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Application of Bejan's Constructal Theory to a Solar Collector System. Part I: The Fundamentals to Define the First Construction*

J. Ojeda[1], F. Méndez[2**]

[1,2]Facultad de Ingeniería, UNAM, 04510, Mexico
Email: [1]jos242@gmail.com, [2]fmendez@servidor.unam.mx

Abstract

In the present work, we develop a theoretical scheme that establishes the fundamentals of the first construction for a solar collector system. In particular, the first construction is divided into two elements: a solar cavity and the fluid that is heated. For the cavity, we assume a natural convective regime that for small values of Rayleigh number heat transfer becomes dominated only by a conductive heat transfer regime. Meanwhile in the other region, where the fluid is circulating in a circular tube, the forced convective mode is dominant. The dimensionless temperature profiles in both regions are calculated theoretically and the minimization conditions for the dimensionless heating time of the fluid is found by this first construction.

Keywords: *Constructal theory; solar collector; optimization; heating.*

1. Introduction

In general, solar collectors have been used in the past as alternative devices for heating water. In addition, the design and construction of these equipment in the world is now very popular and is easy to find practical applications related with conditioning air in buildings, residences and industrial centers by using different arrangements and modules of solar collectors.

In order to find an useful criterion for predicting better efficiencies in solar collectors, in the past, the influence of different geometric factors have been taken into account to enhance the performance of these devices. Among others, the main geometric parameters of the solar collector that play an important role in the above discussion are the distance between the tubes, the diameters and lengths of the tubes, and obviously the number of tubes that participate in collecting the solar energy. However, the above geometric characteristics are not unique and we must include other design parameters. Representative parameters can be the area of the collector and the active area per unit of the annual demand of received solar heat. The above is to have a better idea of the thermal design for absorbing solar devices (Szargut & Stanek, 2007).

On the other hand, there are other physical mechanisms directly involved in the heating of the water. These depend strongly on the simultaneous action of the solar radiation and the forced and natural convection, which operate in a delicate manner for maintaining the efficiency of the system within a tolerable range. These mechanisms are, together with the geometric characteristics of the collector system, the main variables for an appropriate thermal design and control. In this direction, the previous specialized literature has reported well-documented monographs and related studies for calculating the efficiency of conventional solar collectors. However, in these solar energy collection systems the operability and production margins are frequently low; therefore, in this work we have concentrated on developing a theoretical approach to find the thermal optimization of a solar collector system.

In an effort to obtain the above objective, in the present work we apply the Constructal Theory developed by Bejan (1997). Nowadays, it is well-known that the Constructal Theory developed explicitly to optimize complex thermodynamic systems allows us to design new geometries for the optimal heat transfer between a known volume and a concentrated sink of heat. This theory proposes basically the generation or construction of tree networks that satisfy a certain physical factor or restriction. In this manner, we find the more appropriate geometrical configuration that maximizes or minimizes the heat transfer process. In some cases of study, such theory requires a principle of minimization of the resistance to the flow. Typical examples in the specialized literature have appeared with different transport mechanisms: laminar and turbulent fluids, heat, nutrients; and in more complexes cases, the combination of some of them. In nature, Bejan (2000) has clearly identified vascular networks, lungs, etc. as illustrative examples where Constructal Theory can be easily applied. An important characteristic of this theory is that the principle of minimization of flow resistances must always satisfy volume and area restrictions.

The selected physical principle depends on the same natural and physical characteristics of the problem; however, the use of the mass, momentum and energy conservation equations must be taken into account to complete the physical description and optimization of the constructal system. In the present work, some basic concepts of this theory will be applied to define the first construction of a possible network of tubes with a circulating fluid, in such a way that they can receive the largest amount of solar energy. In the specialized literature, however, conductive heat transport processes have dominated the development of the constructal modeling. Nowadays, the fundamentals for studying steady-state cases are sufficiently well understood. However, examples of the use of the Constructal Theory under transient situations is

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**Corresponding Author

scarce. In this direction, Dan & Bejan (1998) suggest the development of Constructal Theory to include transient effects. In the foregoing work, the unsteady cooling of a solid by means of a band of conductive material and under the influence of the time is predicted with possible applications for the cooling of electronic circuits. The main results, among others, for the first construction are the temperature distributions and the minimum cooling time for a prescribed geometrical parameter $\hat{H}_0$, which will be calculated below for our case. Therefore, in the present work we consider the transitory character of the heating process of the fluid. The origin of this heating is basically represented by a transient signal of radiative energy to the first constructal element.

2. First Element or Construction

This first element takes into account the convective characteristics commonly found in solar equipment. Consider a finite size volume that is initially found at a uniform temperature T_0. The transverse area of this volume and plotted in Figure 1, is known and given by $A_0 = H_0 L_0$ (Bejan, 2000). The above assumption is the restriction frequently assumed in Constructal Theory. Obviously, the lengths L_0 and H_0 are unknown and must be determined as a part of the optimization procedure. In accordance with the constructal approach, the total volume of the system is represented by an assembly of building blocks of different sizes and the optimization process begins with the smaller blocks. Consequently, Figure 1 shows a sketch and the dimensions of this first element. The top and bottom regions are separated by an infinitesimal wall with negligible thermal conductivity.

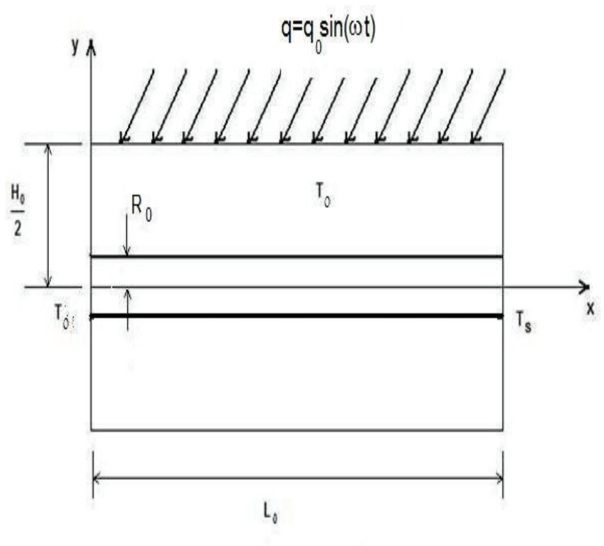

Figure 1. First construction or insert of high dimensionless thermal conductivity κ.

In the transverse direction, at $y = H_0 / 2$, the element receives the solar energy. For simplicity, in our case, we represent the above radiative heat flux with a sinusoidal behavior, given by $q = q_0 sin(\omega t)$, where q_0 is a reference value or amplitude of this periodic heat flux for the present system, ω is the frequency of the solar signal and t is the physical time. The space between the surface where the solar collection occurs and the horizontal element of radius R_0 is considered as a cavity filled with air at temperature T_a. We anticipate that the characteristic times for the existence of the free convection inside the cavity are very large. Therefore, the heat transfer mechanism in this region is basically considered as diffusive. The above is justified for the very small values of the Rayleigh number in the cavity.

Inside the pipe, a fluid is circulating in the x-direction in a steady-state regime. This hypothesis is justified below. The flow is considered Newtonian and fully developed. The fluid enters at temperature T_0 and leaves the element at a higher temperature T_s due to the heating process. The variable temperature of the fluid is T_f. These last two temperatures are unknown and must be determined as part of the problem, together with the lengths L_0 and H_0. It is important to note that the definition of the predefined constructal first element is not a real solar collector and that the existence of this connection is essential to build the entire network constructal. Then the design constructal may be subject to the geometric features of a real collector. However, in this work such considerations are left for further investigation. In addition, we anticipate that the minimization process for the present problem is to find the minimum values for the heating time between the top of the cavity and the tube.

2.1. Mathematical Model

For each region of the first constructal element defined in the previous section, the temperature profiles can be found by making an energy balance. For the tube region, we take into account accumulative, conductive, convective and radiative energy terms, resulting in physical units into the following partial differential equation,

$$\rho_f C_{pf} \frac{\partial T_f}{\partial t} - k_f \frac{\partial^2 T_f}{\partial x^2} + \rho_f C_{pf} u_x \frac{\partial T_f}{\partial x} = \frac{2}{R_0} k_a \left(\frac{\partial T_a}{\partial y} \right)\bigg|_{y=R_0} . \tag{1}$$

Equation (1) takes into account that the transverse temperature variations are smaller than the horizontal temperature variations. It means that, as a first approximation, the temperature of the fluid is function only of the longitudinal coordinate x ; *i. e.*, $T \simeq \mathrm{T}(x)$. This is the well-known thin fin approximation reported elsewhere (Bejan, 1993). In addition, in Eq. (1) we consider that for the infinitesimal surface that separates the cavity from the fluid trapped in the tube, the boundary condition $h(T_{sur} - T_0) = -k_a \left(\partial T_a / \partial y \right)\big|_{y=R_0}$ is fulfilled. It allows us to establish a conjugate heat transfer process between the cavity and the fluid. In this manner, the temperature of the air trapped in the cavity should be determined as part of the problem. This formulation of the conjugate heat transfer process enables us then to interpret properly the "radiative" term. That is, the warming effect of air trapped in the cavity comes from the signal $q = q_0 sin(\omega t)$, which provides an approximation of the radiation energy. Therefore, the corresponding governing equations for the cavity region are presented below. However, first we can use the following dimensionless variables to simplify the number of physical parameters involved:

$$\chi = \frac{x}{A_0^{1/2}}\ ,\ \eta = \frac{y}{A_0^{1/2}}\ ,\ \tau = \frac{t}{A_0/\alpha_a}\ ,\ \theta = \frac{T-T_0}{T_0}\ ,\ \theta_f = \frac{T_f - T_0}{T_0}\ ; \qquad (2)$$

then Eq. (1) can be rewritten as,

$$-\tilde{\beta}\frac{\partial \theta_f}{\partial \tau} + \frac{\kappa\phi_0 \hat{H}_0 \bar{\alpha}}{4}\frac{\partial^2 \theta_f}{\partial \chi^2} - \frac{2}{3}\frac{\partial \theta_f}{\partial \chi} - \kappa\bar{\alpha}^2\left(\frac{\partial \theta}{\partial \eta}\right)\Big|_{\eta=0} = 0\ , \quad (3)$$

where $\tilde{\beta} = \alpha_a \phi_0 \hat{H}_0 / 4\alpha_f Pe \sim 10^{-8}$, and therefore the energy accumulation term can be neglected. Thus, a quasi-stationary state is assumed for the circulating fluid. In addition, $\kappa = k_f / (k_a Pe)$, $\bar{\alpha} = k_a / k_f$, $\phi_0 = R_0 / H_0$, α_a and α_f represent the thermal diffusivities of the air and Newtonian fluid, respectively, and k_a and k_f the corresponding thermal conductivities of the air and fluid. Also, the dimensionless number Pe represents the Péclet number and is defined as $Pe = U_f R_0 / \alpha_f$, where U_f is the characteristic velocity in the fluid and related with the pressure gradient for a fully developed fluid.

The boundary conditions to solve Eq. (3) are the following,

$$\chi = 0:\quad \theta_f = 0 \quad ;\ \chi = \frac{1}{\hat{H}_0} \quad : \quad \frac{\partial \theta_f}{\partial \chi} = 0\ ; \qquad (4)$$

The solution of Eq. (3) subject to the boundary conditions, Eqs. (4), can be easily obtained and is given by:

$$\theta_f(\chi,\tau) = \frac{3}{2}\kappa\bar{\alpha}^2 \times\left[\frac{3}{8}\kappa\bar{\alpha}\phi_0\hat{H}_0 \exp^{-\frac{8}{3\kappa\bar{\alpha}\phi_0\hat{H}_0^2}}\left(\exp^{\frac{8}{3\kappa\bar{\alpha}\phi_0\hat{H}_0}\chi} - 1\right) - \chi\right]\frac{\partial\theta}{\partial\eta}\Big|_{\eta=0} \qquad (5)$$

Similarly, we can carry out an analysis of order of magnitude for the momentum conservation and energy equations for the space defined by the cavity, previously mentioned. After some algebraic manipulations, it can be easily shown that the characteristic velocities of the air into the cavity are of the order of magnitude of the Raleigh number:

$$U_c \sim \alpha_a \frac{L_0}{H_0^{\ 2}} Ra,\ V_c \sim \frac{\alpha_a}{H_0} Ra\ ; \qquad (6)$$

where U_c and V_c are the characteristic air velocities in the x and y directions, respectively. Ra represents the characteristic Rayleigh number for the region occupied by the cavity and is given by $Ra = g\beta_{th} q_0 H_0^4 / \alpha_a \nu_a k_a$. With the aid of this order of magnitude analysis and using the previous dimensionless variables, the energy equation reduces to the following differential equation,

$$\frac{\partial\theta}{\partial\tau} - \frac{\partial^2\theta}{\partial\eta^2} = O(Ra) << 1 \qquad (7)$$

Obviously, the convective terms in the right-hand side of the above equation are of the order of Ra . Taking into account that the analysis of this first element begins with the block of smallest size, we assume that $H_0 << 1$. In addition, the other parameters involved in the definition of the Rayleigh number are not sufficient to dominate the influence of H_0, because $Ra \sim H_0^4$ and thus, $Ra << 1$. This is the argument for neglecting the convective terms, included in the energy equation. Then, the heat transfer process in the cavity, as a first approximation, is governed only by accumulation and diffusive terms, Eq. (7). This last equation must be solved with the following boundary and initial conditions:

$$\tau = 0 \quad : \quad \theta = 0\ ,$$

$$\eta = 0 \quad : \quad \frac{\partial\theta}{\partial\eta} = \frac{16\hat{H}_0}{9\kappa\bar{\alpha}^2\phi_0}\left[\kappa\bar{\alpha}\phi_0\hat{H}_0^2\left(1 - \exp^{\frac{8}{3\kappa\bar{\alpha}\phi_0\hat{H}_0^2}}\right) - \frac{8}{3}\right]^{-1}\theta,$$

$$\eta = \frac{\hat{H}_0}{2} : \quad \frac{\partial\theta}{\partial\eta} = \gamma\sin(\beta\tau) \quad ; \qquad (8)$$

where the dimensionless parameters γ and β are defined as $\gamma = A^{\frac{1}{2}}{}_0 q_0 / k_a T_0$ and $\beta = \omega A_0 / \alpha_a$. The solution of Eq. (7) together with the initial and boundary conditions Eq. (8) can be easily determined by the application of the Duhamel's theorem (Özisik, 1993, p. 692). This is given by the following expression,

$$\theta(\eta,\tau) = \gamma\left\{\eta + \frac{9\kappa\bar{\alpha}^2\phi_0}{16\hat{H}_0}\left[\kappa\bar{\alpha}\phi_0\hat{H}_0^2\left(1 - e^{\frac{8}{3\kappa\bar{\alpha}\phi_0\hat{H}_0^2}}\right) - \frac{8}{3}\right]\right\}\sin\beta\tau + \beta\gamma\sum_{m=1}^{\infty}\frac{\beta_m\cos\beta_m\left(\frac{\hat{H}_0}{2} - \eta\right)}{\beta_m\hat{H}_0 + \sin\left(\beta_m\hat{H}_0\right)} \times\left[\frac{\beta^2{}_m\cos\beta\tau + \beta\sin\beta\tau}{\beta^4{}_m + \beta^2} - \frac{\beta^2{}_m e^{-\beta^2_m\tau}}{\beta^4{}_m + \beta^2}\right] \times\left[\frac{1 - \left(\cos\beta_m\hat{H}_0/2 + \left(\beta_m\hat{H}_0/2\right)\sin\beta_m\hat{H}_0/2\right)}{\beta_m} - \frac{9\kappa\bar{\alpha}^2\phi_0}{16\hat{H}_0\beta_m}\left[\kappa\bar{\alpha}\phi_0\hat{H}_0^2\left(1 - e^{\frac{8}{3\kappa\bar{\alpha}\phi_0\hat{H}_0^2}}\right) - \frac{8}{3}\right]\sin\beta_m\hat{H}_0/2\right] \qquad (9)$$

In the above equation, the β_m's represent the characteristic or eigenvalues and they are determined by the following transcendental equation,

$$\beta_m \tan\left(\beta_m\frac{\hat{H}_0}{2}\right) = \frac{16\hat{H}_0}{9\kappa\bar{\alpha}^2\phi_0}\left[\kappa\bar{\alpha}\phi_0\hat{H}_0^2\left(1 - e^{\frac{8}{3\kappa\bar{\alpha}\phi_0\hat{H}_0^2}}\right) - \frac{8}{3}\right]^{-1} \qquad (10)$$

The details necessary to derive the above eigenvalues relationship are omitted for simplicity and can be found elsewhere (Özisik, 1993).

2.2. Results

The temperature profiles for the Newtonian fluid circulating into the tube are shown in Figures 2 and 3. In these figures, as illustration, we use a fixed value of $\hat{H}_0 = 0.7$. In both figures, we show the dimensionless temperature of the fluid θ_f versus the dimensionless coordinate χ for different values of dimensionless time, τ. Specifically, in Figure 2, we take the following values: $\phi_0 = 0.1$ and $\kappa = 100$. It can be appreciated that for increasing values of the dimensionless time τ and for an arbitrary and prescribed value of the dimensionless coordinate χ, the temperature is always oscillating. Specifically, for a time increment $\Delta\tau = 0.15$, the values for the temperature profile with $\tau = 0.2$ are practically the same values that the temperature profile has for $\tau = 0.05$. This behavior is a direct consequence of the periodic radiative heat transfer that occurs at the top of the cavity. On the other hand, in the same figure and for a fixed value of the dimensionless time τ, the dimensionless temperature of the fluid, θ_f, increases along the x axis. This result is a direct consequence of the heating process to the fluid.

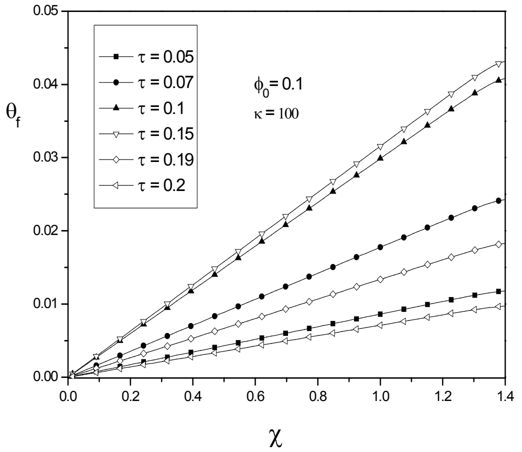

Figure 2. Dimensionless fluid temperature θ_f as a function of the coordinate χ and different values of the dimensionless time τ.

Meanwhile in Figure 3, we use the following values: $\phi_0 = 0.001$ and $\kappa = 100$. Now, the influence of the dimensionless parameter ϕ_0 is revealed. In this case, the periodic character of the temperature prevails; however, for the same increasing values of the dimensionless time τ, the corresponding values of the temperature profile are larger than for the case shown in Figure 2 only for values of $\tau < 0.1$. Otherwise, for $\tau \geq 0.1$ the temperature behavior is inverted. The above result can be interpreted in the following manner: for $\tau < 0.1$ and $\phi_0 = 0.001$ the volume of the heated fluid is reduced in comparison with the case of Figure 2, where $\phi_0 = 0.1$. It implies that higher values of dimensionless temperature are reached. Otherwise, for $\tau > 0.1$, the situation is inverted. In both cases it should be noted that the volume of the fluid region is given by the following relationship $V = \pi R_0^2 L_0 = (\pi H_0^2 L_0)\phi_0^2$. For fixed values of H_0 and L_0, the volume is a function only of the dimensionless variable ϕ_0. However, the cyclic character of the temperature is maintained due to that $\Delta\tau$ remains unaltered.

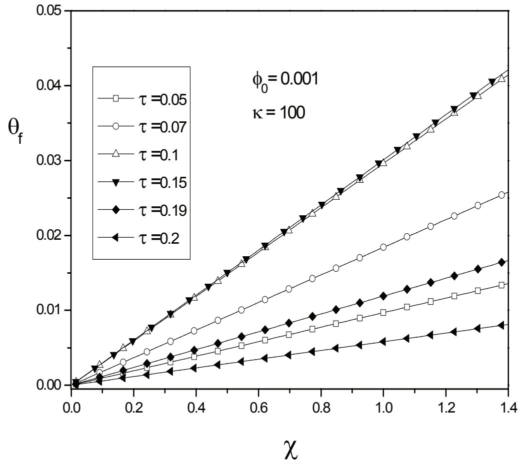

Figure 3. Dimensionless fluid temperature θ_f as a function of the coordinate χ and different values of the dimensionless time τ.

In Figures 4 and 5, we show the transient temperature profiles at the top of the cavity ($\eta = \widehat{H}_0 / 2$). Each figure is composed, in turn, by two figures a) and b). For instance, in Figure 4a) we have shown the transient temperature as a function of the dimensionless time τ for different values of the parameter κ and $\phi_0 = 0.1$. In this Figure, the heating-cooling process is evident: the dimensionless temperature increases for $\tau \leq 0.11$. For larger values than to $\tau \sim 0.11$, the process is inverted and the temperature begins to decrease up to complete a cycle when $\theta(\tau \sim 0.19) = 0$. On the other hand, the influence of the parameter κ on the transient temperature is clarified in Figure 4a). These results are shown in Figure 4b), exclusively for the cooling process, *i. e.*, for $\tau > 0.11$. Here, for increasing values of the parameter $\kappa = k_f / (k_a Pe)$, the dimensionless temperature of the air cavity increases also. The above indicates a logical result. Starting from the definition of the parameter κ, we can appreciate that for increasing values of this parameter there are two options: we can decrease the Peclet number, increase the ratio k_f / k_a or both. In any case, the use of high-thermal conductivity inserts (which means to increase the values of κ), here represented by the parameter κ, actually results in higher values of temperature. This is correct and can be seen in the following manner: $\kappa \gg 1$ is equivalent, for example, to take $Pe \ll 1$. In this limit, the fluid is practically at rest and the energy accumulation into the cavity is easier to reach; therefore, the temperature increases. In this form, the optimization process for this first element satisfies the criterion of the Constructal Theory: the inserts of high thermal conductivity for the cooling process serve just to modulate the heat transfer process. Obviously, similar considerations can be obtained

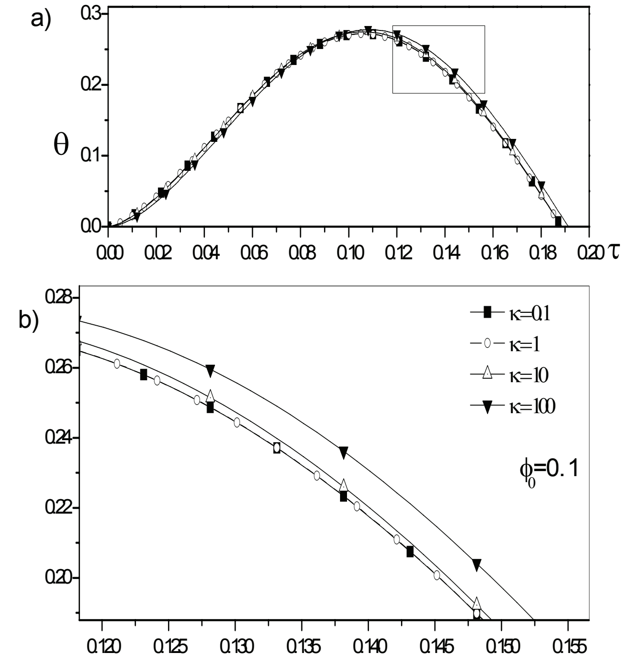

Figure 4. Dimensionless air temperature at $\eta = \widehat{H}_0 / 2$ as a function of the dimensionless time τ and different values of the Constructal or insert parameter κ. Case: a) a complete solution of the above transient evolution; b) detailed solution to reveal the influence of the parameter κ.

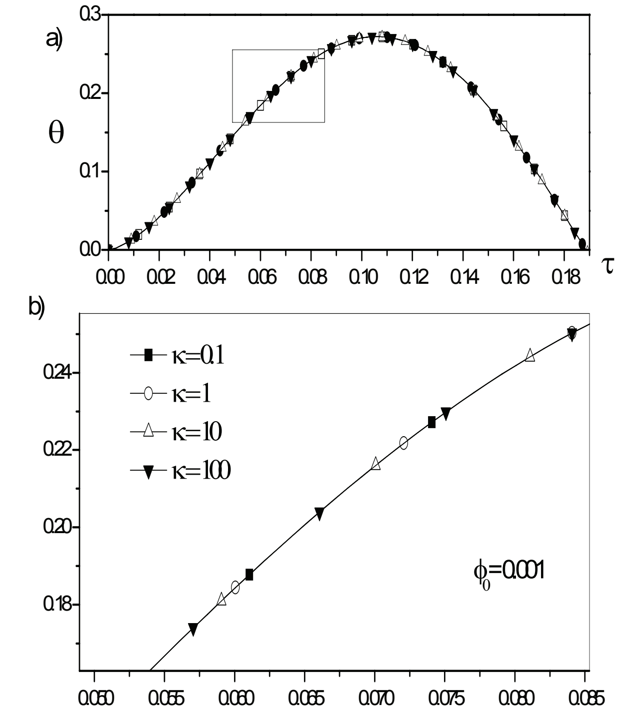

Figure 5. Dimensionless air temperature at $\eta = \widehat{H}_0 / 2$ as a function of the dimensionless time τ and different values of the constructal or insert parameter κ. a) a complete solution of the above transient evolution; b) detailed solution to reveal the influence of the parameter κ.

for the heating process. It is important to note that transient temperature profiles into the cavity for other positions such that $0 < \eta < \widehat{H}_0 / 2$ present similar results and are not shown here. In Figure 5 we show, again, the transient temperature as a function of the dimensionless time τ for different values of the parameter κ; however, now $\phi_0 = 0.001$. In Figure 5a) a similar behavior appears and comparing quantitatively with Figure 4a), the transient temperature distributions are practically the same. In addition, from Figure 5b) it is evident that the influence of the dimensionless parameter κ in this case is insignificant; however, this is a direct consequence of the assumed value of ϕ_0. Therefore, for both Figures 4 and 5, we can say that the optimization for this first element enables that the inserts of high thermal conductivity improve the heat transfer process from the top of the cavity to the surface of the tube.

The above results are completed with Figures 6 and 7, adding the corresponding transient temperature profiles of the fluid. Again, each figure is separated in two. For instance, in Figure 6a) we show the transient temperature of the fluid evaluated only at $\chi = 1/\hat{H}_0$, that is, at the end of the tube where the fluid has accumulated the largest amount of energy due to solar radiation. In this figure, we can appreciate that the influence of the parameter κ is very sensitive: for increasing values of this parameter, the temperature increases abruptly. Again, the influence of the physical characteristics of the insert, here represented by the dimensionless parameter κ, is evident and is consistent with the results of Figure 4. In addition, we must remember that the value of the fluid temperature at $\chi = 1/\hat{H}_0$ is not

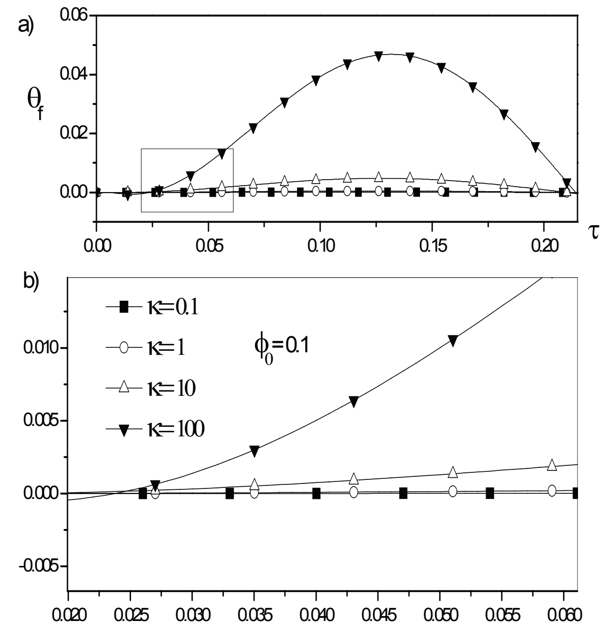

Figure 6. Dimensionless fluid temperature at $\chi = 1/\hat{H}_0$ as a function of the dimensionless time τ and different values of the constructal or insert parameter κ. a) a complete solution of the above transient evolution; b) detailed solution to reveal the influence of the constructal or insert parameter κ.

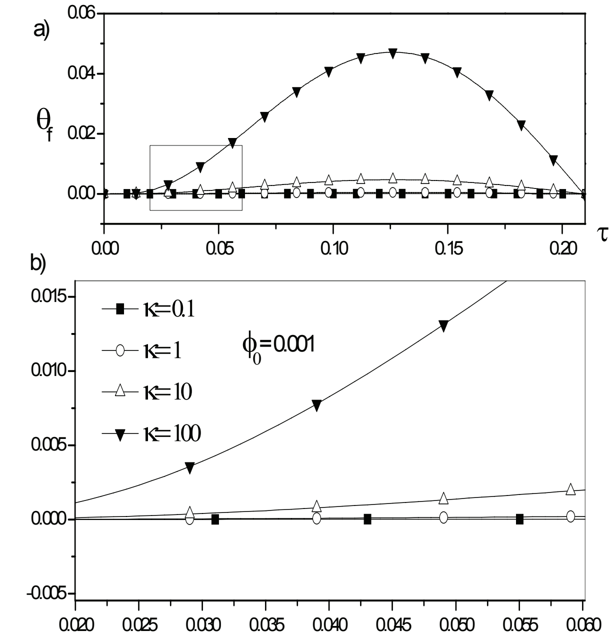

Figure 7. *Dimensionless fluid temperature at* $\chi = 1/\hat{H}_0$ *as a function of the dimensionless time* τ *and different values of the constructal or insert parameter* κ *. a) a complete solution of the above transient evolution; b) detailed solution to reveal the influence of the constructal or insert parameter* κ .

known in advance because we use as a boundary condition for this coordinate the well-known adiabatic condition given by Eq. (4). Therefore, the results presented in these figures serve as a thermal design criterion to reach the largest values of the fluid temperature for this first constructal element. The above figure is complemented with Figure 6b) by showing a detail of the curve in order to clarify the role of the parameter κ . In addition, for both figures we use the value of $\phi_0 = 0.1$. In comparison, in Figures 7a) and 7b) we present similar results for the case of $\phi_0 = 0.001$. However, in this case the influence of the parameter ϕ_0 is practically irrelevant because the physical behavior of the fluid temperature is similar to Figures 6a) and 6b).

Finally in Figure 8, we predict the cooling or heating time for different values of the dimensionless constructal parameter $\hat{H}_0$, including the corresponding minimum values of the optimal time. Again, we present results for two different values of the dimensionless parameter ϕ_0; that is, $\phi_0 = 0.001$ and $\phi_0 = 0.1$ .The results are clear: for increasing values of the parameter ϕ_0, the value of the minimum time is drastically reduced indicating a favorable heating or cooling process. In addition, we assign a pre-selected value of θ, given by Eq. (8), such that this value is extremely small, i.e., $\theta = \varepsilon_1 = 0.0001$. The foregoing procedure to find the optimum time of heating or cooling for cyclical conditions is very simple: we are numerically finding the values of $\hat{H}_0$ that make the temperature θ at the top of the cavity, i. e., $\theta_{top} = \theta(\hat{H}_0, \tau)$, tend to zero. This means that the temperature in physical units, T_a , again reaches the initial temperature T_0 only for specific values of $\hat{H}_0$. From the set of points that define the plots of Figure 8, in particular, there is only a value of $\hat{H}_0$ for which the optimal time has a minimum value. Mathematically, this is equivalent to finding the roots of the relationship between τ and $\hat{H}_0$. This criterion can be found elsewhere [the details are in Dan & Bejan (1998)].

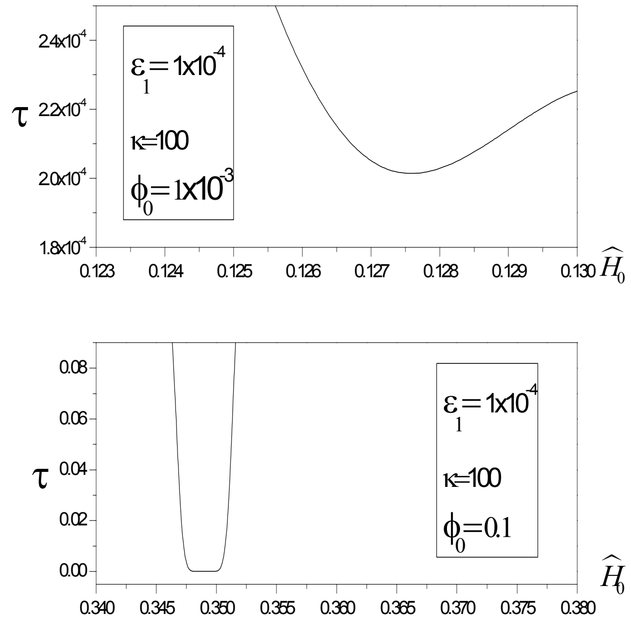

Figure 8. *Dimensionless heating or cooling time* τ *as a function of the dimensionless constructal parameter* $\hat{H}_0$ *and two different values of the dimensionless parameter* ϕ_0

3. Conclusions

In the present work, we have developed the fundamentals for deriving the optimal and basic characteristics of the heat transfer for the first element or construction, according to the Constructal Theory. It is well-known in this theory that using an element or insert of high thermal conductivity is sufficient to improve the heat transfer process. This affirmation was proved by Dan & Bejan (1998) when the constructal element has an insert with high thermal conductivity. However, in the present work, we have extended this hypothesis considering a constructal or insert element with a “high thermal conductivity” based on the convective transport and represented here with the parameter κ . The novel part of the present manuscript is based on the use of an oscillatory boundary condition. This boundary condition, here represented by the radiative signal $q = q_0 sin(\omega t)$, has not been previously considered in Constructal Theory, although the practical interest for those cases for which a dependence on time of the boundary conditions are inevitable. This is the case of the solar radiation. The present mathematical model offers a set of multiple solutions due to the dimensionless parameters $\gamma, \beta, \hat{H}_0, Pe$ and ε_1 . Obviously in the present work, we have chosen arbitrary values for these parameters; however, the set of solutions can be wider. Perhaps, the most relevant conclusion is addressed to the existence of a minimum value for the cooling time. Therefore, we consider that even with the inclusion of periodic boundary conditions, the essential characteristics of the Constructal Theory remains unaltered. In addition, we appreciate that the oscillatory effect of the solar reception yields the cooling or heating of the first construction element.

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Nomenclature

A_0	transverse area of the constructal element or insert (m^2)
H_0	height of the constructal element or insert (m)
$\widehat{H}_0$	dimensionless parameter
k_a	thermal conductivity of the air (W/mK)
k_f	thermal conductivity of the fluid (W/mK)
L_0	length of the tube (m)
Pe	Péclet number defined after Eq. (2)
q_0	characteristic heat flux (W/m^2)
q	periodic heat flux (W/m^2)
R_0	radius of the tube (m)
Ra	Rayleigh number defined after Eq. (5)
t	physical time (sec)
T_0	initial temperature of the constructal element (K)
T_f	temperature of the fluid (K)
T_a	temperature of the air cavity(K)
U_c, V_c	characteristic velocities defined in Eqs. (5), (m/sec)
U_f	characteristic velocity of the fluid (m/sec)
x, y	Cartesian coordinates (m)

Greek symbols

α_a	thermal diffusivity of the air (m^2/sec)
α_f	thermal diffusivity of the fluid (m^2/sec)
$\bar{\alpha}$	dimensionless parameter defined after Eq. (2)
β	dimensionless parameter defined after Eq. (7)
β_{th}	thermal coefficient defined after Eq. (5)
$\tilde{\beta}$	dimensionless parameter defined after Eq. (2)
β_m	eigenvalues characteristic defined in Eq. (9)
ε_1	small parameter needed to find the relationship between τ and $\widehat{H}_0$, see Eq. (8)
γ	dimensionless parameter defined after Eq. (7)
ϕ_0	dimensionless parameter defined after Eq. (2)
κ	dimensionless parameter defined after Eq. (2)
η	dimensionless coordinate defined in Eq. (1)
ν_a	kinematic viscosity of the air defined after Eq. (5)
τ	dimensionless time defined in Eq. (1)
θ	dimensionless air temperature defined in Eq. (1)
θ_f	dimensionless fluid temperature defined in Eq. (1)
ω	frequency of the solar signal (sec^{-1})
χ	dimensionless longitudinal coordinate defined in Eq. (1)

Subscripts

a	indicates air properties
f	indicates fluid properties
0	indicates initial conditions or characteristic values of the lengths H_0 and L_0 of the area A_0; and of the dimensionless parameter $\widehat{H}_0$

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