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Improving Mathematical Optimization Techniques with the Aid of Exergy-Based Variables*

Marc Jüdes[1]** and George Tsatsaronis
Institute for Energy Engineering, Technische Universität Berlin
Marchstraße 18, 10587 Berlin, Germany
E-mail: [1]juedes@iet.tu-berlin.de; Phone: ++49-30-314-21738; Fax: ++49-30-314-21683

Abstract

The design optimization of energy conversion plants requires sophisticated optimization techniques. The usefulness of mathematical programming approaches has been discussed in several papers. Usually, the quality of the computed solutions, concerning global optimality and the convergence speed, is not discussed in these papers and even the existence of local optimal solutions is not mentioned. Indeed, the optimization of nonconvex mixed integer non-linear problems (MINLP), such as the structural and design optimization of power plants, is a very difficult problem. However, knowledge of the real optimization potential can assist the design engineer in better understanding the optimization procedure. This article deals with the use of exergetic variables for improving the quality of results obtained from mathematical optimization techniques and their convergence speed. LaGO, the solver used to compute the discussed results, can evaluate the obtained solution of the discussed minimization problems by calculating lower bounds of the original problem based on a relaxed convex objective function. Here, the use of exergetic variables can help to increase the lower bounds significantly and thus, to improve the evaluation of the computed solutions and the convergence speed. The method is applied to different optimization tasks.

Keywords: Cost minimization, design optimization, cogeneration plant, mixed-integer nonlinear programming, exergy, convergence speed.

1. Introduction

In this article, we discuss the potential of using exergy-based variables to improve mathematical optimization techniques applied to the optimization of the design of energy conversion plants.

Particularly with regard to deregulated energy markets (e.g., Jopp, 2004; DEWI et al., 2005) optimization of the design and the operation of energy conversion plants becomes increasingly important. Knowledge of a global optimal solution and the value of its objective function can provide important information to decision makers regarding the construction of a new power plant.

Various publications, such as Ahadi-Oskui (2006), Savola (2005), Frangopoulos (1992), and Muñoz and von Spakovsky (2001), present mathematical programming approaches for optimization problems concerning energy conversion plants. Their use requires a complete mathematical formulation of the optimization problem (e.g. thermodynamic model *and* cost equations). If this information is available, mathematical programming techniques seem to be a good choice: Compared to other methods, such as evolutionary algorithms (e.g., Ahadi-Oskui, 2006; Axmann et al., 1997), mathematical programming techniques offer the advantage that the obtained results are reproducible and their quality can be evaluated with regard to global optimality. Only mathematical methods are theoretically appropriate to produce evidence of global optimality. A comparison of different solvers and an evaluation of their capability to find global optimal solutions is presented in LaGO (2009). Ahadi-Oskui (2006) compares and evaluates mathematical optimization techniques and evolutionary algorithms.

The advantages of evolutionary algorithms concerning design optimization are not an integral part of this article. Indeed, in the literature, only a few articles (e.g., Ahadi-Oskui, 2006) deal with a detailed evaluation of the computed solutions concerning complex energy engineering problems (nonconvex mixed integer non-linear problems, thus MINLP problems). An analysis and evaluation of the influence of constraints on the quality of the computed solution and the convergence speed is not available for these kinds of problems.

This article deals with the use of exergetic variables in mathematical optimization techniques to improve the quality of the computed solutions and the convergence speed of the solver by a stronger limitation of the optimization problem's search space. The purpose of this article is not to evaluate the quality of the solver used (LaGO: see Nowak 2005, Vigerske, 2008), but to give a more general insight into the underlying problem. Indeed, Jüdes et al. (2009) show that all available state-of-the-art solvers have to handle similar problems. To demonstrate the strength of this method, different optimization problems are solved and different MINLP solvers are compared.

2. Problem Description

Typically, energy engineering problems are non-linear and nonconvex and often require the use of integer variables to build mathematical models. A general formulation of such an optimization problem, here a minimization problem, is given with Eq.(1).

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**Corresponding Author

$$
\begin{aligned}
\min \quad & \mathrm{TRR}_{\mathrm{lev}}(T,p,\dot{m},x,y) \\
\text{s.t.} \quad & f_{i,j,k}(T_i,p_i,m_i,x_{i,j}) = 0 \quad \forall i,j,k \\
& g_{i,j,n}(T_i,p_i,m_i,x_{i,j}) \le 0 \quad \forall i,j,n \\
& y_m \in \{0,1\} \quad \forall m
\end{aligned} \tag{1}
$$

Here, f indicates mass, energy, and momentum balances, equations describing the performance of components, as well as equations for the calculation of working fluid properties (k different equations). The n different constraints g denote, for example, minimum temperature differences. Binary variables are indicated by *y*. While i represents the different working fluid streams, j denotes the different chemical components of these streams. Structural decisions are indicated by the index m.

Nonconvex MINLP problems according to Eq. (1) may have several local optimal solutions (cf. Table 4). To evaluate the quality of an obtained solution for the design of a power plant, it is necessary to obtain information about the maximal optimization potential. Usually, the value of the objective function at the optimal solution is not known a priori and we need methods that allow us to compute absolute lower bounds (if the objective function has to be minimized) of the original problem.

In the optimization of energy conversion plants, we must deal with two major problems. The first one concerns the structure of the plant: if the structure is part of the design optimization, the use of a so-called superstructure is necessary (e.g., Ahadi-Oskui, 2006). The resulting mixed integer optimization problem requires the use of binary variables that indicate the existence and operation of single components or component groups. In the model used here, binary variables were considered using the so-called Big-M-constraints, i.e., inequalities that replace equations.

The second problem concerns the working fluid properties and the performance of the plant components. Typically, the describing equations are non-linear and some of these equations are nonconvex. Hence, the resulting problem becomes a nonconvex MINLP problem, the solution of which requires powerful mathematical solvers, whereas the analytical evidence of global optimality is still an unsolved problem.

Indeed, today's MINLP solvers cannot guarantee the generation of reliable *global* optimal solutions of complex energy engineering problems. Expert knowledge is still very important to enable the solver to compute in a manageable time good solutions of the considered optimization problem. Here, the use of expert knowledge in terms of exergy-based variables for a better formulation and limitation of the mathematical program is the focus of this article.

To evaluate the quality of a computed solution, the solver used here (LaGO) generates convex relaxations (i.e. a convex simplification of the original problem), that underestimate the original problem in a convex way (see Figure 1). The minimum of all values obtained through convex relaxations is computed with local solvers and represents an absolute lower bound of the original problem. White areas in Figure 1 represent the actual search space limited by the lower bound (lo) and a local optimal solution (up) of the original problem. Global optimality has been proven when these two values coincide, i.e., when the gap between the minimum of the relaxation and the absolute minimum of the original function is zero.

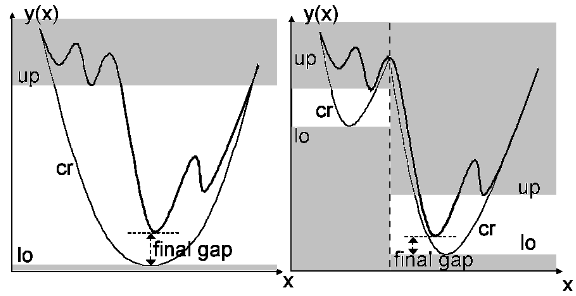

Figure 1. A Nonconvex and Non-linear Problem (bold line). Left: without branching, right: with branching techniques (dashed line). up: upper bounds, lo: lower bounds, cr: convex relaxations.

In practical applications, a major problem arises through the non-convexity of the original problem as shown in Figure 1: If the original function (bold line) is strongly nonconvex, the gap between the original function and the convex relaxation (thin line) may become large.

A block separable formulation of the problem in which we define several blocks within the plants' model connected only by linear constraints helps to reduce this problem and assists in the branch and cut algorithm. This algorithm reduces the search space in a successive way and helps to minimize the gap by splitting the major optimization problem into several sub-problems. In Figure 1, the left part shows a relaxation without the application of branching techniques. The right part shows a relaxation after the application of branching techniques, applied to continuous variables. Even if the final gap becomes smaller, it cannot be closed. As will be shown in Section 4, a branching with respect to the continuous variables increases the number of sub-search-spaces dramatically. To avoid this unwelcome side effect, a strong limitation of single variables has to be preceded. This can be carried out, e.g., by using exergy-based variables.

LaGO generates starting points for a local optimization by solving the relaxed function. Therefore, a good relaxation of the original problem is very important: the computation of a solution becomes easier and faster, and the gap becomes smaller with an increasing quality of relaxation - the better the relaxations, the better the solution.

The computation of the relaxation proceeds in five steps (cf. Figure 2):

a) Initial choice of sample points (black dots) and calculation of a (local) minimum $x_{\min}$ of the original function g(x).
b) Calculation of a quadratic function q(x) that underestimates all sample points. At $\hat{x} = x_{\min}$, $g(\hat{x})=q(\hat{x})$
c) Maximization of the error δ=q(x)-g(x) in-between the sample points leads to x^*.
d) If q(x^*)-g(x^*) > Tolerance $\delta_{\max}$, x^* is added to the sample points and the quadratic underestimator is recalculated.
e) If q(x^*)-g(x^*) < Tolerance $\delta_{\max}$, q(x) is lowered by the actual difference δ.

It is necessary to emphasize that nonconvex and non-linear equations cannot be avoided in such mathematical models: Figure 3 shows exemplarily the logarithmic mean temperature difference

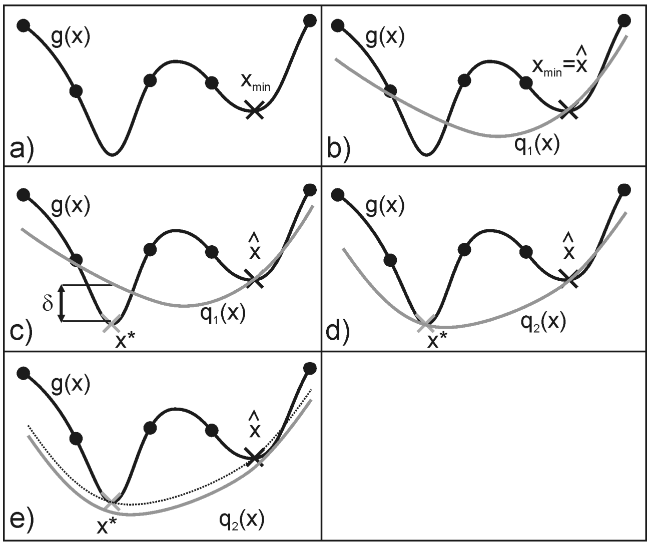

Figure 2. Construction of a Quadratic Underestimator.

$$\Delta T_{\log} = (\Delta T_1 - \Delta T_2)/(\log \Delta T_1 - \log \Delta T_2). \quad (2)$$

A simplification of this function (e.g., a linearization and even a piecewise linearization with few line sections) may lead to a completely wrong calculation of the heat exchanger surface area and, therefore, of the purchased equipment cost. This simple example demonstrates that a strong simplification may not be suitable for all kinds of problems.

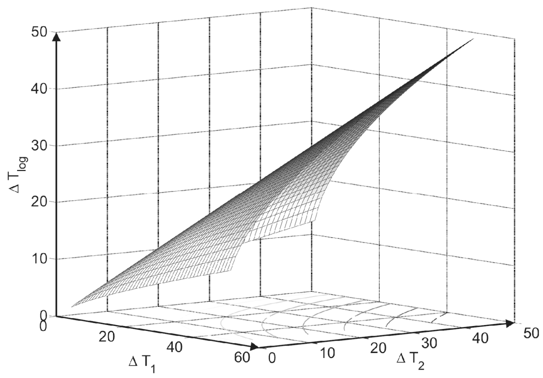

Figure 3.: $\Delta T_{\log}$, a Nonconvex Non-linear Function.

Obviously, it is very helpful to use reasonable variables and constraints that permit a strong limitation of the search space and therefore a strong limitation of the entire optimization problem. Indeed, it is necessary to limit most of the variables used in an adequate way and thus enable today's MINLP solvers to solve complex strongly nonconvex problems. Therefore, not only an understanding of the mathematics involved in the solution, but also a deep understanding of the thermodynamic processes occurring in the plant is essential. Otherwise, the global optimal solution might be excluded a priori by a too strong bounding of the decision variables.

However, a strong limitation of the single variables is not always possible, especially when superstructures are used to optimize the structure of the plant or when partial loads should be considered (Jüdes and Tsatsaronis, 2007; Jüdes et al., 2009; Jüdes, 2009). In these cases, a wider range for the variables is necessary and the design engineer cannot predict these ranges at all points within the process. The solver should have the ability to generate very different solutions with different properties of the working fluid.

In this article, we discuss advantages and disadvantages of using in the mathematical program the exergy of the working fluid and exergy-based variables of the components to improve the optimization process. These variables can often be limited to a narrow range by simple thermodynamic considerations. Without numerous simulations, the definition of lower and upper bounds of some exergy flows, of the exergy destruction and of exergetic efficiencies becomes possible (see also Tables 6 and 7). On the other hand, the calculation of exergy flows makes it necessary to include a large number of additional non-linear and partly nonconvex equations and constraints in the model. For example, the condensating fraction of water at ambient conditions now has to be calculated for the exhaust gas streams. The optimization results and the advantages of using exergy based variables are presented using two different examples.

3. Description of the Plants and Results

The method proposed in this article is applied to two different energy conversion systems: The CGAM problem (e.g.Valero et al., 1994; Figure 4), and a more complex cogeneration plant (Jüdes and Tsatsaronis, 2007; Jüdes et al., 2009; Figure 5). Three different objective functions are used.

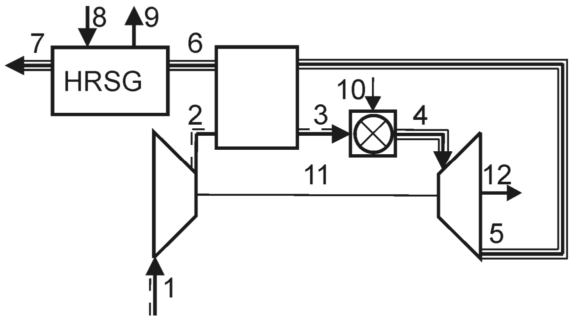

Figure 4. Flow Sheet of the CGAM Problem.

The CGAM problem represents a non-convex non-linear optimization problem (NLP) without any binary variables. This simple example gives insight into the problems of nonconvex optimization techniques and the computed results are comparable to those presented in the literature. The optimization problem is formulated as a mathematical program in GAMS (GAMS, 2009) and the MINLP solver LaGO was used (LaGO, 2009) considering only its NLP solution techniques. Tables 1 and 2 summarize the results of a thermodynamically optimal and cost-optimal design, neglecting income taxes (that is one of the reasons why the cost-optimal design differs from the values given by Bejan et al., 1996). The objective function for the thermodynamically optimal design is the fuel cost $\dot{C}_{\text{fuel}}$, the objective function for the cost-optimum is the levelized total revenue requirement TRR_{lev}.

Table 2 presents the values of the objective function and the lower bounds. Here, the gap is calculated according to Eq. (3):

$$gap = \frac{[\max(\text{UP},\text{LO}) - \min(\text{UP},\text{LO})]}{\min(\text{UP},\text{LO})} \quad (3)$$

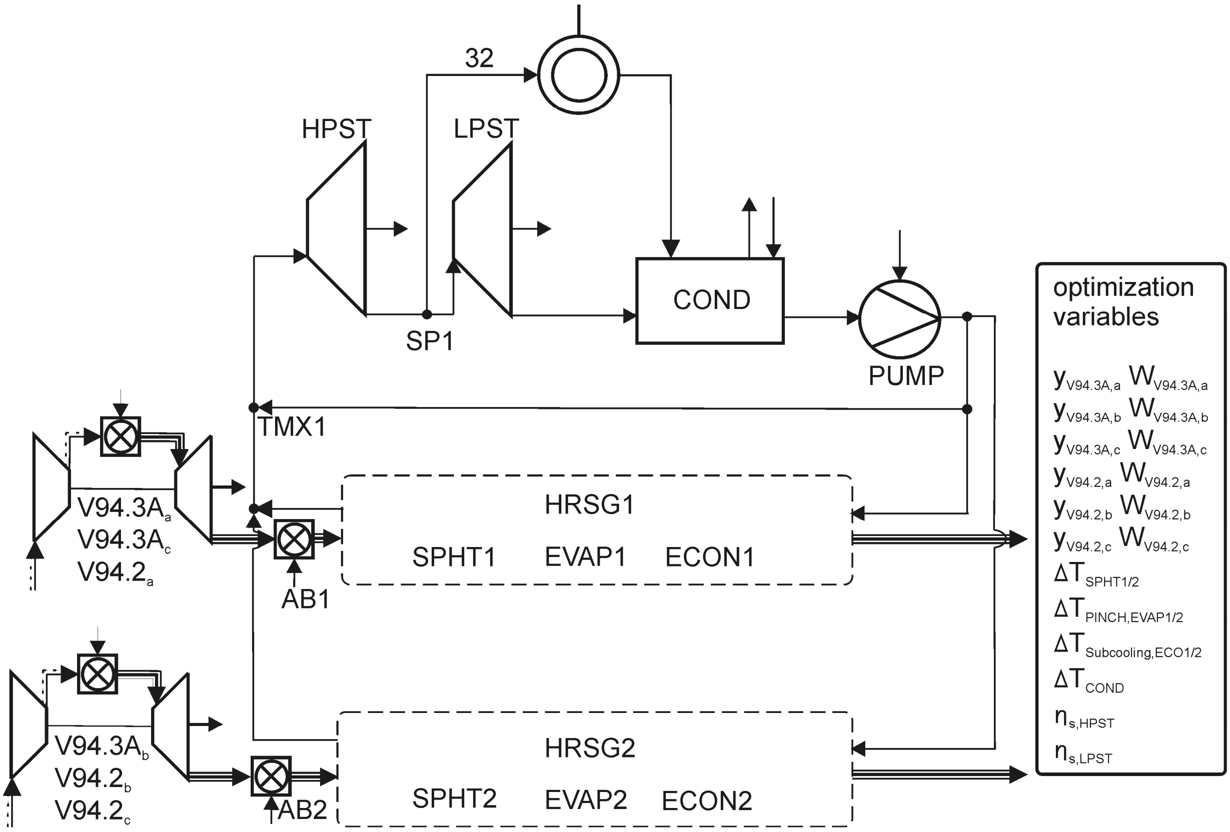

Figure 5. Superstructure and Optimization Variables of the Simplified Energy Conversion Plant. The Decision Variables are related to Table 4.

Table 1. Variables for the Thermodynamically Optimal (TO) and the Cost-optimal (CO) Designs of the CGAM Problem, Neglecting Income Taxes. Economic Assumptions According to Bejan et al. (1996).

Variable	Dimension	TO	CO
p_2 / p_1	-	16.0	6.5
$\eta_{s,C} / \eta_{s,T}$	%	88.0/90.0	82.3/85.8
T_3	K	792.4	923.6
T_4	K	1550.0	1470.7
$\Delta T_{min,HRSG}$	K	15.0	15.0
$\dot{m}_1$	kg/s	74.23	113.56
$\dot{m}_{10}$	kg/s	1.51	1.66

Table 2. CGAM problem: Solutions Computed by LaGO After 1000 Iterations. Variables for the Thermodynamically Optimal (TO) and cost-Optimal (CO) Designs (TOex and COex with Consideration of Exergy-based Variables) Neglecting Income Taxes. In addition, the Iteration of the First Solution and the Lower Bounds are Given. Economic Assumptions from Bejan et al. (1996).

Variable	TO	CO	TOex	COex
Objective function	$\dot{C}_{fuel}$	TRR_{lev}	$\dot{C}_{fuel}$	TRR_{lev}
Solution [Mio.€/a]	5.97	25.1	5.97	25.1
Lower bound [Mio.€/a]	2.01	12.9	5.52	19.67
gap, cf. Eq.(2) [%]	196	94.6	8.2	27.6
1st solution in iteration	1	14	1	1

where UP and LO represent the upper bound (i.e., the solution of the minimization problem) and the lower bound, respectively.

We stopped the optimization after 1000 iterations. In further iterations, the upper bound, i.e. the optimal solution, stays constant and the lower bound increases only marginally. LaGO found the first feasible solution after the first and 14th iteration for the thermodynamically optimal and cost-optimal design, respectively. This information indicates the convergence speed of the solver. More complex problems, such as the cogeneration plant shown in Figure 5, require significantly more iterations (cf. Table 3 and Section 4). Therefore, not only the quality of the solution, but also the convergence speed plays an important role in design optimization problems.

Obviously, LaGO was not able to close the gap according to Eq. (3) and thus prove global optimality for the CGAM problem. It should be mentioned that the NLP-solver CONOPT (ARKI, 2008) was not able to find a feasible solution neither for the original CGAM problem nor for the problem including exergy-based variables without a presetting of good starting values.

The computed cost-optimal results are similar to those presented by Bejan et al. (1996). Only the computed temperature difference at the pinch point $\Delta T_{min,HRSG}$ is noticeably smaller. A recalculation of the objective function with fixed decision variables according to Bejan et al. (1996) results in TRR_{lev} = 25.7 Mio.€/a instead of TRR_{lev} = 25.1 Mio.€/a for the obtained optimal solution discussed in this article.

However, the focus of this article is not on evaluating the quality of the solver LaGO - other solvers have similar problems (see also Jüdes et al., 2009 and Table 5) and different solvers may be more appropriate for solving this problem. One of the purposes of this article is to present a method in which engineering expert knowledge is used to improve mathematical optimization techniques. Thus, the improvement of the convergence speed (number of iterations to find the first feasible solution) and the solution qualities using mathematical programming techniques are evaluated. In this case, LaGO is a representative tool.

The second plant analyzed in this article is the more complex cogeneration plant shown in Figure 5. It is a single-pressure combined cycle plant with a supplementary firing (additional burners AB) for each heat recovery steam generator (HRSG) similar to the plants used in Jüdes and Tsatsaronis (2008), Jüdes et al. (2009) and Jüdes (2009). To demonstrate the new approach, only one steady-state operation point at full load is considered in this case. The required electric power output is 750 MW and the required mass flow rate of process steam is $\dot{m}_{32} = 133.1$ t/h at 120°C. For all economic assumptions refer to Jüdes (2009).

According to Hüttenhofer and Lezuo (2001), we consider different types of gas turbines: each HRSG operates independently and can be fed by any combination of the three Siemens V94.3A and three Siemens V94.2 gas turbines. The two HRSG's consist of an economizer, an evaporator and a superheater. The operation of the additional burners and the water injector (TMX) is optional.

The steam is supplied to the high, intermediate (both indicated with HPST) and low-pressure (LPST) sections of the steam turbine. A process steam extraction SP1 is placed after the intermediate-pressure section of the steam turbine. The condensate returning from this heat sink is mixed with the outlet stream of the low-pressure steam turbine in the condenser (COND).

Tables 3 and 4 show the results of the economic design optimization of the cogeneration power plant. Again, the number of iterations needed to generate the first solution is presented. Additionally, Table 3 presents the lower bounds of the objective function.

Table 4 gives the values of the respective optimization variables. Here, *first solution* denotes the plant's design at the first feasible solution while *optimal solution* indicates the solution after 1000 iterations of LaGO.

Due to a stronger limitation of the variables of the more complex power plant, the gap given in Table 3 is smaller than the gap shown in Table 2 (CGAM problem) even without the use of any exergy-based variables.

Thus, the limitation of variables leads to better solution evaluations and/or a faster convergence of the optimization algorithm. Therefore, further variables should be used to upgrade the quality of the solution. Exergetic variables

Table 3. Cogeneration Plant (cf. Figure 5): Results of the Cost-Optimal Design (objective function: TRR_{lev}) Computed by LaGO after 1000 iterations. "Solution" Represents the Respective Value of the Objective Function. In addition, the First Solution and the Lower Bounds are Presented (see also Jüdes, 2009). CO: without exergy, COex: with exergy-based variables; $c_{\text{fuel}} = 5.5$ €/GJ$_{\text{LHV}}$

Variable	CO	COex
Solution [Mio.€/a]	319.2	319.2
Lower bound [Mio.€/a]	278.7	278.7
gap, cf. Eqn (3) [%]	14.5	14.5
1st solution in iteration	37	25

Table 4. Decision Variables for the Design Optimization of the Cogeneration Plant Shown in Figure 5. Solutions Found by LaGO in the Respective Iteration Presented in Table 3.

Decision variable	Dimension	first solution	optimal solution
$\dot{W}_{94.3\text{A,a}}$	MW	253.5	253.5
$\dot{W}_{94.3\text{A,b}}$	MW	-	253.5
$\dot{W}_{94.3\text{A,c}}$	MW	253.5	-
$\dot{n}_{\text{AB1}}$	kmol/s	0.042	-
$\dot{n}_{\text{AB2}}$	kmol/s	-	0.044
T_{12} / T_{20}	K	823/-	808/859
$\Delta T_{\text{PINCH1/2}}$	K	10/-	10/10
$\Delta T_{\text{S,ECO1/2}}$	K	0/-	0/0
$\dot{n}_{\text{cw}}$	kmol/s	336	337
p_8 / p_{21}	bar	0.05/86.3	0.05/75.7
$\dot{n}_{23}$	kmol/s	0.0	0.0
$\eta_{\text{sHDT/NDT}}$	%	85.5/86.9	85.7/86.2
PEC_{tot}	Mio.€/a	146.0	144.2
$\dot{E}_{\text{P,tot}}$	GWh/a	4363	4363
$\dot{E}_{\text{F,tot}}$	GWh/a	7922	7932
ε_{tot}	%	55.06	55.01
$\dot{C}_{\text{F}}$	Mio.€/a	151.9	152.1

enable a strong limitation of the problem and thus may improve the optimization. It must be emphasized that neither LaGO nor other tested MINLP solvers (AlphaECP, BARON, Bonmin, DICOPT, OQNLP and SBB) can prove the solution's global optimality as has been shown in Jüdes et al. (2009) and Jüdes (2009) for the cogeneration plant considering (c.f. Table 5).

Table 5. Best Objective Function Values TRR_{lev} When Optimizing the Cogeneration Plant (cf. Figure 4), Considering Four Different Operation Points with Different MINLP Solvers and a Time Limit of 3 h. The Third Column Gives the Best Values When a Solution with Objective Function value 236.24 Mio. €/a (LaGO's solution after 10 hours) is Provided as the Starting Point to the Respective solver.

Solver	best solution	best solution with feasible starting value
	Mio. €/a	Mio. €/a
LaGO	244.17	236.24
AlphaECP	--	--
BARON	255.13	236.24
Bonmin	242.50	--
DICOPT	--	--
OQNLP	--	235.97
SBB	236.76	235.73

4. The Influence of Exergy-based Variables

As discussed in the previous sections, for optimization tasks, it is helpful to obtain information about the global optimum of an objective function. However, the available solvers still have difficulties with "real life" problems from the field of design optimization of power plants in proving global optimality.

From the design engineer's point of view, it is desirable to use process knowledge to improve the solution provided by the optimization algorithm. Exergy seems to be a helpful concept in that respect because exergy flows and exergy-based variables can be limited very strongly without excluding feasible and potential parts of the solution space – only the search space is limited. For example, negative exergy flows are not allowed when the pressures are above the ambient pressure nor are negative exergy destructions.

Table 6 shows some representative exergy-based variables that enable the design engineer to limit the search space of the CGAM problem while Table 7 presents some representative exergetic variables of the complex cogeneration plant shown in Figure 5. The range of the bounds is rather wide but still acceptable. A minimal assumed exergetic efficiency of ε_{tot} = 20% results in exergy flows within the cogeneration plant lower than 4000 MW.

Table 6. Exergetic Efficiencies ε, and Exergy flows $\dot{E}$ with Lower (LO) and Upper (UP) Bounds of the CGAM Problem (cf., Figure 4).

Variables	Dimension	LO	UP
ε_{COMP}	%	80	97
ε_{EXP}	%	80	97
ε_{HRSG}	%	50	90
$\varepsilon_{Air\ preheater}$	%	70	95
$\dot{E}_i$	MW	0	150

Table 7. Exergetic Efficiencies ε, Exergy Destructions $\dot{E}_D$ and Exergy flows $\dot{E}$ with Lower (LO) and Upper (UP) Bounds. Nomenclature According to Figure 5.

Variables	Dimension	LO	UP
ε_{AB}	%	60	80
ε_{SPHT}	%	50	90
ε_{EVAP}	%	50	85
ε_{ECON}	%	30	85
ε_{HPST}, ε_{LPST}	%	70	95
$\dot{E}_{D,AB}$	MW	0	50
$\dot{E}_{D,SPHT,EVAP,ECON}$	MW	0	50
$\dot{E}_{D,HPST}$, $\dot{E}_{D,LPST}$	MW	0	50
$\dot{E}_i$	MW	0	4000

Calculating exergy flows requires knowledge of enthalpy and entropy values of all material flows within the plant. The latter are not normally required for a conventional optimization procedure and, therefore, when exergy values are used, the complexity of the resulting mathematical program increases also because of the required calculation of enthalpy and entropy values at ambient conditions (T_0 and p_0).

There is a trade-off between improving the solution's quality due to a stronger limitation of the search space and the integration of additional and partly nonconvex equations and constraints. Especially the nonconvex equations increase the difficulties associated with the solver that has to compute good relaxations for optimal solutions of the original problem.

The last two columns of Table 2 (TOex and COex) show the thermodynamically optimal and cost-optimal results of the CGAM problem obtained using exergy-based variables. Tables 3 and 4 show the results of the cost-optimal design of the complex power plant (cf. Figure 5). Exergy-based variables improve the solution qualities: for the CGAM problem there is a clear increase of the lower bound in both the thermodynamic and the economic optimization cases. For cost-optimal designs, there is an obvious improvement of the convergence speed. However, the obtained solutions are the same (the optimization variables are presented in Table 1).

The increasing convergence speed can be explained as follows: Due to the complexity of the optimization problems, it is necessary not only to branch the binary problem, but to branch also the continuous one. Otherwise, the starting values contributed to the local solver from the solution of the relaxed problem are far away from the real solution and LaGO is not able to compute a solution. However, a branching with respect to the continuous variables increases the number of sub-search-spaces dramatically. Thus, a strong limitation of single variables avoids this unwelcome side effect.

For the cogeneration plant, the benefits are smaller. Both approaches show the same results with respect to the optimal solution and the lower bound. We observe only an increase of the convergence speed: LaGO finds a first solution after 37 and 25 iterations without and with exergy-based variables, respectively. The "optimal" solution is found after 437 and 396 iterations without and with exergy, respectively. Until the end of the optimization procedure, there is no more change, neither with respect to optimal solution nor with respect to considering the lower bound.

5. Conclusions

The method presented in this article enables the design engineer to use his expert knowledge for improving mathematical optimization techniques. The improvement is based on a better limitation of the variables and thus, on a better limitation of the search space, due to well-known ranges of exergy-based variables. It is obvious that the limitation of single exergy flows and exergy-based variables is easier from the engineer's point of view than a limitation of temperatures, mass flow rates and pressures. Especially enthalpies and entropies are hard to limit due to the a priori unknown chemical composition of exhaust gas flows within the process.

The new approach, a combination of mathematical programming techniques and an application of exergy-based variables, was applied to different kinds of problems, the computation of thermodynamically optimal and cost-optimal designs, and two different types of plant models: The CGAM problem (nonconvex NLP model) and a more complex plant (nonconvex MINLP model). In all cases, the MINLP solver LaGO was used to allow easy comparison of the optimization results.

To avoid a unilateral focus on the solver LaGO, we compared the results to the solutions provided by well-known MINLP solvers, such as AlphaECP, BARON, Bonmin, DICOPT, OQNLP and SBB. Some of these solvers are not able to solve the optimization problem. The results of the successful solvers were approximately identical, at least when feasible starting values were provided. It should be noted that the comparison of these solvers was conducted for a more complex optimization problem (the cogeneration plant shown in Figure 5 with consideration of four different operation points). Jüdes et al. (2009) present a more detailed comparison of these MINLP solvers.

In all cases, exergy-based variables improve the quality evaluation of the solution or at least the convergence speed. Of course, the use of these variables does not influence the optimal solution, but the convex relaxations and thus the gap between the solution and the underestimator may be reduced. In addition, the number of sub search spaces that have to be explored during the optimization procedure can be reduced.

Consequently, the use of exergy seems to be an appropriate method to enhance the performance of mathematical solvers for energy engineering problems although additional nonconvex equations and constraints have to be included in the mathematical program.

In future work, exergoeconomic variables may also be included in the mathematical program, the use of which requires linear equations only. Furthermore, the method should be applied to power plants with more than one operating point.

Nomenclature

Variable	Meaning	Dimension
AB	Additional burner	
$\dot{C}_{\text{fuel}}$	Cost flow rate associated with the fuel flow rate	Mio. €/a
c_{fuel}	Specific fuel cost	€/GJ_{LHV}
CO / TO	Cost-optimal / Thermo-dynamically optimal design	
COex / TOex	Cost-optimal / Thermo-dynamically optimal design considering exergy-based variables	
COND	Condenser	
$\dot{E}_{\text{i}}$	Exergy flow rate	MW
$\dot{E}_{\text{D}}$	Exergy destruction	MW
ECON	Economizer	
EVAP	Evaporator	
HPST	High-pressure steam turbine	
HRSG	Heat recovery steam generator	
LO	Lower bound	
LPST	Low-pressure steam turbine	
$\dot{m}$	Mass flow rate	kg/s
MINLP	Mixed integer non-linear programming	
NLP	Non-linear programming	
SPHT	Superheater	
p	Pressure	bar
PEC	Purchased Equipment Cost	Mio. €
T	Temperature	K
TMX	Water injector	
TRR_{lev}	Levelized total revenue requirement	Mio. €/a
UP	Upper bound	
$\dot{W}$	Electric power	MW
y	Binary variable	-
Greek Symbols		
Δ	Difference	
δ	Difference	
η_s	Isentropic efficiency	%
ε	Exergetic efficiency	%
Subscripts		
F	Fuel	
P	Product	

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