calculated within the $\xi$-approximation. $\xi$ is a dimensionless parameter representing the magnitude of the Coulomb energy and is approximated by $1.2 \mathrm{ZA}^{-1 / 3}[1]$. Logft values calculated of first forbidden transitions are found to be in better agreement with measured data.


Keywords: First Forbidden, Beta Transitions

## 1. INTRODUCTION

It is well known that beta decay processes are very important to understand the weak interaction processes and the nuclear structure. Although there are many theoretical and experimental studies about the allowed transitions in literature scientists have not shown the same interest in forbidden transitions. The studies performed recently show that the first-forbidden beta transition process provides useful information in checking the validity of theories related to the r-processes and $2 \nu \beta \beta$ [2-10]. The ground state transitions in allowed Gamow-Teller and FF $\beta$-decays are studied by Suhonen [11]. Various models are used for the calculations of beta decay half-lives. $|\Delta \mathrm{J}|=0,2$ excited states are calculated dependence of spin-isospin on even-even, odd-odd and FF $\beta$-decay transitions for [12]. The $0+\leftrightarrow 0$ - transitions are studied QRPA model for spherical nuclei in the mass range $90-214$. [13] and the results were in better agreement with the experimental results and previous calculations. In the studies of $\beta$-decay the proton-neutron QRPA theory has been widely used and in pn-QRPA a quasiparticle basis using pairing interaction is constructed and schematic first forbidden (FF) residual interaction is solved using RPA equation [14-19]. Sorensen and Halbleib [20] modifying the RPA model for the calculations of relevant transitions developed this model. The RPA, QRPA and pn-QRPA models were extended for spherical nuclei [21-29] and deformed nuclei as well by many authors [30-34]. In total decay rates for the calculation of half-lives, the FF transitions plays an important role.

The present work aims to study the first-forbidden beta transitions of spherical odd-mass nuclei in $1 / 2^{+} \leftrightarrow 1 / 2^{-}$ states for $\Delta \mathrm{J}=0$. The rank 0 FF transitions are studied by using pn-QRPA in the Woods-Saxon potential with the particle-hole term of the effective interactions of the $\beta$-decay.

The first forbidden beta transitions of $\frac{1}{2}^{+} \leftrightarrow \frac{1}{2}^{-}$states for $\Delta \mathrm{J}=0$ have been investigated in spherical odd mass nuclei. The Woods-Saxon potential with Chepurnov parametrization has been used as a mean field basis in numerical calculations. The eigenvalues and eigenfunctions of the Hamiltonian with separable residual GT effective interactions in particle-hole (ph) channel were solved within the framework of pn-QRPA model.

The model Hamiltonian which generates the spin-isospin dependent vibrations modes with $\lambda^{\pi}=0^{-}, 1^{-}$ on odd-odd nuclei in quasi boson approximation is given as

$$
\begin{equation*}
\widehat{H}=\widehat{H}_{s q p}+\hat{h}_{p h} \tag{1}
\end{equation*}
$$

The single quasi-particle Hamiltonian of the system is given by

$$
\begin{equation*}
\widehat{H}_{s q p}=\sum_{j_{r}} \varepsilon_{j_{r}} \alpha_{j_{r} m_{r}}^{\dagger} \alpha_{j_{r} m_{r}(\tau=p, n)} \tag{2}
\end{equation*}
$$

where $\varepsilon_{\mathrm{j}_{\mathrm{r}}}$ and $\alpha_{j_{r} m_{r}}^{\dagger}\left(\alpha_{j_{r} m_{r}}\right)$ are the single quasi-particle energy of the nucleons with angular momentum $j_{r}$ and the quasi-particle creation (annihilation) operators, respectively.
The $\hat{h}_{p h}$ is the spin-isospin effective interaction Hamiltonian which generates $0^{-}, 1^{-}$vibration modes in particle-hole channel and given as

$$
\begin{equation*}
\hat{h}_{p h}=2 \chi_{p h} \sum_{\mu} T_{\lambda \mu}^{+} T_{\lambda \mu}^{-}, \quad T_{\lambda \mu}^{-}=\left(T_{\lambda \mu}^{+}\right)^{+} \tag{3}
\end{equation*}
$$

where $T_{\lambda \mu}^{ \pm}$is the first forbidden beta decay operator and $\chi_{p h}$ is particle-hole effective interaction constant.

$$
\mathrm{T}_{\lambda \mu}^{ \pm}=\left\{\begin{array}{cc}
\mathrm{g}_{\mathrm{V}} \sum_{\mathrm{k}} \mathrm{t}_{ \pm}(\mathrm{k}) \mathrm{r}_{\mathrm{k}} \mathrm{Y}_{1 \mu}\left(\theta_{\mathrm{k}}, \varphi_{\mathrm{k}}\right) & \text { for dipole interactions }  \tag{4}\\
\mathrm{g}_{\mathrm{A}} \sum_{\mathrm{k}} \mathrm{t}_{ \pm}(\mathrm{k}) \mathrm{r}_{\mathrm{k}}\left[\mathrm{Y}_{1 \mu}\left(\theta_{\mathrm{k}}, \varphi_{\mathrm{k}}\right), \sigma_{1}(\mathrm{k})\right]_{\lambda} & \text { for spin - dipole interactions }
\end{array}\right.
$$

The beta decay operator in quasiparticle space is written as follows

$$
\begin{align*}
& T_{\lambda \mu}^{+}=\sum_{p, n}\left[\bar{b}_{p n} C_{n p}^{+}(\lambda, \mu)+(-1)^{\lambda+\mu+1} b_{p n} C_{n p}(\lambda,-\mu)\right]+ \\
& \sum_{p, n}\left[d_{p n} D_{n p}^{+}(\lambda, \mu)+(-1)^{\lambda+\mu} \bar{d}_{p n} D_{n p}(\lambda,-\mu)\right] . \tag{5}
\end{align*}
$$

The $C_{n p}^{+}(\lambda, \mu)$ and $D_{n p}^{+}(\lambda, \mu)$ are the quasi boson operators and given as

$$
\begin{align*}
& C_{n p}^{+}(\lambda, \mu)=\sqrt{\frac{2 \lambda+1}{2 j_{n}+1}} \sum_{m_{n}, m_{p}}(-1)^{j_{p}-m_{p}}\left(j_{p} m_{p} \lambda \mu / j_{n} m_{n}\right) \alpha_{j_{n} m_{n}}^{+} \alpha_{j_{p}-m_{p}}^{+}  \tag{6}\\
& D_{n p}^{+}(\lambda, \mu)=\sqrt{\frac{2 \lambda+1}{2 j_{n}+1}} \sum_{m_{n}, m_{p}}\left(j_{p} m_{p} \lambda \mu / j_{n} m_{n}\right) \alpha_{j_{n} m_{n}}^{+} \alpha_{j_{p} m_{p}} \tag{7}
\end{align*}
$$

### 2.1 The pn-QRPA Equation In Even Mass Nuclei

As the Hamilton operator of even-even mass nuclei is known, it contains the multiplicative parts of the effective interaction $C_{n p}(\lambda \mu)$ and $C_{n p}^{+}(\lambda, \mu)$ operators. Other parts become zero at the end of the commutation operations. Thus, the Hamilton operator of even-even mass nuclei is written as

$$
\begin{equation*}
H_{0}=H_{S Q P}+h_{c c} \tag{8}
\end{equation*}
$$

The charge-exchange interaction forms the excited $\lambda_{i}^{-}$phonon states in neighboring nuclei. The phonon operator in pn-QRPA method given as

$$
\begin{equation*}
Q_{i}^{+}(\lambda, \mu)=\sum_{p, n}\left[\mathrm{r}_{n p}^{i} C_{n p}^{+}(\lambda, \mu)-(-1)^{\lambda+\mu+1} \mathrm{~s}_{n p}^{i} C_{n p}(\lambda,-\mu)\right] \tag{9}
\end{equation*}
$$

The excited states $\omega_{i}$ energies and the amplitudes of the $\mathrm{r}_{n p}^{i}, \mathrm{~s}_{n p}^{i}$ wave functions are found by solving the following equation of motion, taking into account the normalization condition:

$$
\begin{equation*}
\left[H_{0}, Q_{i}^{+}(\lambda, \mu)\right]|0\rangle=\omega_{i} Q_{i}^{+}(\lambda, \mu)|0\rangle \tag{10}
\end{equation*}
$$

The beta matrix elements of even-even mass nuclei from ground state to neighbour nuclei $\lambda_{i}^{-}$excited states are given as follows

$$
M_{i}^{ \pm}\left(0^{+} \rightarrow \lambda_{i}^{-}\right)=\langle 0|\left[Q_{i}(\lambda, \mu), \mathrm{T}_{\lambda \mu}^{ \pm}\right]|0\rangle\left\{\begin{array}{c}
\sum_{p n}\left(\bar{b}_{p n} \mathrm{r}_{n p}^{i}+b_{p n} \mathrm{~s}_{n p}^{i}\right)  \tag{11}\\
(-1)^{\lambda+\mu+1} \sum_{p n}\left(b_{p n} \mathrm{r}_{n p}^{i}+\bar{b}_{p n} \mathrm{~s}_{n p}^{i}\right)
\end{array}\right.
$$

### 2.2 The pn-QRPA Equation In Odd Mass Nuclei

The first term of $\mathrm{T}_{\lambda \mu}^{ \pm}$operator (belonging to C) are given as follows

$$
\begin{align*}
& T_{\lambda \mu}^{+}(C)=\sum_{i}\left[M_{i}^{+}\left(0^{+} \rightarrow \lambda_{i}^{-}\right) Q_{i}^{+}(\lambda, \mu)+M_{i}^{-}\left(0^{+} \rightarrow \lambda_{i}^{-}\right) Q_{i}(\lambda, \mu)\right],  \tag{12a}\\
& T_{\lambda \mu}^{-}(C)=\sum_{i}\left[M_{i}^{-}\left(0^{+} \rightarrow \lambda_{i}^{-}\right) Q_{i}^{+}(\lambda, \mu)+M_{i}^{+}\left(0^{+} \rightarrow \lambda_{i}^{-}\right) Q_{i}(\lambda, \mu)\right] \tag{12b}
\end{align*}
$$

Firstly, the nucleus consisting of odd-neutron, even-proton investigated. The wavefunction of this nuclei is composed of one-quasiparticle + one quasi particle phonon terms in the pn-QRPA method and are given as follows

$$
\begin{equation*}
\left|j_{k} m_{k}\right\rangle=\Omega_{j_{k} m_{k}}^{j+}=N_{j_{k}}^{j} \alpha_{j_{k} m_{k}}^{+}+\sum_{j_{v}^{\prime}, m_{v, i, \mu}^{\prime}} \mathrm{R}_{j i}^{k v^{\prime}}\left(j_{v}^{\prime} m_{v}^{\prime} \lambda \mu / j_{k} m_{k}\right) Q_{i}^{+}(\lambda, \mu) \alpha_{j_{v} m_{v}^{\prime}}^{+} \tag{13}
\end{equation*}
$$

The Hamilton operator of even mass nuclei and the effective interactions in the particle-hole channel are given

$$
\begin{align*}
& H=H_{0}+h_{C D},  \tag{14}\\
& h_{C D}=2 \chi_{p h} \sum_{\mu}\left[T_{\lambda \mu}^{+}(C) T_{\lambda \mu}^{-}(D)+T_{\lambda \mu}^{+}(D) T_{\lambda \mu}^{-}(C)\right] \tag{15}
\end{align*}
$$

The equation of motion the pn-QRPA may be written as follow

$$
\begin{equation*}
\left[\mathrm{H}, \Omega_{j_{k} m_{k}}^{j+}\right]=W_{j_{k}}^{j} \Omega_{j_{k} m_{k}}^{j+} \tag{16}
\end{equation*}
$$

The secular equation for the $\omega_{n}^{k}$ energies is found as follows

$$
\begin{equation*}
W_{j_{k}}^{j}-\varepsilon_{j_{k}}=\frac{2 \lambda+1}{2 j_{k}+1} \sum_{i, j_{v}} \frac{\left\{2 \chi_{p h}\left[M_{i}^{+}\left(0^{+} \rightarrow \lambda_{i}^{-}\right) d_{v k}+M_{i}^{-}\left(0^{+} \rightarrow \lambda_{i}^{-}\right) \bar{d}_{v k}\right]\right\}^{2}}{W_{j_{k}}^{j}-\omega_{i}-\varepsilon_{j_{v}}} \tag{17}
\end{equation*}
$$

also, quasiparticle + phonon amplitudes for each energy value are found by the following

$$
\begin{equation*}
\mathrm{R}_{j i}^{k \nu}=2 \chi_{p h} \frac{2 \lambda+1}{2 j_{k}+1} \frac{M_{i}^{+}\left(0^{+} \rightarrow \lambda_{i}^{-}\right) d_{v k}+M_{i}^{-}\left(0^{+} \rightarrow \lambda_{i}^{-}\right) \bar{d}_{v k}}{w_{j_{k}}^{j}-\omega_{i}-\varepsilon_{j v}} N_{j_{k}}^{j} \tag{18}
\end{equation*}
$$

And

$$
\begin{equation*}
\left(N_{j_{k}}^{j}\right)^{2}+\sum_{i, j_{v}}\left(\mathrm{R}_{j i}^{k v}\right)^{2}=1 \tag{19}
\end{equation*}
$$

### 2.3 One Proton State

One particle and one hole nuclei allow of the simplest possible theoretical description of their states. The structure of one-particle nuclei within the simple mean field approximation is the following. One-proton states $|\nu\rangle$ and one-neutron states $|k\rangle$ are described as

$$
\begin{equation*}
\left.\left.|\nu\rangle=\mathrm{C}_{v}^{\dagger} \mid \text { core }\right\rangle, \quad|k\rangle=\mathrm{C}_{\mathrm{k}}^{\dagger} \mid \text { core }\right\rangle \tag{20}
\end{equation*}
$$

where $\mid$ core $\rangle$ is the core with its Fermi level at some magic number. In one proton state are done the $\mathrm{k} \leftrightarrow \nu$ transformation for all of above equations.

The negative beta decay transition matrix element $\left|\mathrm{j}_{\mathrm{k}} \mathrm{m}_{\mathrm{k}}\right\rangle^{\mathrm{n}} \xrightarrow{\beta^{-}}\left|\mathrm{j}_{v} \mathrm{~m}_{v}\right\rangle^{\mathrm{n}}$ (state non-changing the number of pair) from a neutron state $j_{k} m_{k}$ to proton state $j_{v} m_{v}$ are calculated as follow

$$
\begin{equation*}
\left\langle\left\langle j_{v} m_{v}\right|\left[\Omega_{j_{v} m_{v}}^{j}, T_{\lambda \mu}^{-}\right] \mid j_{k} m_{k}\right\rangle=\frac{\left\langle j_{v}\left\|T_{\lambda}^{-}\right\| j_{k}\right\rangle}{\sqrt{2 j_{v}+1}}\left\langle j_{k} m_{k} \lambda \mu / j_{v} m_{v}\right\rangle \tag{21}
\end{equation*}
$$

and the reduced matrix element is written

$$
\begin{equation*}
<j_{v}\left\|T_{\lambda}^{-}\right\| j_{k}>=N N+N R+R N+R R . \tag{22}
\end{equation*}
$$

The reduced matrix element is composed of four terms and every term was separately considered. These terms are given as

$$
\begin{align*}
& \mathrm{RR}=\mathrm{N}_{\mathrm{j}_{\mathrm{k}}}^{\mathrm{j}} \mathrm{~N}_{\mathrm{j}_{v}}^{\mathrm{j}} \sqrt{\frac{(2 \lambda+1)\left(2 \mathrm{j}_{v}+1\right)}{2 j_{k}+1}} \mathrm{~d}_{v \mathrm{k}}  \tag{23a}\\
& N R=N_{j_{v}}^{j} \sqrt{2 j_{v}+1} \sum_{i, v} \mathrm{R}_{j i}^{k v} M_{i}^{+}\left(0^{+} \rightarrow \lambda_{i}^{-}\right)  \tag{23b}\\
& R N=(-1)^{j_{v}-j_{k}+\lambda+1} N_{j_{k}}^{j} \frac{2 j_{v}+1}{\sqrt{2 j_{k}+1}} \sum_{i, k} \mathrm{R}_{j i}^{v k}(-1)^{\lambda+\mu+1} M_{i}^{-}\left(0^{+} \rightarrow \lambda_{i}^{-}\right)  \tag{23c}\\
& R R=N_{j_{v}}^{j} \sqrt{2 j_{v}+1} \sum_{i, v^{\prime}, k^{\prime}}(-1)^{j_{v}+j_{k}+\lambda} \sqrt{(2 \lambda+1)\left(2 j_{k}^{\prime}+1\right)\left(2 j_{v}+1\right)} \mathrm{R}_{j i}^{k \prime^{\prime} \prime^{\prime}} \mathrm{R}_{j i}^{v k^{\prime}} \bar{d}_{v^{\prime} k^{\prime}}\left(\begin{array}{ccc}
j_{k}^{\prime} k_{k} \lambda j_{v} \\
j_{k} & \lambda & \lambda j_{v}^{\prime}
\end{array}\right) \tag{23d}
\end{align*}
$$

The beta decay transition matrix element $\left|\mathrm{j}_{\mathrm{k}} \mathrm{m}_{\mathrm{k}}\right\rangle^{\mathrm{n}} \xrightarrow{\beta^{+}}\left|\mathrm{j}_{v} \mathrm{~m}_{v}\right\rangle^{\mathrm{n}}$ (state changing the number of pair) for an odd neutron state $\mathrm{j}_{\mathrm{k}} \mathrm{m}_{\mathrm{k}}$ changing the number of pair are done the $\mathrm{d}_{\mathrm{vk}} \leftrightarrow \overline{\mathrm{d}}_{\mathrm{vk}}$ and $\mathrm{M}_{\mathrm{i}}^{+}\left(0^{+} \rightarrow \lambda_{\mathrm{i}}^{-}\right) \leftrightarrow$ $\mathrm{M}_{\mathrm{i}}^{-}\left(0^{+} \rightarrow \lambda_{\mathrm{i}}^{-}\right)$transformations in the above equations.

### 2.4 Investigation Of The Relativistic Moment Matrix Elements

The relativistic beta moment matrix elements has an important role when the non relativistic beta moment matrix elements is small, depending on microscopic structure of the states. The relativistic beta moment matrix element for the $0^{-} \leftrightarrow 0^{+}$and $1^{-} \leftrightarrow 0^{+}$transitions have been investigated. These matrix elements are given as follows [1]

$$
\begin{align*}
& M\left(\rho_{A}, \lambda=0\right)=\frac{1}{\sqrt{4 \pi}} \frac{g_{A}}{c} \sum_{k} t_{-}(\vec{\sigma}(k) \cdot \vec{v}(k)),  \tag{24a}\\
& M\left(\rho_{A}, \lambda=0\right)=\frac{1}{\sqrt{4 \pi}} \frac{g_{A}}{m c} \sum_{k} t_{-}(\vec{\sigma}(k) \cdot \vec{p}(k))  \tag{24b}\\
& M^{\mp}\left(j_{v}, \kappa=0, \lambda=1, \mu\right)=\frac{g_{A}}{\sqrt{4 \pi c}} \sum_{k} t_{\mp}(k) r_{k}\left(\vartheta_{k}\right)_{1 \mu},  \tag{25a}\\
& M^{\mp}\left(\rho_{v}, \lambda=1, \mu\right)=g_{A} \sum_{k} t_{\mp}(k) r_{k} Y_{1 \mu}\left(r_{k}\right), \tag{25b}
\end{align*}
$$

$$
\begin{equation*}
M^{\mp}\left(j_{v}, \kappa=1, \lambda=1, \mu\right)=g_{A} \sum_{k} t_{\mp}(k) r_{k}\left\{Y_{1}\left(r_{k}\right) \sigma_{k}\right\}_{1 \mu}, \tag{25c}
\end{equation*}
$$

The reduced matrix elements are needed to calculate the beta transition matrix element $\left.\left(<j_{2}\|\vec{p}\| j_{1}\right\rangle\right)$. But, Pyatov et al. used a different approach. Using the following commutation condition, the momentum reduced matrix elements can be expressed in terms of the reduced matrix elements of the dipole moment [35].

This commutation is written

$$
\begin{equation*}
[H, \vec{R}]=-\frac{i \hbar}{m A} \vec{P} \tag{26}
\end{equation*}
$$

also, the matrix form of this commutation is given as

$$
\begin{equation*}
\left(j_{2}\|\vec{p}\| j_{1}\right)=i \frac{m}{\hbar} \sqrt{\frac{4 \pi}{3}}\left(\varepsilon_{j_{2}}-\varepsilon_{j_{1}}\right)\left(j_{2}\left\|r Y_{1}\right\| j_{1}\right) \tag{27}
\end{equation*}
$$

and this equation can be written with pairing interaction terms

$$
\begin{equation*}
\left(u_{j_{2}} v_{j_{1}}-u_{j_{1}} v_{j_{2}}\right)\left(j_{2}\|\vec{p}\| j_{1}\right)=i \frac{m}{\hbar} \sqrt{\frac{4 \pi}{3}}\left(E_{j_{1}}+E_{j_{2}}\right)\left(u_{j_{2}} v_{j_{1}}+u_{j_{1}} v_{j_{2}}\right)\left(j_{2}\left\|r Y_{1}\right\| j_{1}\right) \tag{28}
\end{equation*}
$$

where $\mathrm{A}, \mathrm{n}, \varepsilon_{j}, \mathrm{E}_{\mathrm{j}}, u$ and $v$ are the atomic mass of nucleus, the mass of nucleon, the single particle energy, the one quasi particle $u$ and $v$ energies, the Bogoliubov coefficient, respectively.

The transition probabilities $B\left(\lambda^{\pi}=0^{-}, 1^{-} ; \beta^{\mp}\right)$ are calculated by

$$
\begin{align*}
& B\left(\lambda^{\pi}=0^{-}, \beta^{\mp}\right)=\left|\left\langle 0_{i}^{-}\left\|M_{\beta^{\mp}}^{0}\right\| 0^{+}\right\rangle\right|^{2}, \\
& M_{\beta^{\mp}}^{0}=\mp M^{\mp}\left(\rho_{A}, \lambda=0\right)-i \frac{m_{e} c}{\hbar} \xi M^{\mp}\left(\rho_{A}, \kappa=1, \lambda=0\right)  \tag{29}\\
& B\left(\lambda^{\pi}=1^{-}, \beta^{\mp}\right)=\left|\left\langle 1_{i}^{-}\left\|M_{\beta^{\mp}}^{1}\right\| 0^{+}\right\rangle\right|^{2} \\
& M_{\beta^{\mp}}^{1}=M^{\mp}\left(j_{v}, \kappa=0, \lambda=1, \mu\right) \pm i \frac{m_{e} c}{\sqrt{3} \hbar} M^{\mp}\left(\rho_{A}, \lambda=1, \mu\right)+ \\
& i \sqrt{\frac{2}{3}} \frac{m_{e} c}{\hbar} \xi M^{\mp}\left(j_{A}, \kappa=1, \lambda=1, \mu\right) \tag{30}
\end{align*}
$$

In eqs. (29) and (30), the upper and lower signs refer to $\beta^{-}$and $\beta^{+}$decays, respectively

The $f t$ values are given by the equation

$$
\begin{equation*}
(f \mathrm{t})_{\beta^{\mp}}=\frac{D}{\left(\frac{g_{A}}{g_{v}}\right)^{2} 4 \pi B\left(I_{i} \rightarrow I_{f}, \beta^{\mp}\right)} \tag{31}
\end{equation*}
$$

where

$$
D=\frac{2 \pi^{3} \hbar^{2} \ln ^{2}}{g_{v}^{2} m_{e}^{5} c^{4}}=6250 \text { sec. } \quad \frac{g_{A}}{g_{v}}=-1.254
$$

## 3. RESULTS AND CONCLUSIONS

The corresponding life-times for the first-forbidden beta decay of some odd-mass spherical nuclei are obtained within the framework of pn-QRPA (WS) with a separable residual schematic particle-hole interaction. The logft values of beta decay transitions have been calculated for the states of $\chi_{p p}=0$. The pairing correlation constants were taken a $C_{p}=C_{n}=12 / \sqrt{A}$. The strength parameters of the effective interaction are $\chi_{\text {rank } 0}=30 A^{-5 / 3} \mathrm{MeVfm}^{-2}$ and $\chi_{\text {rank } 1}=55 A^{-5 / 3} \mathrm{MeVfm}^{-2}$ for rank0 and rank1, respectively. The relativistic beta decay matrix elements have been calculated without any assumption. The calculated results are compared with the other calculations and available experimental data. The first forbidden energy values ( $\omega_{\mathrm{i}}$ ) in intermediate nuclei for dipole and spin-dipole transitions and the FF $\beta$-decay calculated $\operatorname{logft}$ values are given in table 1 . As seen from the table, the calculated theoretical logft values are closer to the experimental data.

Table 1. First Forbidden Beta Transitions of $1 / 2^{+} \rightarrow 1 / 2^{-}$states for $\Delta \mathrm{J}=0$ in some odd mass nuclei. The experimental values are taken from B. Singh et al. [36]. The E, $\theta$ and $\omega_{\mathrm{i}}$ are used in the MeV unit.

| Transitions | Parent |  | Daughter |  | $\begin{gathered} \text { States } \\ n \rightarrow p \end{gathered}$ | $\theta$ exp. | logft exp. | 0- (SD) |  | $1{ }^{-}$(D) |  | 1 ( SD ) |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | E | $J^{\pi}$ | E | $J^{\pi}$ |  |  |  | $\omega \mathrm{i}$ | logft | $\omega \mathrm{i}$ | logft | $\omega^{\text {i }}$ | logft |
| ${ }_{38}^{95} \mathrm{~S}_{57} \rightarrow{ }_{39}^{95} Y_{56}$ | 0 | $1 / 2^{+}$ | 0 | 1/2- | $3001 \rightarrow 2101$ | 6.08 | 6.16 | 5.93 | 6.01 | 6.12 | 6.55 | 6.01 | 6.44 |
| ${ }_{47}^{111} A g_{64} \rightarrow{ }_{48}^{111} C d_{63}$ | 0 | 1/2- | 0 | 112+ | $2101 \rightarrow 3001$ | 1.03 | 7.3 | 1.07 | 6.22 | 1.07 | 6.40 | 1.06 | 5.92 |
|  |  |  |  |  |  |  |  |  |  | 4.53 | 7.45 | 4.53 | 6.92 |
| ${ }_{49}^{115} n_{66} \rightarrow{ }_{50}^{115} 5 n_{65}$ | 0.32 | $1 / 2-$ | 0 | 1/2+ | $2101 \rightarrow 3001$ | 0.49 | 6.7 | 1.27 | 6.01 | 1.25 | 6.17 | 1.59 | 5.69 |
| ${ }_{49}^{117} n_{68} \rightarrow{ }_{50}^{117}$ S $n_{67}$ | 0.31 | $1 / 2-$ | 0 | 1/2+ | $2101 \rightarrow 3001$ | $1.45$ | 6.71 | 1.13 | 5.80 | 1.13 | 5.97 | 1.11 | 5.49 |
|  |  |  |  |  |  |  |  |  |  |  |  | 3.47 | 6.64 |
| ${ }_{49}^{121} / n_{72} \rightarrow{ }_{50}^{121} S n_{71}$ | 0.31 | $1 / 2-$ | 0 | $1 / 2+$ | $2101 \rightarrow 3001$ | 3.36 | 6.22 | 5.77 | 5.29 | 3.36 | 5.96 | 3.31 | 6.82 |

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