

Stopping Power and Range Calculations of Electrons For Some Human Body Tissues

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Abstract

This study presents calculation results of the stopping power and range of electrons of kinetic energy from 20 eV to 10 MeV for some human body tissues. The method is based on utilization of the modified Bethe-Bloch stopping power expression and analytical expression for effective atomic electron number and effective mean excitation energies of target atoms, and for effective charge of incoming electrons. For this aim, Sugiyama's semi-empirical formula from Petersen and Green is embedded in the formula. An analytical expression for the practical stopping power calculations using Bethe approximation and Thomas-Fermi Model of atom is taken from the previous study. The calculated results of the stopping power and range for electrons in some materials, such as adipose tissue, bone and water are compared with a number of other calculation such as the Penelope 14 code and ESTAR results. The present electron stopping power and range calculation method should be useful in nuclear medicine, radiation treatment, and biomedical dosimetry.

Keywords: Effective Charge, Effective Mean Excitation Energy, CSDA Range, Lenz-Jensen screening function, Tietz Screening function.

1. INTRODUCTION

The electron stopping power above 10 keV energies is theoretically well described and can be found in tables given in Berger and Seltzer¹, Pages et al.² and ICRU 37 Report³. The semiempirical effective charge expression for incoming electrons was used to fit the Peterson and Green⁴ method of Sugiyama⁵⁻⁶.

In the previous study we used the Lenz⁷ Jensen⁸ (LJ) screening function and Tietz⁹ screening function for electron-positron stopping power and heavy ion stopping power and range calculations¹⁰⁻¹⁴. When calculating the stopping power (SP), any screening effects are considered by introducing an effective charge of both incident particle and target, and an effective mean ionization energy for the target. These values affect the stopping power, especially below 10 keV. For molecular targets, Bragg's rule¹⁵ is employed.

The purpose of this study is to present stopping power and range for electrons in some tissue materials and to compare with a number of other calculation such as ESTAR¹⁶ and the Penelope 2014 code results¹⁷, stopping power and the Continuous Slowing Down Approximation (CSDA) range are calculated by following the procedure described in Gümüş^{12,13}. Here, results of calculations are shown performed for incident projectile energies from 20 eV to 10 MeV. A brief explanation of the theory is given, and tables on the obtained values are provided and discussed.

2. STOPPING POWER EQUATIONS

The modified collision SP formula for incoming electrons can be written as ^{5,12,13,18}

$$S_{\xi} = -\frac{dE}{d\xi} = \frac{dE}{\rho dx} = \frac{4\pi e^4 z^{*2}}{mv^2} \frac{N_0}{A} Z_2^* \left\{ \ln\left(\frac{E}{I^*}\right) - F(\tau)/2 \right\} \quad (1)$$

where

$$F(\tau) = 1 - \beta^2 + [(\tau^2/8) - (2\tau + 1)\ln 2]/(1 + \tau)^2, \quad (2)$$

$$\frac{4\pi e^4 z^{*2}}{mv_1^2} \frac{N_0}{A} Z_2^* = (k/A\beta^2) z^{*2} Z_2^*, \quad (3)$$

m is the electron mass, $N = N_0/A$ is the density of target atoms, z^* is the effective charges of incident electrons, Z_2^* is the effective number of target electrons, A is the atomic weight of the target element, N_0 Avogadro's number, β is the ratio of v_1/c , where c is the light velocity, $k = 4\pi e^4 N_0/mc^2 = 0.307075$ MeVcm² and I^* is the effective mean excitation energy of the target atom.

For compound targets, Bragg's rule¹⁵ is used i.e., the stopping power of a compound is calculated by the linear combination of the stopping powers of its individual elements Eq. (1):

$$(S/\rho)_{comp.} = w_1 \left(\frac{S_1(E)}{\rho} \right) + w_2 \left(\frac{S_2(E)}{\rho} \right) + \dots \quad (4)$$

where w_1, w_2, \dots, w_{15} are the atomic rates of element in compound.

Effective charge, Z^* and effective mean excitation ionization energies (EMEE), I^* of the target atom can be obtained from Bohr's stripping criterion¹⁹⁻²⁰, and from the Lindhard and Schraff theory²¹, Effective Mean Excitation Energies (EMEE) is given by Sugiyama⁵⁻⁶.

$$Z^* = \int_{r_c}^{\infty} 4\pi r^2 n(r) dr \quad (5)$$

$$\ln I_2^* = \frac{1}{Z_2^*} \int_{r_c}^{\infty} \ln\{\gamma \hbar \omega_p(r)\} 4\pi r^2 n(r) dr \quad (6)$$

Where r is the distance from the nucleus and r_c is determined from adiabatic Bohr criterion¹⁹⁻²⁰. I^* and Z^* can be obtained analytically¹² from these expressions by using Lenz⁷ and Jensen⁸ or Tietz⁹ screening function of the Thomas Fermi (TF) atom. Lenz-Jensen Screening Function^{7,8} is given by

$$\Phi(x) = (1 + y + 0.3344y^2 + 0.0485y^3 + 0.002647y^4) e^{-y} \quad (7)$$

Here is $y = \sqrt{9.67x}$.

According to Tietz⁹, the screening function for the TF atom can be written as $x_c = r_c/\lambda$ and they obtained,

$$\Phi(x) = \frac{b^2}{(x+b)^2} \quad (8)$$

where b is chosen as $b=(8/\pi)^{2/3}$ to normalize the electronic density²².

In this study, the effective charge, Z^* and mean excitation energy of the target atom, I^* was calculated directly following the procedures described by Gümüş¹⁰⁻¹³ for LJ and for Tietz screening function. The effective charge of incident electrons is a phenomenological parameter, which was determined by Sugiyama⁵ by fitting the semiempirical formula of Peterson and Green⁴.

The semiempirical effective charge of incident electrons z^*e is used with z^* being given by

$$z^* = 1 - \exp(-2200\beta^{1.78}) \quad (9)$$

where $\beta=v_1/c$ is the ratio of v_1/c , with c the velocity of light⁶.

In order to calculate the electronic SP of the compound the effective charge of the incident electron, z^* was obtained primarily using Eq. (9) and then the effective electron number of the target atoms, Z^* and in the compound were obtained using Eq. (4).

3. RANGE CALCULATIONS

An electron follows a tortuous path undergoing many interactions before coming to a stop. The furthest distance radiation travels in a medium is called the range. Since we know the energy loss or stopping power we can calculate the range (pathlength) a charged particle travels before stopping. This is called the CSDA (Continuously Slowing Down Approximation) range

$$R(E_0) = \int_{E_T}^{E_0} \frac{dE}{(S/\rho)} \quad (10)$$

Here (S/ρ) is collision stopping power. If you SP choose in [MeV.cm²/g] or [keV.cm²/mg] units and energy in keV you find, the range in mg/cm² units. Numeric integration can be performed by using Simpson's or Trapezoidal formulae.

4. RESULTS AND DISCUSSION

The collision stopping power for incident electrons was obtained by considering the effective charge and effective mean ionization energy of the target, by using Lenz-Jensen screening function¹² and by using Tietz screening function¹³ and the effective charge of the electron⁶.

The chemical contents of the tissues used in this study are given in the table Penelope pdcompos.pen (Penelope, 2014¹⁶), This file contains composition data and physical parameters for 280 materials, taken from the database of the ESTAR¹⁶ program of Berger and Seltzer²³. Following the procedures described by Gümüş¹⁰⁻¹³, for the quantities in Eq. (1), the collision stopping power was calculated.

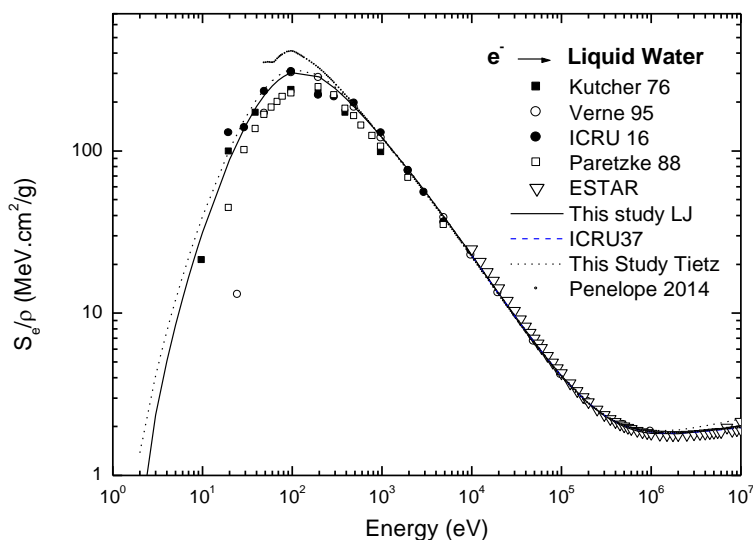


FIG. 1. Comparison of mass stopping power results for incident electron energies in liquid water. Solid curves, — are the present calculations by using Lenz-Jensen screening function, dashed line; - -, are the present calculations by using Tietz screening function; ■ semiempirical formula by Kutcher and Green²⁴; ○ the results of ICRU and Report No. 16²⁶; □, from Paretzke²⁷; ▽, values obtained from ESTAR package¹⁶; - -, data from ICRU 37 Report³; ○, results calculated by Penelope 2014 computer code¹⁷.

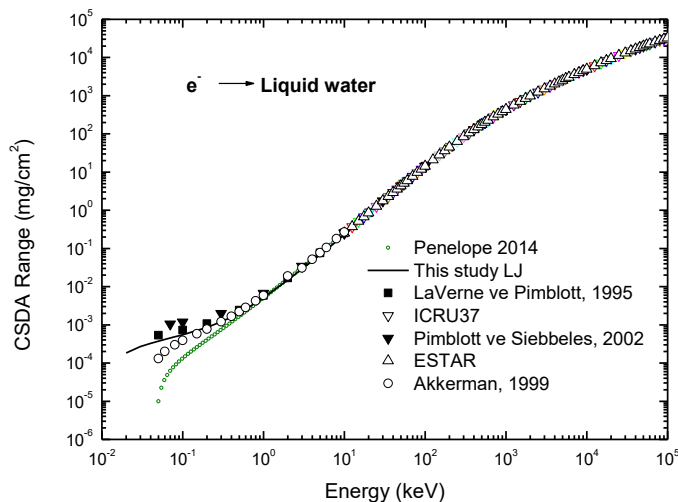


FIG. 2. Comparison of CSDA range results for incident electron energies in liquid water. Solid curves, — are the present calculations by using Lenz-Jensen screening function; ○, results calculated by Penelope 2014 computer code¹⁷; ■ taken from La Verne and Pimblott²⁵; ▽, are taken from ICRU 37 Report³; ▼ are taken from Pimblott and Siebbeles²⁸; Δ, are taken from ESTAR, and ○, the results Akkerman and Akkerman²⁹.

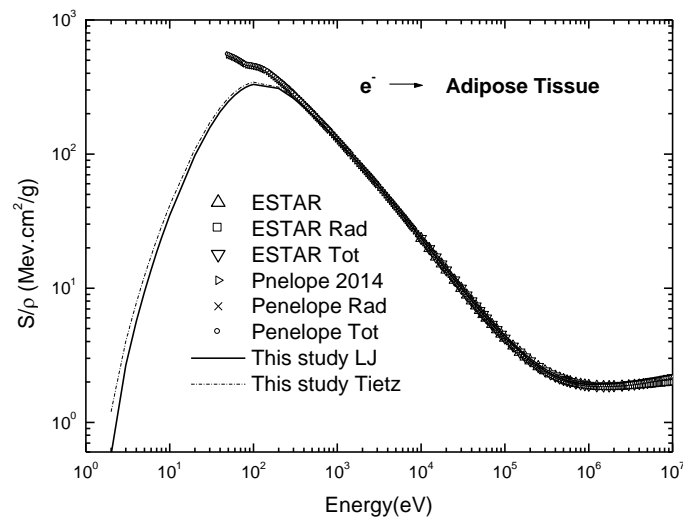


FIG. 3. Comparison of stopping power values (Mev.cm²/g) of adipose tissue for incoming electrons.

TABLE I. Comparison of stopping power values (Mev.cm²/g) of adipose tissue for incoming electrons

Energy (eV)	This study LJ fonk.	This study Tietz fonk.	ESTAR Col.	Pen 014 Col.	Energy (eV)	This study LJ fonk.	This study Tietz fonk.	ESTAR Col.	Pen 014 Col.
2	0.5768	1.19932			2000	79.14873	79.50427		79.0382
3	2.66057	4.02097			3000	58.93789	59.13892		59.0289
4	5.66142	7.80524			4000	47.54661	47.67612		47.6131
5	9.39568	12.32763			5000	40.14518	40.23527		40.2068
6	13.72055	17.42593			6000	34.91231	34.97831		34.9481
7	18.52281	22.97704			7000	30.99838	31.04859		31.0146
8	23.71096	28.88437			8000	27.9505	27.98981		27.9549
9	29.20996	35.07042			9000	25.50387	25.53536		25.5
10	34.95749	41.47189			10000	23.4928	23.51849	23.47	23.4837
20	97.67708	109.04904			20000	13.67314	13.67902	13.65	13.653
30	157.0311	170.95314			30000	10.00069	10.00278	9.984	9.98156
40	206.6054	221.65603			40000	8.04761	8.04844	8.034	8.02916
50	245.7853	261.10797		547.856	50000	6.8271	6.82741	6.816	6.81041
60	275.6607	290.74789		521.666	60000	5.98922	5.98926	5.979	5.97335
70	297.7366	312.2977		494.897	70000	5.3773	5.37721	5.369	5.36244
80	313.4849	327.36464		468.43	80000	4.91038	4.9102	4.903	4.89607
90	324.1972	337.32439		457.996	90000	4.54222	4.54199	4.535	4.52813
100	330.9524	343.30784		453.121	100000	4.24447	4.24422	4.238	4.23008
200	308.3288	314.9087		355.674	200000	2.87444	2.87416	2.871	2.85862
300	260.3465	264.33499		282.736	300000	2.42062	2.42037	2.418	2.40074
400	223.7633	226.50337		237.104	400000	2.20604	2.20581	2.204	2.18225
500	196.8930	198.94253		205.285	500000	2.08789	2.08767	2.081	2.05889
600	176.4958	178.11873		182.148	600000	2.01747	2.01726	2.081	1.98372
700	160.4487	161.78513		164.79	700000	1.97369	1.9735	1.954	1.93456
800	147.4459	148.57759		150.704	800000	1.94604	1.94585	1.921	1.90203
900	136.6601	137.63842		139.045	900000	1.92871	1.92852	1.897	1.87912
1000	127.5441	128.40317		129.22	1000000	1.91825	1.91807	1.880	1.86361

The difference between SP values obtained by using LJ and Tietz function in 2 eV was 48 %, 20 % in 20 eV and in 2000 eV this difference decreased to 0.4 %. The coincidence of the LJ model results with the results of Penelope 2014 is better than that obtained with the Tietz screening function. In larger energies, the results obtained in this study are better than 0.1 % with both ESTAR and Penelope code results.

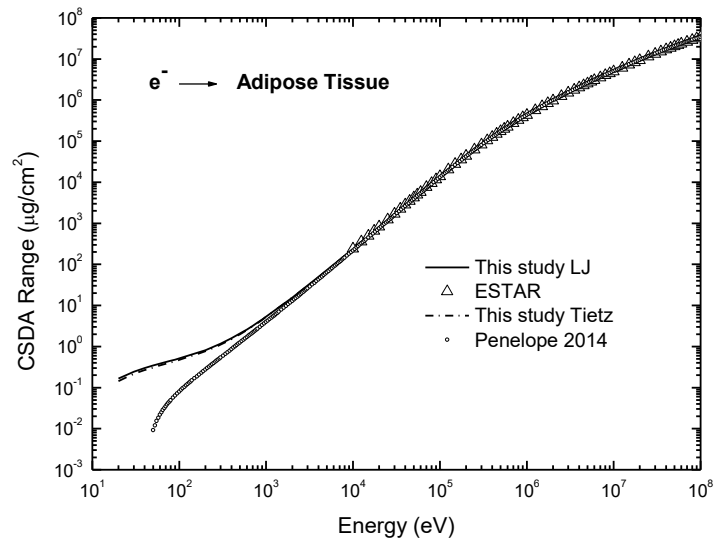


FIG. 4. Continuous slowing down approximation (CSDA) range for electrons, in Adipose Tissue as a function of the electron energy.

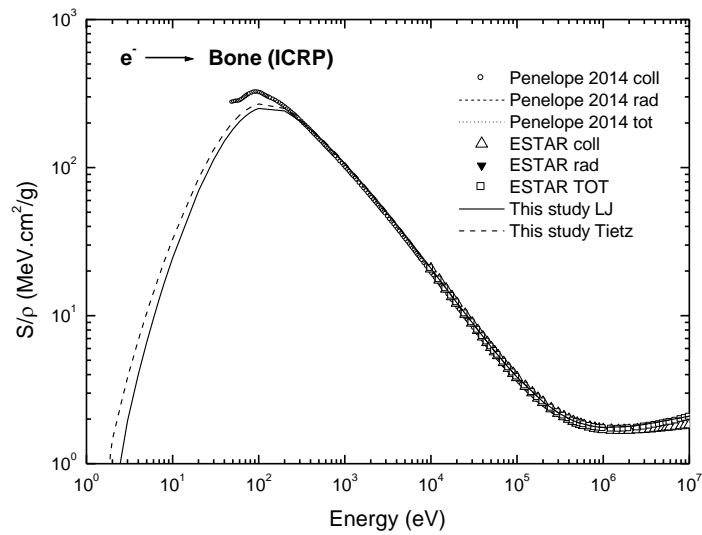


FIG. 5. Comparison of stopping power values (MeV.cm²/g) of adipose tissue for incoming electrons.

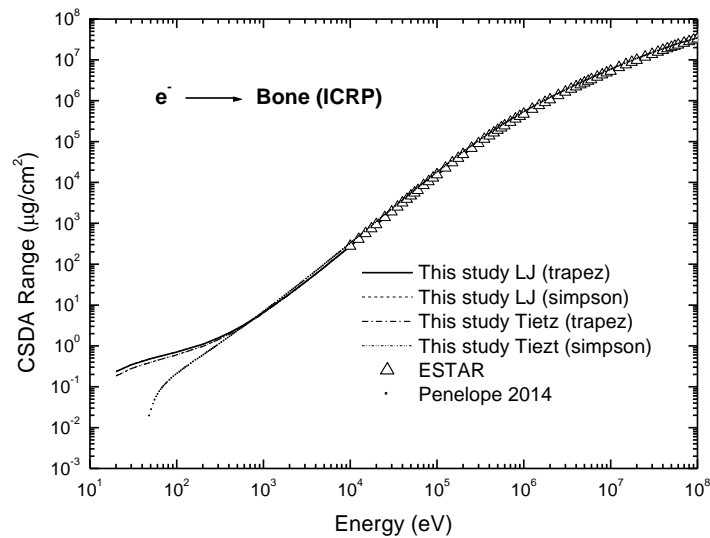


FIG. 6. Continuous slowing down approximation (CSDA) range results obtained from this calculation procedure for incident electron on bone tissue target.

The calculated results in this study are in good agreement with the ESTAR¹⁶ results above 10 keV and are in good agreement Penelope 2014 results above 300 keV energies. The calculated results of this study for stopping power and range are good agreement better than other theoretical calculations with the experimental data in low energy as in the water sample.

5. CONCLUSION

In the present study, we presented stopping power and CSDA-range values for electrons in water and some tissue materials, such as adipose tissue and bone by using Lenz Jensen and Tietz Screening functions. The calculated values was compared with a number of other calculation such as ESTAR¹⁶ and the Penelope 2014 code¹⁷ results,

The results obtained by using LJ screening function are in good agreement with the ESTAR¹⁶ and Penelope calculation results above high energy region, but it is easy to use the Tietz function. The calculated results are in good agreement with experimental data (for water) and the other theoretical calculations in general. The simple effective charge, effective atomic number and EMEE expressions are found to be successful, and are useful for practical computation of the SP and ranges for incident electron in tissues.

The procedure presented in this study does not need to solve any equation. Therefore it can be used directly in SP and range calculations, and should be useful for Monte Carlo calculations. The absorption dose of any tissue for incoming electrons can be calculated by using this calculation procedure.

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