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Exergetic Optimization of a Refrigeration Cycle for Re-Liquefaction of LNG Boil-Off Gas

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Abstract

The development of a liquefaction process for liquefied natural gas boil-off re-liquefaction plants will be addressed to provide an environmentally friendly and cost effective solution for gas transport. Onboard boil-off gas (BOG) re-liquefaction is a new technology that liquefies BOG and returns it to the cargo tanks instead of burning it. Exergetic efficiency optimization for a cryogenic refrigeration cycle for re-liquefaction of LNG boil-off gas is performed. Thermodynamic modeling has been performed based on energy and exergy analyses. Objective problem is developed based on maximization of the plant exergetic efficiency and selected decision variables and constraints. Optimization process is performed using MATLAB genetic algorithm optimization toolbox. The results of exergetic efficiency optimization are compared with the corresponding results of the base case system obtained in the previous study. Finally, effects of some operating parameters on the exergetic efficiency are discussed by sensitivity analysis.

Keywords: *Liquefied Natural Gas; boil-off gas (BOG); exergetic analysis; optimization, refrigeration; genetic algorithm; sensitivity analysis.*

1. Introduction

Recently, there has been a significant increase in the level of interest in environmentally friendly and economically solutions for the transport of Liquefied Natural Gas (LNG). A steam turbine and the boil-off gas (BOG) have driven LNG carriers from the LNG cargo. The high consumption of the steam turbine as compared to last generation diesel engines in addition to environmental problems finally will motivate their replacement. Alternative systems such as the diesel engine are equipped on the LNG carriers for better fuel economy. Instead of the common application of using the boil-off gas as fuel, the LNG BOG re-liquefaction system provides a solution to liquefy the boil-off gas back to the cargo tanks. The LNG re-liquefaction system has merit in the large savings in the total fuel consumption and improved propulsion redundancy.

The reverse Brayton refrigeration cycle is widely used as the LNG re-liquefaction plant. Many important works about Brayton power or refrigeration cycles have been published in recent years. De Vos (1985) investigated the efficiency of some heat engines at maximum power conditions. Bejan (1989) built the theory of heat transfer-irreversible refrigeration plants. Sahin, Kodal, Yilmaz, & Yavuz (1996) analyzed the maximum power density of an irreversible Joule–Brayton engine. Chen, Sun, Wu, & Kiang (1997) analyzed the performance of a regenerative closed Brayton cycle. Chen, Wu, & Sun (1997) analyzed the cooling load and COP performance of regenerated Brayton refrigeration. Cheng & Chen (1997) determined the maximum power output and the corresponding thermal efficiency for an irreversible closed-cycle Brayton heat engine. Later, Cheng & Chen (1998) calculated the maximum thermal efficiency and the corresponding power output for the same system. Chen, Wu, & Sun (1998) analyzed the cooling load and COP performance of an irreversible simple Brayton cycle. Sahin, Kodal, & Kaya (1998) analyzed a comparative performance of irreversible regenerative reheating Joule–Brayton engines. Chen, Ni, Wu, & Sun (1999) analyzed the performance of a closed regenerated Brayton heat pump with internal irreversibilities via methods of entropy generation minimization. Huang, Hung & Chen (2000) carried out an exergy analysis based on an ecological optimization criterion for an irreversible Brayton engine with an external heat source. Luo, Chen, Sun & Wu (2002) optimized the cooling load and COP performance of an irreversible simple Brayton refrigeration cycle coupled to constant-temperature heat reservoirs.

Dimopoulos & Frangopoulos (2008) present the thermoeconomic model of the cryogenic liquefaction of a LNG vessel. In this work the authors investigated the LNG boil-off gas cryogenic system from an economic and thermodynamic view together. In fact, the thermoeconomic model was subsequently used for synthesis, design and operation optimization of the system.

In the case of the proposed reverse Brayton cycle in this paper, the turbo-expander is used as the main component to remove energy from the nitrogen gas stream. However, the Refrigeration system has two expansion units, a turbo expander and an expansion valve. Higher system efficiency is achieved by using the expander and lower temperature is obtained by using the expansion valve. The refrigeration cycle is analyzed based on energy and exergy rules. Finally, the objective function, exergetic efficiency, is maximized using a stochastic/deterministic approach namely as genetic algorithm. The thermodynamic features of the base case and optimized system are compared. Finally, the sensitivity of the exergetic efficiency to operating parameters of the system is reviewed. It is required to mention that the optimization in this paper with the aim of maximizing the exergetic efficiency of the proposed cycle has been performed. The economic feature of the optimized system is beyond the scope of this work; however, the authors performed thermoeconomic optimization of this cycle in another work (Sayyaadi & Babaelahi, 2010).

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2. Modeling

2.1 Description of the proposed cycle

The design of the LNG boil-off gas re-liquefaction plant has been performed based on the nominal LNG boil-off gas rate (BOR) of 0.15% per day for a cargo capacity of 220,000 m^3 (Full tank filling) LNG carriers. The characteristics of the BOG are assumed 100 percent methane and a boil-off gas rate of 5640 kg/hr. A schematic diagram of the proposed BOG cycle and enthalpy-entropy diagram are shown in Figs.1 and 2.

First, the nitrogen gas (stream 10) is compressed in a 3-stage compressor (c1, c2, c3) and then passes through a heat exchanger (H-E1). A portion of nitrogen gas (stream 9) is then bifurcated from the main stream (stream 2), expanded through an expander, and reunited with the return stream below the second heat exchanger (H-E2). The stream to be supplied to the BOG condenser (stream 5) continues through the second (H-E2) and third (H-E3) heat exchangers and is finally expanded through an expansion valve to the BOG condenser. In the BOG condenser, the cold nitrogen stream (stream 5) undergoes heat exchange with the BOG (stream b) and liquefies the BOG into liquid LNG. The cold nitrogen vapor (stream 6) from the BOG condenser is returned through the heat exchangers in order to cool the incoming gas (streams 1, 2, 3). To clarify the system design, the inlet temperature of the 3-stage compressor (c1, c2, c3) is set to 35ºC. The lower pressure of the nitrogen cycle (stream 10) is set to 14 bars. The pressure drop through each heat exchanger (HE1, HE2, and HE3) and intercooler (I-C1, I-C2, and I-C3) are fixed to 0.1 bars. The adiabatic efficiency of the turbo expander and the compressor of the nitrogen cycle (c1, c2 and c3) are evenly set to 0.7. In the BOG cycle, the inlet temperature of the BOG to the compressor 4 (c4) is adjusted to -120ºC, the output temperature of BOG in the condenser is set to -161ºC (stream c), the pressure drop in the condenser is set to 0.03 bars, and the adiabatic efficiency of the compressor 4 (c4) is assumed to 0.7.

2.2. Decision variables and constraints

Energy and exergy evolution are done based on decision and input variables. In this paper the decision variables consist of:

- pinch temperature difference in heat exchanger 1 (ΔT_{H-E1})
- pinch temperature difference in heat exchanger 2 (ΔT_{H-E2})
- pinch temperature difference in heat exchanger 3 (ΔT_{H-E3})
- pinch temperature difference in intercoolers ($\Delta T_{in-cool}$)
- pinch temperature difference in condenser (ΔT_{con})
- ratio of the expander mass flow rate to the total flow rate through the turbo-expander ($x_{\exp}$)

Although the decision variables may be varied in the optimization procedure, each decision variable is normally required to be within a given range as follow:

$$\Delta T_{H-E1} > 5$$

$$\Delta T_{H-E2} > 5$$

$$\Delta T_{H-E3} > 5$$

$$\Delta T_{in-cool} > 5$$

$$\Delta T_{con} > 5$$

$$0.1 < x_{\exp} < 0.7$$

These ranges are defined by economic or thermodynamic constrains. Reference values are set to $P_0 = 0.1043\,MPa$ and $T_0 = 298.15\,K$.

2.3. Energy analysis

Steady state energy balance equations for each component can be written in terms of specific enthalpy:

$$\sum_{in} \dot{m}_{in} h_{in} - \sum_{out} \dot{m}_{out} h_{out} - \dot{W} + \dot{Q} = 0 \qquad (1)$$

The energy balance of each component is defined based on decision and input variables. Energy balance equations for each component are presented in Appendix A.

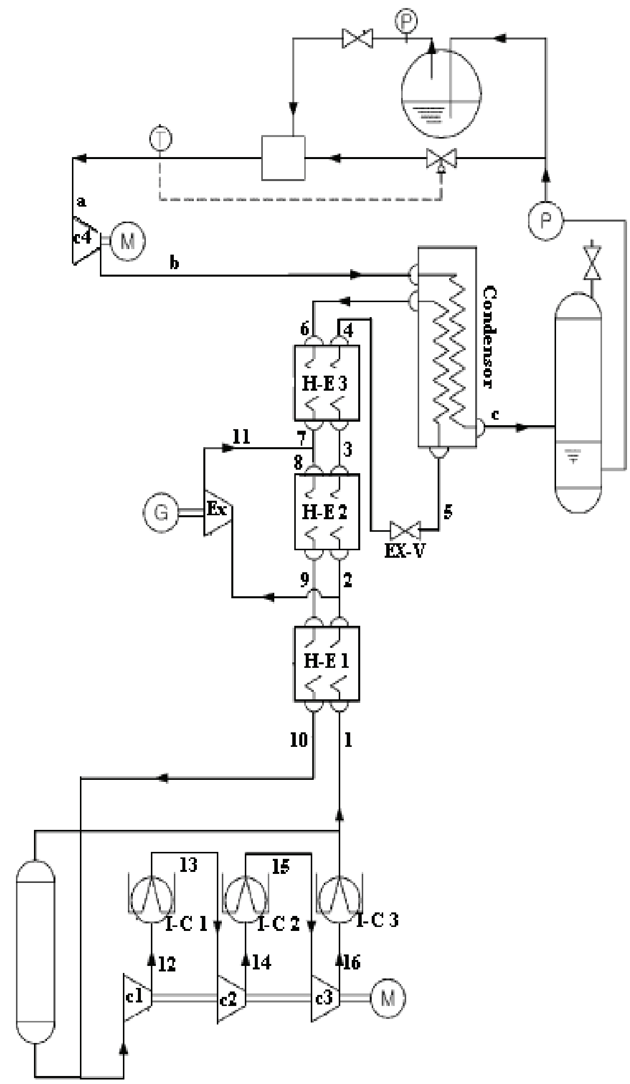

Figure 1. Schematic of the Claude cycle refrigerator with a turbo expander.

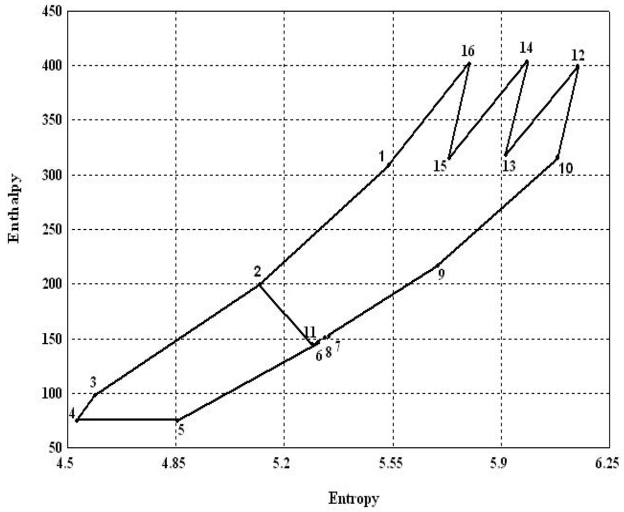

Figure 2. Enthalpy-Entropy diagram of proposed cycle.

2.4. Exergy analysis

The exergy flow equation for each part of the BOG liquefaction system is defined as below:

$$\sum \dot{Q}(1-\frac{T_0}{T}) - \dot{W} + \sum_{in} \dot{m}_{in} e_{in} - \sum_{out} \dot{m}_{out} e_{out} = \dot{E}_D \quad (2)$$

Assuming adiabatic behaviors for all components of the proposed system, the term $\dot{E}_Q$ is neglected in exergy balance equations. $\dot{E}_D$ is exergy destruction due to the system irreversibilities. The magnitude of specific exergy at every state is determined from the following equation,

$$e = e_{ph} + e_k + e_p + e_{ch} \quad (3)$$

in which, e_{ph}, e_k, e_p and e_{ch} represent physical exergy, kinetic exergy, potential exergy and chemical exergy, respectively. In this study, the magnitudes of the kinetic and potential exergy have been neglected. Assuming constant chemical composition of streams, the chemical exergy term is cancelled out in the exergy balance equations. Therefore, specific exergy at every arbitrary state can be assumed equivalent with the physical exergy. Physical exergy is obtained from the following equation.

$$e = e_{ph} = (h - h_0) - T_0(s - s_0) \quad (4)$$

Exergy balance equations of each component are presented in Appendix A.

2.5. Objective function

The Objective of this paper is maximization of the exergetic efficiency of the BOG-LNG re-liquefaction cycle. The exergetic efficiency in this case is defined as follow:

$$\varepsilon_{total} = \frac{\dot{E}_{product}}{\dot{E}_{fuel}} = \frac{\dot{E}_c - \dot{E}_b}{\dot{W}_{com,1} + \dot{W}_{com,2} + \dot{W}_{com,3} + \dot{W}_{com,4} - \dot{W}_{ex}} \quad (5)$$

3. Result and discussion

3.1. Optimization

The optimization problem of this work is a non-linear optimization type; therefore there is risk of finding a local optimum instead of the global optimum if the optimization is performed using conventional deterministic mathematical approaches. In order to avoid this problem, the optimization process is performed using a stochastic/deterministic approach namely as genetic algorithm (Holland, 1975). The tuning of the evolutionary algorithm is performed according to the values indicated in Table 1. States of various points including the magnitudes of the temperature, pressure, enthalpy, exergy and thermodynamic state for the optimized cycle are indicated in Table 2. Table 3 compares the magnitudes of decision variables for optimized and base case systems.

Table 3 indicates that pinch temperature differences for the heat exchangers #1 and #2 in the optimum design are lower than the corresponding values of the base design. In addition, pinch temperature differences for the heat exchanger #3 and condenser in the optimum design are higher than the base case design. Indeed, the magnitudes of the pinch points for these two exchangers as reported by Moon, Lee, Jin, Hong & Chang (2007) are too low and impractical. This problem has been resolved in our optimized system using appropriate constraints considered in optimization process. Energy components for the optimized and base case designs are compared in Table 4.

The results in Table 4 indicate that in the optimum design, the nitrogen flow rate and the input work to compressors and the output work from the expander are lower than in the base case design. Table 5 indicates the main exergy component of the base case and optimized system. Result shows that in the optimized system the exergetic efficiency is increased from the value of 0.2965 in the base case to 0.3159. In fact, a 6.5% improvement in the exergetic efficiency of the optimized system is obtained. Furthermore, from the results in Table 5 it can be found that in the optimized system 12.5%, 15.9% and 10.8% reductions are obtained respectively for the rate of fuel exergy (electricity), exergy destruction, and exergy loss, respectively.

In summary, our optimization approach leads to a number of thermodynamic and engineering advantage in the optimal design of the BOG re-liquefaction plant. For example, in the optimized system, the flow rate of nitrogen as a refrigerant is reduced. It means lower heat exchanger surfaces and smaller size of piping, valves and so on, and therefore probably lowers installation costs. Furthermore, the optimized system has compressors with lower capacities and lower electric power consumption. This may lead to a lower capital investment for compressors and a lower operating cost. Moreover, pinch temperature differences of heat exchangers in the optimized system are more rational and practical than the corresponding values of the base design.

Table 1. Tuning parameters in GA optimization program.

TUNING PARAMETERS	VALUE
Population size	500
Maximum number of generations	300
Pc (probability of crossover)	70(%)
Pm (probability of mutation)	1(%)
Number of crossover point	2
Selection process	Tournament
Tournament size	2

Table 2. Thermodynamic states at various locations in the optimized LNG BOG re-liquefaction plant.

Stream No.	Pressure [MPa]	Temperature [K]	Enthalpy [kJ/kg]	Entropy [kJ/kg/K]	Exergy [kW]	State
1	8.104	313.15	310.4	5.539	10858.85	Dense Fluid ($T>T_c$)
2	8.084	223.15	200	5.122	11248.81	Dense Fluid ($T>T_c$)
3	8.074	163.15	99.11	4.589	7338.95	Dense Fluid ($T>T_c$)
4	8.064	159.72	74.88	4.530	7230.93	Dense Fluid ($T>T_c$)
5	1.440	111.37	74.88	4.856	5673.98	Saturated (quality=0.92)
6	1.430	153.75	147.7	5.309	4672.90	Dense Fluid ($T>T_c$)
7	1.420	158.15	153	5.345	4585.68	Dense Fluid ($T>T_c$)
8	1.420	156.55	151.1	5.333	8096.56	Dense Fluid ($T>T_c$)
9	1.410	215.79	217.8	5.697	6917.97	Dense Fluid ($T>T_c$)
10	1.400	308.15	317	6.082	6478.74	Dense Fluid ($T>T_c$)
11	1.420	151.41	145	5.293	3554.59	Dense Fluid ($T>T_c$)
12	2.520	388.34	400.5	6.148	8277.11	Dense Fluid ($T>T_c$)
13	2.510	312.47	319.4	5.917	7932.57	Dense Fluid ($T>T_c$)
14	4.518	393.78	404.4	5.983	9773.22	Dense Fluid ($T>T_c$)
15	4.508	313.06	316.4	5.733	9393.87	Dense Fluid ($T>T_c$)
16	8.114	394.53	402.1	5.800	11245.84	Dense Fluid ($T>T_c$)

Table 3. Comparison of decision variables in the base case and optimum designs.

PARAMETER	SYMBOL	OPTIMUM VALUE	BASE DESIGN VALUE (Moon et al., 2007)
Pinch temperature difference in intercoolers	$\Delta T_{in-cool}$	5.00	–
Pinch temperature difference in heat exchanger #1	ΔT_{H-E1}	5.00	8.00
Pinch temperature difference in heat exchanger #2	ΔT_{H-E2}	7.36	17.20
Pinch temperature difference in heat exchanger #3	ΔT_{H-E3}	5	0.10
Pinch temperature difference in condenser	ΔT_{con}	6.78	2.20
Expanded mass ratio in expander	x_{ex}	0.4303	0.5000

Table 4. Energy parameters in the base case and optimum design.

PARAMETER	SYMBOL	OPTIMUM VALUE	BASE DESIGN VALUE (Moon et al., 2007)
Heat transfer in condenser	$\dot{Q}_{con}[MW]$	1.1244	1.1244
Input work to compressor 1	$\dot{W}_{com1}[MW]$	2.3532	2.7579
Input work to compressor 2	$\dot{W}_{com2}[MW]$	2.3922	2.8320
Input work to compressor 3	$\dot{W}_{com3}[KW]$	2.4139	2.8580
Input work to compressor 4	$\dot{W}_{com4}[MW]$	0.2211	0.2211
Output work of expander	$\dot{W}_{ex}[MW]$	0.6685	0.9970
Nitrogen flow rate	$\dot{m}_{N2}[kg/s]$	28.178	33.023

Table 5. Comparison of the thermodynamic features of the base case and optimum BOG-LNG system.

PARAMETER	SYMBOL	OPTIMUM VALUE	BASE DESIGN VALUE (Moon et al., 2007)
Exergetic efficiency	ε_{total}	0.3159	0.2965
Rate of Fuel exergy (the rate of net work)	$\dot{W}net = \sum_{i=1}^{4}\dot{W}_{com,i} - \dot{W}_{ex}\ [MW]$	6.7125	7.6719
Rate of Product exergy (the rate of condensing exergy)	$\dot{E}_c - \dot{E}_a [MW]$	2.1992	2.1992
Rate of Exergy Destruction	$\dot{E}_D [MW]$	3.6356	4.3253
Rate of Exergy Loss	$\dot{E}_L [MW]$	0.9561	1.0721

3.2. Sensitivity analysis

In this section, the sensitivities of the exergetic efficiency of the optimized system with respect to parameters of design are discussed in several figures. Fig. 3 shows the sensitivity of the exergetic efficiency with respect to the pressure ratio of the nitrogen compressors. These results show that the exergetic efficiency increased by increasing in the pressure ratio.

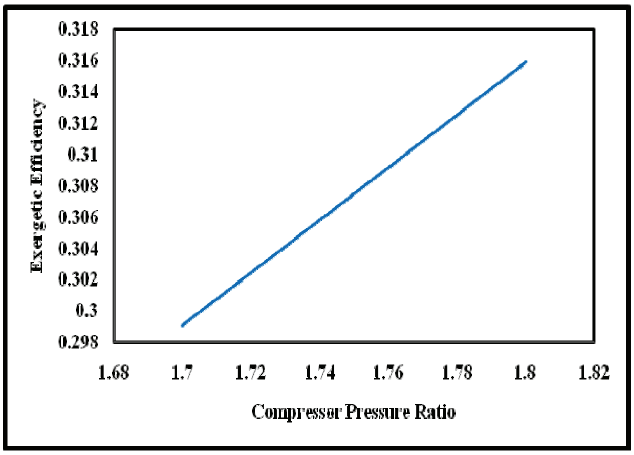

Figure 3. Sensitivity of the exergetic efficiency respect to the pressure ratio of the nitrogen compressor.

Fig. 4 shows the sensitivity of the exergetic efficiency to the pressure ratio of compressor number 4 (BOG compressor). The results indicate that the exergetic efficiency is decreases as the compressor pressure ratio is increased.

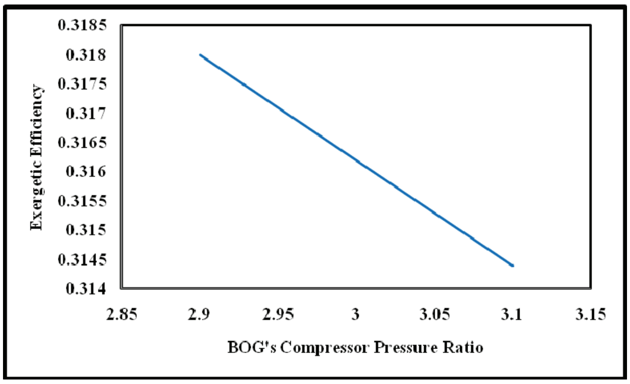

Figure 4. Sensitivity of the exergetic efficiency respect to the pressure ratio of BOG compressor.

The effect of the expander mass ratio on the plant exergetic efficiency is shown in Fig. 5. The expander mass ratio is defined as the ratio of the mass flow rate of stream expanded through the turbo-expander to the total nitrogen flow rate. Based on this figure, the exergetic efficiency is increased by increasing the expander mass ratio.

4. Conclusion

An optimization of the LNG-BOG re-liquefaction system is presented based on an exergetic objective. Thermodynamic modelling was developed based on the energy and exergy analyses in the proposed cryogenic system. An optimization process was conducted with six decision variables and the appropriate constraints using a Genetic Algorithm. The features of the base case and optimized plants were surveyed. Improvements in all thermodynamic features of the system were obtained in the optimized design. Finally, the sensitivity of the objective function (exergetic efficiency) with respect to some design parameters was reviewed. It was found that increases in the pressure ratios of the nitrogen compressor and compressor 4 (BOG compressor), and increases in the expander mass ratio lead to increases in the plant exergetic efficiency.

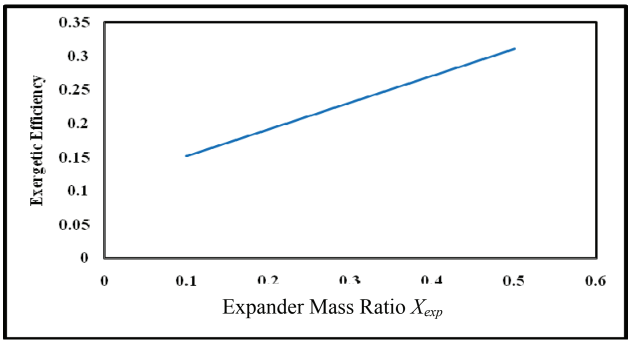

Figure 5. Sensitivity of the exergetic efficiency respect to the expander mass ratio

Nomenclature

BOG	Boil-off gas
$\dot{E}$	the rate of exergy
$\dot{E}_D$	the rate of exergy destruction
$\dot{E}_L$	the rate of exergy loss
e	specific molar exergy
h	enthalpy
LNG	Liquefied Natural Gas
$\dot{m}$	flow rate
s	molar specific entropy
T	temperature
$\dot{W}$	power
X_{exp}	Expander mass ratio
Greek letters:	
ε	exergetic efficiency
η_{is}	isentropic efficiency
Subscripts	
1,2,..,16,a, b,c	states in cryogenic system
H-E	Heat exchanger
com	Compressor
ex	Expander
Ex-v	Expansion valve
in-cool	Intercooler
con	Condenser

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Appendix A: Energy and exergy balance

Energy balance equations

Eqs. (1-A)-(6-A) show energy balances of compressors in the following manner.

$$h_{12} = \frac{h_{12s} - h_{10}}{\eta_{s,com1}} + h_{10} \quad (1\text{-A})$$

$$\eta_{s,com1} = \frac{h_{12s} - h_{10}}{h_{12} - h_{10}} \quad (2\text{-A})$$

$$\eta_{is,com2} = \frac{h_{14s} - h_{13}}{h_{14} - h_{13}} \quad (3\text{-A})$$

$$h_{14} = \frac{h_{14s} - h_{13}}{\eta_{is,com2}} + h_{13} \quad (4\text{-A})$$

$$\eta_{is,com3} = \frac{h_{16s} - h_{15}}{h_{16} - h_{15}} \quad (5\text{-A})$$

$$h_{16} = \frac{h_{16s} - h_{16}}{\eta_{is,com3}} + h_{15} \quad (6\text{-A})$$

Neglecting heat losses, energy balance for intercoolers and heat exchangers are written as below:

$$\dot{m}_{N_2}(h_{12} - h_{13}) = \dot{m}_{water}(h_{ow1} - h_{iw}) \quad (7\text{-A})$$

$$T_{12} = T_{ow1} + \Delta T_{in-cool} \quad (8\text{-A})$$

$$\dot{m}_{N_2}(h_{14} - h_{15}) = \dot{m}_{water}(h_{ow2} - h_{iw}) \quad (9\text{-A})$$

$$T_{14} = T_{ow2} + \Delta T_{in-cool} \quad (10\text{-A})$$

$$\dot{m}_{N_2}(h_{16} - h_{1}) = \dot{m}_{water}(h_{ow3} - h_{iw}) \quad (11\text{-A})$$

$$T_{16} = T_{ow3} + \Delta T_{in-cool} \quad (12\text{-A})$$

$$h_1 - h_2 = h_{10} - h_9 \quad (13\text{-A})$$

$$T_1 = T_{10} + \Delta T_{H-E1} \quad (14\text{-A})$$

$$(1 - x_{ex}) \times (h_2 - h_3) = h_9 - h_8 \quad (15\text{-A})$$

$$T_2 = T_9 + \Delta T_{H-E2} \quad (16\text{-A})$$

$$h_3 - h_4 = h_7 - h_6 \quad (17\text{-A})$$

$$T_3 = T_7 + \Delta T_{H-E3} \quad (18\text{-A})$$

Eqs. (19-A)-(20-A) are the energy balance for the turbo-expander.

$$\eta_{is,ex} = \frac{h_{11} - h_2}{h_{11s} - h_2} \quad (19\text{-A})$$

$$h_{11} = (h_{11s} - h_2) \times \eta_{is,\mathrm{ex}} + h_2 \tag{20-A}$$

In the expansion valve, there is no heat and work transfer. Here, the energy balance equation can be written as:

$$h_4 = h_5 \tag{21-A}$$

In the proposed system, the condenser is the major component for condensing of the BOG. Energy balances for the condenser are shown in Eqs. (22-A)-(23-A).

$$\dot{m}_{BOG}(h_b - h_c) = \dot{m}_{N_2}(h_6 - h_5) \tag{22-A}$$

$$T_5 = T_c - \Delta T_{con} \tag{23-A}$$

$$\dot{W}_{com1} = \dot{m}_{N_2}(h_{12} - h_{10}) \tag{24-A}$$

$$\dot{W}_{com2} = \dot{m}_{N_2}(h_{14} - h_{13}) \tag{25-A}$$

$$\dot{W}_{com3} = \dot{m}_{N_2}(h_{16} - h_{15}) \tag{26-A}$$

$$\dot{Q}_{in-cool1} = \dot{m}_{N_2}(h_{12} - h_{13}) \tag{27-A}$$

$$\dot{Q}_{in-cool2} = \dot{m}_{N_2}(h_{14} - h_{15}) \tag{28-A}$$

$$\dot{Q}_{in-cool3} = \dot{m}_{N_2}(h_{16} - h_1) \tag{29-A}$$

$$\dot{W}_{ex} = x_{ex}\dot{m}_{N_2}(h_2 - h_{11}) \tag{30-A}$$

$$\dot{W}_{com4} = \dot{m}_{BOG}(h_b - h_a) \tag{31-A}$$

$$\dot{Q}_{con} = \dot{m}_{BOG}(h_b - h_c) \tag{32-A}$$

Exergy balance equations

Compressor 1:

$$\dot{E}_{D,com1} = \dot{E}_{10} + \dot{E}_{w,com1} - \dot{E}_{12} \tag{33-A}$$

Compressor 2:

$$\dot{E}_{D,com2} = \dot{E}_{13} + \dot{E}_{w,com2} - \dot{E}_{14} \tag{34-A}$$

Compressor 3:

$$\dot{E}_{D,com3} = \dot{E}_{15} + \dot{E}_{w,com3} - \dot{E}_{16} \tag{35-A}$$

Compressor 4:

$$\dot{E}_{D,com4} = \dot{E}_a + \dot{E}_{w,com4} - \dot{E}_b \tag{36-A}$$

Intercooler 1:

$$\dot{E}_{D,in-cool1} = \dot{E}_{12} - \dot{E}_{13} - \dot{E}^{Q,in-cool1} \tag{37-A}$$

Intercooler 2:

$$\dot{E}_{D,in-cool2} = \dot{E}_{14} - \dot{E}_{15} - \dot{E}^{Q,in-cool2} \tag{38-A}$$

Intercooler 3:

$$\dot{E}_{D,in-cool3} = \dot{E}_{16} - \dot{E}_1 - \dot{E}^{Q,in-cool3} \tag{39-A}$$

Heat exchanger 1:

$$\dot{E}_{D,H-E1} = \dot{E}_1 + \dot{E}_9 - \dot{E}_2 - \dot{E}_{10} \tag{40-A}$$

Heat exchanger 2:

$$\dot{E}_{D,H-E2} = \dot{E}_8 + (1 - x_{ex}) \times \dot{E}_2 - \dot{E}_3 - \dot{E}_9 \tag{41-A}$$

Heat exchanger 3:

$$\dot{E}_{D,H-E3} = \dot{E}_6 + \dot{E}_3 - \dot{E}_7 - \dot{E}_4 \tag{42-A}$$

Condenser:

$$\dot{E}_{D,con} = \dot{E}_5 + \dot{E}_b - \dot{E}_6 - \dot{E}_c \tag{43-A}$$

Expansion valve:

$$\dot{E}_{D,\mathrm{ex}-v} = \dot{E}_4 - \dot{E}_5 \tag{44-A}$$

Expander:

$$\dot{E}_{D,ex} = x_{ex}\dot{E}_2 - \dot{E}_{11} - \dot{W}_{ex} \tag{45-A}$$