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An Equivalent Mechanical Model for Representing the Entropy Generation in Heat Exchangers. Application to Power Cycles

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Abstract

One of the most common difficulties students face in learning Thermodynamics lies in grasping the physical meaning of concepts such as lost availability and entropy generation. This explains the quest for new approaches for explaining and comprehending these quantities, as suggested by diagrams from different authors. The difficulties worsen in the case of irreversibilities associated with heat transfer processes driven by a finite temperature difference, where no work transfer takes place. An equivalent mechanical model is proposed in this paper. Heat exchangers are modelled by means of Carnot heat engines, heat pumps and mechanical transmissions; the use of mechanical models allows a thorough visualization of exergy quantities involved in heat transfer processes. The proposed model is further applied to a power cycle, thus obtaining an "equivalent arrangement" where irreversibilities become understood by means of a mechanical analogy.

Keywords: *Entropy generation, Irreversibility, Heat exchanger.*

1. Introduction

The concept of irreversibility, or exergy destroyed, is often difficult to understand, particularly if there is no work transfer involved. Many authors (Bejan 1997, Branco et al. 2002) have developed diagrams in order to illustrate its physical meaning. In some texts (Bejan et al. 1996) can be found a split of different causes of irreversibilities in heat exchangers, even including leakages to the environment. Other studies (Bejan 1996, White and Shamusundar 1983) define certain non-dimensional numbers related to entropy generation. This paper proposes a model equivalent to the real process where Carnot machines are used to represent fictitious work transfer processes which help in interpreting the physical meaning of lost availability.

Figure 1 represents a heat exchanger. Heat losses are neglected and steady state is considered. Homogeneous fluid temperatures are assumed. An application of the Second Law to both currents and to the exchanger considered as a whole yields

$$\dot{S}_{gen,tot} = \dot{S}_{gen,h} + \dot{S}_{gen,c} + \dot{Q}\left(\frac{1}{\overline{T}_c} - \frac{1}{\overline{T}_h}\right) \quad (1)$$

where the Average Entropic Temperature (Caputa 1967) is defined by

$$\overline{T} = \frac{Q_{io}}{\int_i^o \frac{\delta Q}{T}} \quad (2)$$

Eqn. (1) shows that the entropy generation in the heat exchanger assembly has two components: entropy generation inside the ducts, caused by friction losses (the two first terms in the right-hand side of the equation) and entropy generation caused by the heat transfer through a finite temperature difference. This latter component is usually difficult to understand, mainly because it is not intuitively related to a loss in work capability. This paper will deal with this component in particular.

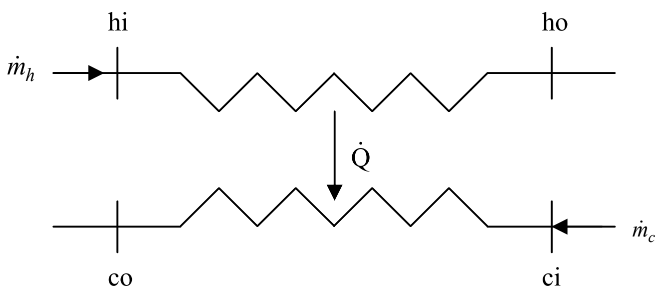


Figure 1. Conceptual sketch of a heat exchanger.

2. Methodology

Figure 2 shows the model used to represent the heat exchanger sketched in Figure 1. Mass sources A and C supply the fluid stream entering the heat exchanger; mass sinks B and D receive fluid streams leaving the heat exchanger. A heat sink at temperature T_o is used to represent the environment. Two Carnot machines have been incorporated: engine CE and heat pump CHP, connected by means of transmission T (a simple gearbox).

The Second Law applied to both Carnot machines yields:

$$\dot{W}_1 = \dot{Q}\left(1 - \frac{T_o}{\overline{T}_H}\right) \quad (3a)$$

$$\dot{W}_2 = \frac{\dot{Q}}{\overline{T}_L}\left(\overline{T}_L - T_o\right) \quad (3b)$$

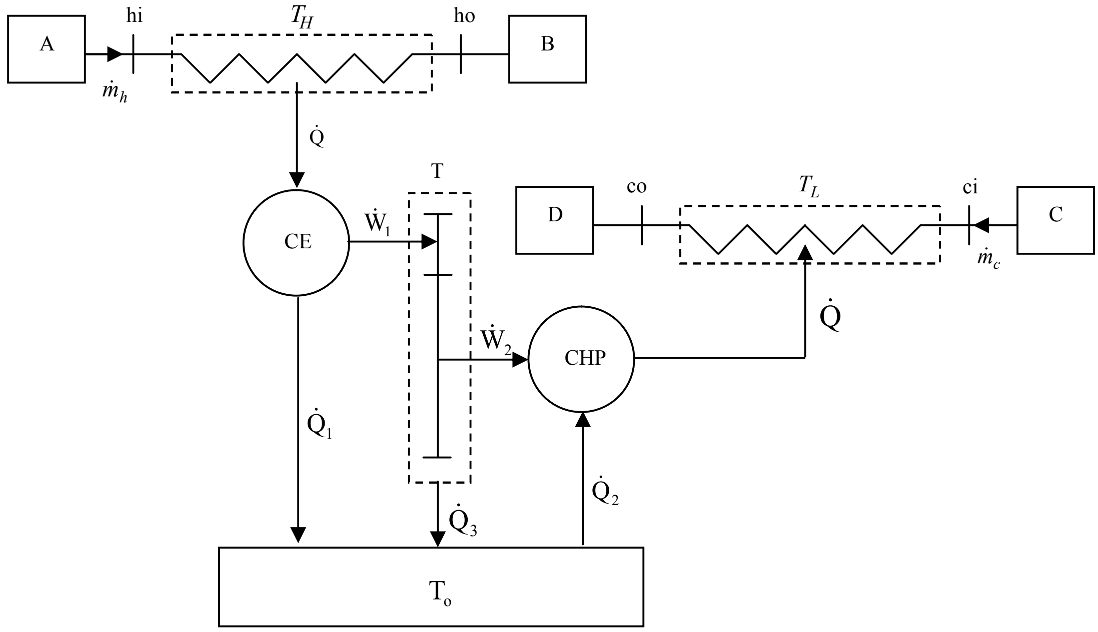


Figure 2. Equivalent model of a heat exchanger.

$$\dot{Q}_1 = T_o \frac{\dot{Q}}{\overline{T}_H} \tag{4a}$$

$$\dot{Q}_2 = T_o \frac{\dot{Q}}{\overline{T}_L} \tag{4b}$$

From First Law applied to the transmission T it follows:

$$\dot{Q}_3 = \dot{W}_1 - \dot{W}_2 = \dot{Q}\, T_o \left(\frac{1}{\overline{T}_L} - \frac{1}{\overline{T}_H} \right) \tag{5}$$

Eqn. (5) shows that heat dissipation in the transmission (in the equivalent model) represents the irreversibility associated with heat transfer (in the actual heat exchanger) from the hot fluid to the cold fluid. Average temperatures for both fluids have been assumed. It is also possible to show that the transmission efficiency is identical with the Second-Law efficiency of the heat exchanger:

$$\eta_T = \frac{\dot{W}_2}{\dot{W}_1} = \frac{\dot{Q}\left(1 - \frac{T_o}{\overline{T}_L}\right)}{\dot{Q}\left(1 - \frac{T_o}{\overline{T}_H}\right)} = \varepsilon \tag{6}$$

In order that the proposed model can be considered equivalent to the actual heat exchanger, it follows that the environment in the equivalent model must not be altered. Thus, from Eqns. (4) and (5):

$$\dot{Q}_1 + \dot{Q}_3 = \dot{Q}_2 \tag{7}$$

Furthermore, the entropy increase of the Universe can be calculated by adding together the terms of entropy generation for each of the subsystems, as shown in Eqn. (8), which also reveals that the irreversibilities caused by fluid flow have been taken into account.

$$\begin{aligned} \frac{dS_u}{dt} &= \dot{S}_{gen,h} + \dot{S}_{gen,c} + \dot{S}_{gen,T} \\ &= \underbrace{\frac{\dot{Q}}{\overline{T}_H} + \dot{m}_h\left(s_{ho} - s_{hi}\right)}_{stream\ "h"} + \underbrace{\dot{m}_c\left(s_{co} - s_{ci}\right) - \frac{\dot{Q}}{\overline{T}_L}}_{stream\ "c"} + \underbrace{\frac{\dot{Q}_3}{T_o}}_{T} \end{aligned} \tag{8}$$

Eqns. (5) and (6) show the relevance of the transmission in the model and the necessity of including the environment and the Carnot heat pump in order to make all the exergy concepts evident: exergy flux due to heat transfer in both streams (mechanical power at both sides of the transmission), Second-Law efficiency of the heat exchanger (transmission efficiency) and exergy destroyed due to heat transfer (energy dissipated by transmission). In the case where the heat was released to the environment the transmission could be transformed into a brake and the Carnot heat pump could be dispensed with. This special case will be dealt with in Section 3.3 (Figure 6) to model external irreversibilities between one power cycle and the environment .

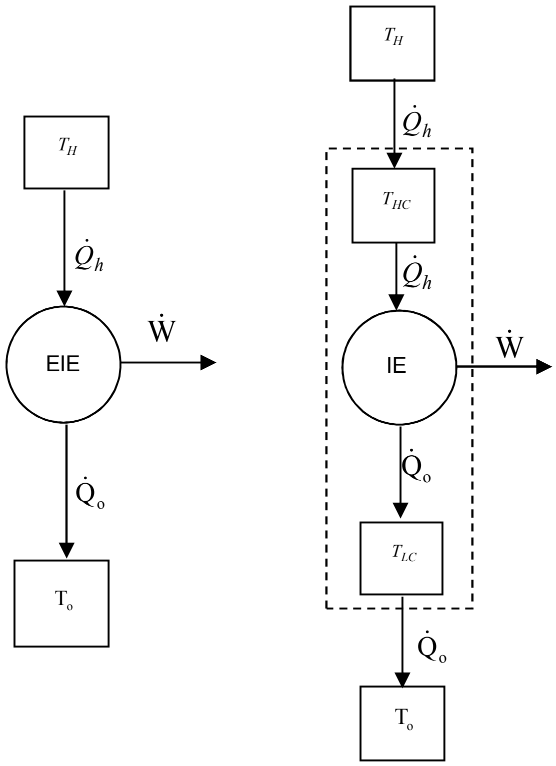
3. Results. Application to power cycles

The proposed model can be generalized and applied to power cycles, such as that sketched in Figure 3. Part (a) represents a power cycle with both external and internal irreversibilities (Externally and Internally irreversible Engine); external irreversibilities have been taken out of the cycle in part (b), so that IE (Internally irreversible Engine) is an externally reversible cycle, which takes in heat at an average entropic temperature T_{HC} and rejects it at T_{LC} . All components enclosed within the dotted line in part (b) are equivalent to the EIE heat engine in part (a).

The Second Law applied to Figure 3a yields:

$$\dot{S}_{gen,tot} = \frac{-\dot{Q}_h}{T_H} + \frac{\dot{Q}_o}{T_o} = \dot{Q}_h \left(\frac{1}{T_o} - \frac{1}{T_H} \right) - \frac{\dot{W}}{T_o} \tag{9a}$$

$$\dot{I}_{tot} = T_o\, \dot{S}_{gen,tot} = \dot{Q}_h \left(1 - \frac{T_o}{T_H} \right) - \dot{W} \tag{9b}$$



(a) (b)

Figure 3. Conceptual sketch of a power cycle

whereas the Second Law applied to Figure 3b yields:

$$\dot{S}_{gen,tot} = \frac{-\dot{Q}_h}{T_H} + \frac{\dot{Q}_o}{T_o} \tag{10a}$$

$$\frac{dS_{EIE}}{dt} = \frac{\dot{Q}_h}{\overline{T}_{HC}} - \frac{\dot{Q}_o}{\overline{T}_{LC}} + \dot{S}_{gen,\text{int}} = 0 \tag{10b}$$

$$\dot{S}_{gen,tot} = \frac{-\dot{Q}_h}{T_H} + \frac{\dot{Q}_h}{\overline{T}_{HC}} - \frac{\dot{Q}_o}{\overline{T}_{LC}} + \dot{S}_{gen,\text{int}} + \frac{\dot{Q}_o}{T_o} \tag{10c}$$

$$\begin{aligned} \dot{I}_{tot} &= T_o\,\dot{S}_{gen,tot} \\ &= \underbrace{T_o\,\dot{Q}_h\left(\frac{1}{\overline{T}_{HC}} - \frac{1}{T_H}\right) + T_o\,\dot{Q}_o\left(\frac{1}{T_o} - \frac{1}{\overline{T}_{LC}}\right)}_{\dot{I}_{ext}} + \\ &\quad + \underbrace{T_o\,\dot{S}_{gen,\text{int}}}_{\dot{I}_{\text{int}}} \end{aligned} \tag{11}$$

Eqn. (11) shows that there are irreversibilities associated with the heat transfer between the heat sinks and the engine. No other irreversibilities can however be visualized with the proposed equivalent model. As a result, the proposed model must be enhanced in order that internal irreversibilities in the engine can be made evident.

Following the configuration sketched in Figure 3b, three submodels are set up: external irreversibilities between the hot heat sink and the engine, external irreversibilities between the engine and the environment, and internal irreversibilities in the engine.

3.1. External irreversibilities between the hot heat sink and the engine

Figure 4 depicts the equivalent model, identical with that shown in Figure 2. Eqn. (5), applied to the transmission shown in Figure 4, yields Eqn. (12) which gives the heat dissipated in the transmission (equivalent model) and at the same time represents the external irreversibilities caused by the heat transfer between the hot heat sink and the actual power cycle. The mechanical efficiency of the transmission represents the Second-Law efficiency associated with the heat exchange between the hot heat sink and the engine. This model also demonstrates that there is no change in the extensive properties of the environment, since the heat exchange with the environment is null.

$$\dot{Q}_2 = \dot{Q}_h\,T_o\left(\frac{1}{\overline{T}_{HC}} - \frac{1}{T_H}\right) \tag{12}$$

3.2. Internal irreversibilities in the engine

Figure 5 represents the model used, similar to that in Figura 4, except for the work supplied by the engine. The Second Law, applied to both Carnot heat engines, yields:

$$\dot{W} + \dot{W}_3 = \dot{Q}_h\left(1 - \frac{T_o}{\overline{T}_{HC}}\right) \Leftrightarrow \dot{W}_3 = \dot{Q}_h\left(1 - \frac{T_o}{\overline{T}_{HC}}\right) - \dot{W} \tag{13a}$$

$$\dot{W}_4 = \frac{\left(\dot{Q}_h - \dot{W}\right)}{\overline{T}_{LC}}\left(\overline{T}_{LC} - T_o\right) \tag{13b}$$

$$\dot{Q}_4 = T_o\,\frac{\dot{Q}_h}{\overline{T}_{HC}} \tag{14a}$$

$$\dot{Q}_6 = T_o\,\frac{\dot{Q}_h - \dot{W}}{\overline{T}_{LC}} \tag{14b}$$

The First Law, applied to the transmission T, yields:

$$\dot{Q}_5 = \dot{W}_3 - \dot{W}_4 = \dot{Q}_h\,T_o\left(\frac{1}{\overline{T}_{LC}} - \frac{1}{\overline{T}_{HC}}\right) - \dot{W}\,\frac{T_o}{\overline{T}_{LC}} \tag{15}$$

Eqn. (15) represents the heat dissipation in the transmission (in the equivalent model), which in turn represents the internal irreversibilities (in the actual engine). Should the transmission be ideal ($\dot{Q}_5 = 0$), the work supplied by the power cycle would be that of an internally reversible cycle. Since in this case a work transfer exists, the interpretation of the mechanical efficiency of the transmission as the Second-Law efficiency of the whole device, Eqn. (16), becomes somewhat more complicated. This is due to the fact that the equivalent model was originally proposed for a heat exchanger. Eqn. (16) can be interpreted in terms of availability as the ratio between the availability associated with the heat leaving the model in Figure 5 and the availability associated with the heat leaving the model minus the work output supplied by the model. That is, the model interprets the heat supplied to the cold heat sink (at $\overline{T}_{LC}$) as a product, and the availability associated with the heat transferred and the work output as

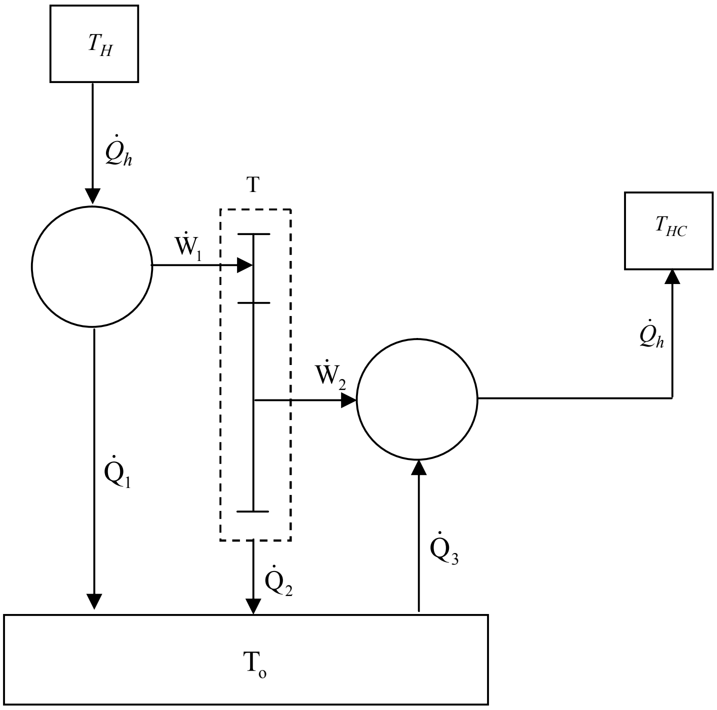


Figure 4. Equivalent model representing external irreversibilities caused by the hot heat sink

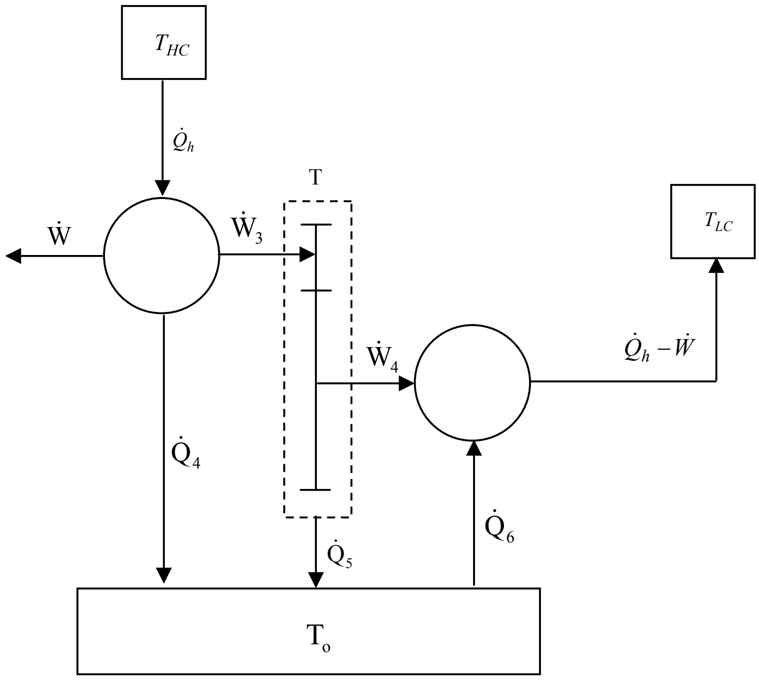


Figure 5. Equivalent model representing internal irreversibilities in the engine

a resource. Finally, it can be shown that there is no overall change in the extensive properties of the environment.

$$\eta_T = \frac{\dot{W}_4}{\dot{W}_3} = \frac{\left(\dot{Q}_h - \dot{W}\right)\left(1 - \frac{T_o}{\overline{T}_{LC}}\right)}{\dot{Q}_h\left(1 - \frac{T_o}{\overline{T}_{HC}}\right) - \dot{W}} \tag{16}$$

3.3. External irreversibilities between the engine and the environment

Figure 6 represents the equivalent model. The Second Law, applied to both Carnot heat engines, yields:

$$\dot{W}_5 = \left(\dot{Q}_h - \dot{W}\right)\left(1 - \frac{T_o}{\overline{T}_{LC}}\right) \tag{17}$$

$$\dot{Q}_7 = T_o \frac{\dot{Q}_h - \dot{W}}{\overline{T}_{LC}} \tag{18}$$

The transmission has been replaced by brake B in this case, so that all the work produced is dissipated in the form of heat to the environment. The First Law applied to brake B yields:

$$\dot{Q}_8 = \left(\dot{Q}_h - \dot{W}\right)\left(1 - \frac{T_o}{\bar{T}_{LC}}\right) = \left(\dot{Q}_h - \dot{W}\right)T_o\left(\frac{1}{T_o} - \frac{1}{\bar{T}_{LC}}\right) \qquad (19)$$

Eqn. (19) gives the heat dissipated in the brake (in the equivalent model), which represents the external irreversibilities associated with the heat transferred to the environment, in the same way as Eqn. (5) did in the model of the heat exchanger. If the brake were to be interpreted as a transmission, its efficiency would be null, as would be the Second-Law efficiency of the model, since all heat produced is taken by the environment. In this case there is a heat exchange with the environment, which equals that exchanged by the power cycle:

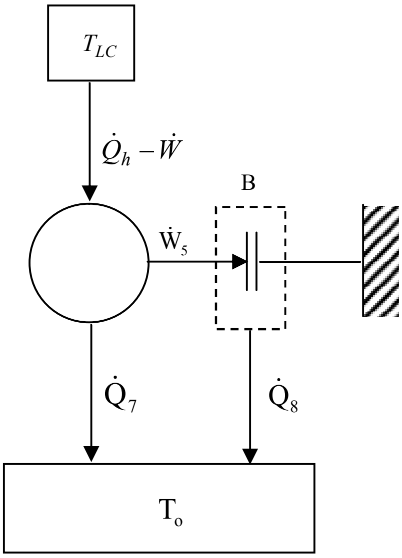


Figure 6. Equivalent model representing external irreversibilities with the environment

$$\dot{Q}_{env} = \dot{Q}_7 + \dot{Q}_8 = \dot{Q}_h - \dot{W} \qquad (20)$$

The overall irreversibility for the power cycle can be calculated through the equivalent model. This can be made by calculating the entropy increase of the Universe, by adding together the entropy increase for each component:

$$\begin{aligned}\frac{dS_u}{dt} &= \frac{dS_H}{dt} + \frac{dS_{IE}}{dt} + \frac{dS_o}{dt} \\ &= \underbrace{\frac{-\dot{Q}_h}{T_H} + \frac{\dot{Q}_h}{\bar{T}_{HC}}}_{hot\ sink} \underbrace{- \frac{\dot{Q}_h}{\bar{T}_{HC}} + \frac{\dot{Q}_h - \dot{W}}{\bar{T}_{LC}}}_{extern.\ rev.\ engine} - \\ &\quad \underbrace{- \frac{\dot{Q}_h - \dot{W}}{\bar{T}_{LC}} + \frac{\dot{Q}_h - \dot{W}}{T_o}}_{environment} = \frac{\dot{Q}_h}{T_o}\left(1 - \frac{T_o}{T_H}\right) - \frac{\dot{W}}{T_o}\end{aligned} \qquad (21)$$

Alternatively, if calculation is made by adding the irreversibilities in each model:

$$\begin{aligned}\dot{I}_{tot} &= \dot{Q}_2 + \dot{Q}_5 + \dot{Q}_8 = \underbrace{\dot{Q}_h T_o\left(\frac{1}{\bar{T}_{HC}} - \frac{1}{T_H}\right)}_{hot\ sink} \\ &\quad \underbrace{+ \dot{Q}_h T_o\left(\frac{1}{\bar{T}_{LC}} - \frac{1}{\bar{T}_{HC}}\right) - \dot{W}\frac{T_o}{\bar{T}_{LC}}}_{extern.\ rev.\ engine} + \underbrace{\left(\dot{Q}_h - \dot{W}\right)T_o\left(\frac{1}{T_o} - \frac{1}{\bar{T}_{LC}}\right)}_{environment} \\ &= \dot{Q}_h\left(1 - \frac{T_o}{T_H}\right) - \dot{W}\end{aligned} \qquad (22)$$

It can be shown that Eqn. (21) times the environment temperature is identical with Eqn. (22), and both coincide with Eqn. (9).

4. Conclusions

An equivalent mechanical model for a heat exchanger has been proposed, in order to represent the irreversibilities associated with heat transfer. This model has been extended to a power cycle, and can represent both external and internal irreversibilities. Irreversibility associated with heat transfer in the actual system is related to a mechanical energy degradation in the equivalent model. This approach makes for a clearer interpretation of its effects, allowing the student to visualize irreversibility as a mechanical loss in an understandable way. Furthermore, the proposed model leads to a better understanding of the concept of exergy destroyed, which is sometimes difficult to grasp when it cannot be traced to other phenomena which students are more familiar with, such as mechanical friction and losses in a gearbox.

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