# Investigation of binding energies of hypernuclei 

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#### Abstract

In high energy nuclear reactions one may observe the production of manifold hypernuclei as a result of the capture of hyperons by nuclear residues. We have investigated the statistical disintegration of such hyper nuclear systems and its connection to the binding energies of hyperons. In the calculations we use the statistical multifragmentation model (SMM) adopted for the formation of hypernuclei in the high energy nuclear multifragmentation reaction events. It was shown within this model that the mass distributions are quite different for the fragments with different strangeness contents.


Key Words: Hypernuclei, Binding Energy, Multifragmentation

## 1. INTRODUCTION

Relativistic heavy-ion collisions offer the possibility of creating excited nuclear systems including hyperons. Some hyperons can be absorbed in the spectator part of the colliding nuclei. Then singleand multi-lambda hypernuclei can be produced after multifragmentation of this spectator. The SMM for hypernuclei, which was first proposed by Botvina and Pochodzalla [1,2] for the formation of hypernuclei, has been used to determine a possible relative yield of hypernuclei in nuclear fragmentation reactions.

The model assumes that a hot nuclear spectator with total mass (baryon) number $A_{0}$, charge $Z_{0}$, number of $\Lambda$ hyperons $H_{0}$, and temperature $T$ expands to a low-density freeze-out volume, where the system is in chemical equilibrium. According to the model, the statistical ensemble includes all breakup channels composed of nucleons and excited fragments with mass number $A$, charge number $Z$, and number of $\Lambda$ 's $H$. The primary fragments are assumed to be formed in the freeze-out volume $V_{f}$. We use the excluded volume approximation $V=V_{0}+V_{\mathrm{f}}$, where $V_{0}=A_{0} / \rho_{0}\left(\rho_{0} \approx 0.15 \mathrm{fm}^{-3}\right.$ is the normal nuclear density), and parametrize the free volume $V_{f}=\kappa V_{0}$, with $\kappa \approx 2$. It is assumed that a statistical equilibrium is reached at low density freeze-out region. It is also assumed that all breakup channels are composed of nucleons, and the laws of conservation of energy, momentum, angular momentum, mass number A and charge number Z are considered. Besides the breakup channels, the compound-nucleus channels at low excitations are also included. Therefore, the SMM covers the conventional evaporation and fission processes occurring at low excitation energy as well as the transition region between the low and high energy de-excitation regimes. In the thermodynamic limit, SMM is consistent with liquid-gas phase transition when the liquid phase is represented by infinite nuclear cluster.

Nuclear clusters in the freeze-out volume are described as follows. Light fragments with mass number $A<4$ are treated as elementary particles with corresponding spin and translational degrees of freedom ("nuclear gas"). Their binding energies were taken from experimental data [3-5]. Fragments with $A=4$ are also treated as gas particles with table masses. Fragments with $A>4$ are treated as heated liquid
drops (hot fragments). In this way one can study the nuclear liquid-gas coexistence of hypermatter in the freeze-out volume. The internal free energies of these fragments are parametrized as the sum of the bulk ( $F_{B}$ ), the surface ( $F_{S}$ ), the symmetry ( $F_{\text {sym }}$ ), the Coulomb ( $F_{C}$ ), and the hyper ( $F_{\text {hyp }}$ ) energies:

$$
\begin{equation*}
F_{A z H}(T, V)=F_{B}+F_{S}+F_{\text {sym }}+F_{C}+F_{\text {hyp }} \tag{1}
\end{equation*}
$$

The first three terms are written in the standard liquid-drop form [1,2]. The model parameters $w_{0}=16$ MeV , $\beta_{0}=18 \mathrm{MeV}, T c=18 \mathrm{MeV}$, and $\gamma=25 \mathrm{MeV}$ were extracted from nuclear semi-emprical mass formula, and provide a good description of multifragmentation data [6-9]. The Coulomb interaction of the fragments is described within the Wigner-Seitz approximation, and $F_{C}$ is taken as in Ref. [6]. The new term is the free hyper energy $F_{\text {hyp }}$. We assume that it does not change with temperature, i.e., it is determined solely by the binding energy of the hyper fragments. We have suggested the liquid-drop hyper energy term [ 1,2$]$ as follows,

$$
\begin{equation*}
F_{\text {hyp }}(A H)=(H / A)\left(-10.68 A+21.27 A^{2 / 3}\right) . \tag{2}
\end{equation*}
$$

In this formula the binding energy is proportional to the fraction of hyperons in the system $(H / A)$. The second part represents the volume contribution reduced by the surface term and thus resembles a liquid-drop parametrization based on the saturation of the nuclear interaction. It was demonstrated in Ref. [10] that we can directly connect the double yield ratio of isotopes with the hyperon binding energy of hypernuclei. Namely,

$$
\begin{align*}
\Delta \mathrm{E}_{\mathrm{bh}}= & \mathrm{T} \cdot\left[\operatorname { l n } \left(\left(\left(\mathrm{~g}_{\mathrm{A} 1, \mathrm{Z} 1, \mathrm{H}} / \mathrm{g}_{\mathrm{A} 1-1, \mathrm{Z1,H-1}}\right) \cdot\left(\mathrm{A}_{1}{ }^{3 / 2} /\left(\mathrm{A}_{1}-1\right)^{3 / 2}\right) /\left(\left(\mathrm{g}_{\mathrm{A} 2, Z 2, \mathrm{H}} / \mathrm{g}_{\mathrm{A} 2-1, \mathrm{Z2,H-1}}\right) \cdot\left(\mathrm{A}_{2}{ }^{3 / 2} /\left(\mathrm{A}_{2}-1\right)^{3 / 2}\right)\right)\right)\right.\right. \\
& \left.-\ln \left(\left(\mathrm{Y}_{\mathrm{A} 1, \mathrm{Z}, \mathrm{H}} / \mathrm{Y}_{\mathrm{A} 1-1, \mathrm{Z} 1, \mathrm{H}-1}\right) /\left(\mathrm{Y}_{\mathrm{A} 2,22, \mathrm{H}} / \mathrm{Y}_{\mathrm{A} 2-1, \mathrm{Z2}, \mathrm{H}-1}\right)\right)\right] . \tag{3}
\end{align*}
$$

In this short presentation, we do not aim to give the details of our derivations and results, instead we refer the reader to Refs. [1,2,10].

## 2. CALCULATIONS AND RESULTS

The multifragmentation of excited hypernuclear systems proceeds in a different way when compared to conventional nuclei. The reason is the additional binding energy of hyperons in nuclear matter. It was also shown that the yields of fragments with two Lambda's depend essentially on the binding energy formulae (i.e., on details of hyperon-nucleon and hyperon-hyperon interactions) used for the calculations [1,2]. Therefore, an analysis of double hypernuclei can help to improve these mass formulae and reveal information about the hyperon-hyperon interaction. In this work, we made our analysis to systems containing up to two hyperons, which may be produced during the dynamical stage of relativistic heavy-ion collisions.

In figure 1 , we show the results obtained in the present calculations within this model concerning the fragments produced after multifragmentation. In this case, the hyperons are kept mostly by heavy nuclei because of their large binding energies and the yield of lightest hypernuclei are essentially low. One can see from this figure a very broad mass distribution and isotope distributions for the produced fragments. This is similar to the results obtained for normal multifragmentation yields and we expect to obtain similar results in hypernuclear experiments too, since the hyperon-nucleon interaction is of the same order as the nucleon-nucleon one. It is important that the statistical description of these reactions are suitable not only for reasonable nuclear fragment production but also for the production of hypernuclei. In the light of this figure, one can understand the fragment and hyper-fragment yields behaviour which can be used to back-trace the information about unknown properties of nuclei.

In figure 2, we show how the secondary de-excitation can modify the results on the binding energy difference $\Delta \mathrm{E}_{\mathrm{bhA} 1 \mathrm{~A} 2}$ (in the figure noted again as $\Delta \mathrm{E}_{\mathrm{bh}}$ ) versus $\Delta \mathrm{A}$, corresponding to single hypernuclei. For clarity, we use the different mother nuclei in the calculations. The typical temperature $\mathrm{T}=4 \mathrm{MeV}$ is taken for initial fragment yields. We have found that the results obtained from double ratios of
primary yields of nuclei change slightly with variation of the temperature in the multifragmentation region.


Figure 1. Characteristics of all fragments (solid lines) and hyper-fragments (dashed and dashed-dotted lines, with $\Lambda$ hyperons) produced after disintegration of an excited hypernuclear system with initial mass (baryon) number A $0=200$, charge $Z 0=70$, temperature $T=4 \mathrm{MeV}$, and containing two $\Lambda$ hyperons ( $\mathrm{H} 0=2$ ). Top panel: Yields (per disintegration event) of fragments versus their mass number.

Bottom left panel: binding energies per nucleon and bottom right panel: isotope yields for Molibdenium (Mo, $\mathrm{Z}=42$ ) versus the neutron number.

For the realistic values of internal excitation energies of these fragments, $\mathrm{E}_{\mathrm{x}}=1.5$ and 2 MeV per nucleon, are assumed. In the beginning, we determined which daughter nuclei can be produced after the evaporation, by taking into account their maximum yield. As is known, the same daughter nuclei can be produced after evaporation of other nuclei close to A and Z . We also calculated the deexcitation of such nuclei, as their mass and charge numbers can reach few tens at the highest excitation energy. We take into account the weight of all primary nuclei after multifragmentation in the freezeout volume and evaluated their contribution in the final daughter yield. Afterwards, according to the formula (3) we found a new $\Delta$ Ebh value.


Figure 2. Influence of the secondary de-excitation on the difference of binding energies of hyperons in nuclei $\Delta \mathrm{E}_{\mathrm{bh}}$ as function of their mass number difference $\Delta \mathrm{A}$, by taking single hypernuclei. The calculations of double ratio yields for primary hot nuclei are shown for temperature 4 MeV (solid line, color circle symbols). The triangles and squares show the calculations with modified double ratios after the secondary deexcitation (via nuclear evaporation) of primary nuclei at excitation energies of 1.5 and $2.0 \mathrm{MeV} /$ nucleon, respectively. The same color symbols show the modification of $\Delta E b h$ and $\Delta \mathrm{A}$ after the de-excitation evolution of many nuclei leading to the same daughter ones.

## 3. CONCLUSIONS

As a result, it was demonstrated that the hyperon binding energies can be effectively evaluated from the different isotopic yields of hypernuclei. This kind of studies would be possibly verified at the new generation of ion accelerators of intermediate energies, at FAIR (Darmstadt), NICA (Dubna), and similar facilities [9,11]. The sophisticated new experimental installations for the fragment detection for hypernuclei would be available at these research centers, soon. The analyses and obtained information of this kind are expected to be useful in investigating the astrophysical processes such as type IIsupernova explosions and modelling of hyperon-rich matter in neutron star crusts, as well.

## 4. ACKNOWLEDGEMENTS

Many helpful discussions with A. S. Botvina are gratefully acknowledged. We also acknowledge the Turkish Scientific and Technological Research Council of Turkey (TUBITAK) support under Project No. 118 F 111 . This work is performed in the framework of COST Action CA15213 THOR.

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