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Dynamic Operation Optimization of a Trigeneration System

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Abstract

It is considered that the electric, thermal and cooling loads of a building complex are covered by a tri-generation system consisting of a gas engine with heat recovery, an absorption chiller driven by thermal energy, electrically driven compression chillers and two thermal storage tanks (one with hot and one with cold water). Supplementary electricity is supplied by the local network. The behavior of such a system during transients is characterized primarily by the transient performance of the storage tanks and the absorption chiller (start-up and shut-down). The objective of the work reported here is the operation optimization of the system under load-varying conditions taking into consideration the transient behavior of the three aforementioned components. The simulation of the dynamic behavior of the absorption chiller, in particular, is based on the Gompertz function (sigmoid curve). Minimization of the total cost for covering the energy needs of the building is selected as the objective function, while the operating point of each component is to be determined by optimization under specified constraints. The optimal values of the control variables are determined using a multi–stage control vector parameterization method and optimization software based on Sequential Quadratic Programming. The solution of the optimization problem is accompanied by a sensitivity analysis. Suggestions are written for continuation and improvement of the work.

Keywords: *Tri-generation system; dynamic optimization; transient behavior; Gompertz equation.*

1. Introduction

The economic feasibility of a cogeneration system depends strongly on the length of the operating period, i.e. the number of operating hours per year, and on the mode of operation, i.e. the power output of the system at each instant of time, taking into consideration that the loads vary with time. In areas with mild winters the heating period is short, unless there are thermal loads independent of the weather. Thus, in order the system to become economically viable, there is need to use the heat in summer for cooling by means, e.g., of an absorption chiller, in other words to convert the system to a trigeneration system. On the other hand, the fact that the peak of the electric load often does not coincide with the peak of the thermal or cooling loads makes the installation of storage tanks for hot and cold water important, if not necessary. Such a system is studied here (Figure 1).

The interest in applying trigeneration and the complexities of these systems gave the impetus for research work aiming at system simulation, performance evaluation and optimization (e.g., Cardona & Piacentino, 2007; Chico & Mancarella, 2009; Kavvadias & Maroulis, 2010; Lozano, Ramos & Serra, 2010; Arosio, Guilizzoni & Pravettoni, 2011; Carvalho, Serra & Lozano, 2011; Rubio-Maya, Uche & Martínez, 2011).

The work presented here is focused on the dynamic operation optimization of a specified system. A clarification is needed: The word "dynamic" is frequently used to imply change with respect to time. However, this is not sufficient in order an optimization problem to be characterized as dynamic. For example, if the period of operation can be decomposed into time intervals of steady state operation independent of each other, then the optimization problem is decomposed into a series of static independent optimization problems, which can be solved relatively easily. However in the system of Figure 1, due to the existence of storage, the power output in a certain time interval affects and is affected by the power output in other time intervals. Furthermore, the existence of an absorption chiller introduces two complexities due to its time constant, which is longer than the time constant of the cogeneration unit: (i) the transient operation during load increase and decrease makes the assumption of steady state operation not accurate; (ii) the time needed for the absorption chiller to go from full load to zero load and back again has to be taken into consideration before deciding to turn the unit off and on. In summary, the facts that (a) the operation in a time interval affects and is affected by the operation in other time intervals, and (b) the transient behavior of at least one important component of the system cannot be ignored, make the optimization problem a dynamic one. The solution of such a problem requires the use of dynamic optimization techniques.

Various methods and applications of dynamic optimization of energy systems can be found in the literature (e.g. Stoecker, 1989; Bausa & Tsatsaronis, 2001; Calise, 2011; Munoz & von Spakovsky, 2001; Kim, von Spakovsky, Wang & Nelson, 2011). The present work is a contribution to this effort.

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2. Description of the System

The energy needs of the building complex under consideration are covered by electricity coming from the local network and by its own energy system consisting of the following main components (Figure 1):

- one gas-engine cogeneration unit,
- natural gas boilers,
- electrically driven compression chillers,
- one absorption chiller,
- one hot water storage tank with its own natural gas burner,
- one cold water storage tank.

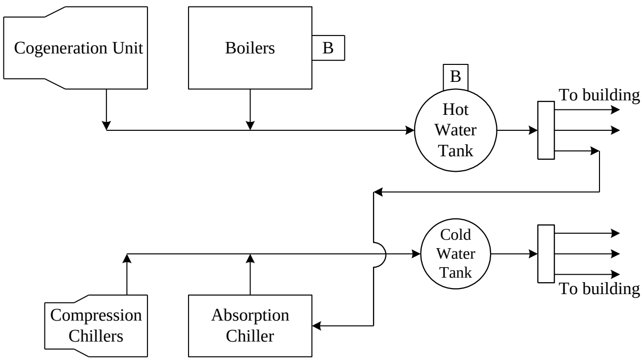

Figure 1. Simplified diagram of the energy system (only the supply lines of hot and cold water are depicted).

Table 1. Level and duration of the building energy needs in a typical day of January and July. [1]

Time period	Δt (h)	$\dot{W}_{cons}$ (kW_e)	$\dot{Q}_{cons}$ (kW_{th})	$\dot{\Psi}_{cons}$ (kW_c)
January				
00-06	6	0	0	0
06-08	2	455	1380	700
08-10	2	580	1690	700
10-12	2	625	1530	700
12-14	2	650	1360	700
14-16	2	635	1320	700
16-18	2	605	1420	700
18-20	2	465	1210	700
20-24	4	0	0	0
July				
00-06	6	0	0	0
06-08	2	455	465	1130
08-10	2	580	465	1650
10-12	2	625	465	2060
12-14	2	650	465	2430
14-16	2	635	465	2620
16-18	2	605	465	2490
18-20	2	465	465	2070
20-24	4	0	0	0

The mass of water in the storage tanks remains constant, because the tanks are components of closed circuits.

The boilers and the compression chillers have sufficient capacity to cover the needs of the building, even when the cogeneration unit and the absorption chiller are not operating.

Table 2. Main specifications of the energy system components at the design point.

Item	Value
Electric power of the cogeneration unit	540 kW_e
Thermal power of the cogeneration unit	759 kW_{th}
Thermal efficiency of the boilers	0.91
Cooling power of the absorption chiller	206 kW_c
Coefficient of performance of the absorption chiller at nominal load	0.7

Table 3. Nominal values of economic parameters.

Item	Value
Specific installed cost of boiler	25 €/kW_{th}
Specific maintenance cost of the boiler	0,001 €/kWh_{th}
Specific installed cost of cogeneration unit	1300 €/kW_e
Specific maintenance cost of the cogeneration unit	0,010 €/kWh_e
Specific installed cost of the compression chillers	140 €/kW_c
Specific maintenance cost of the compression chillers	0,0023 €/kWh_c
Specific installed cost of the absorption chiller	250 €/kW_c
Specific maintenance cost of the absorption chiller	0,0026 €/kWh_c
Personnel overtime cost	15 €/h
Cost of natural gas for the cogeneration unit	0,032 €/kWh_f
Cost of natural gas for the boilers	0,040 €/kWh_f
Unit cost of electricity supplied by the network	0,10039 €/kWh_e
Value Added Tax	0,13
Market interest rate	0,08

Hot water, coming either from the cogeneration unit or from the boilers, is stored in the hot water tank wherefrom it is supplied to the building and to the absorption chiller. A natural gas burner installed on the tank can compensate for thermal losses up to a certain extent, if needed. Cold water, coming either from the compression chillers or from the absorption chiller, is stored in the cold water tank wherefrom it is supplied to the building.

The building operates Monday through Friday. For the purposes of this work satisfactory accuracy is achieved, if it is considered that the change of the energy needs with time is represented with one typical day for each month and nine time intervals of constant needs in each one during each typical day. As a sample, the loads for the typical days of January and July are given in Table 1. The electric load, $\dot{W}_{cons}$, does not include the electric power required by the compression chillers. If this power is added, then the electric power output of the cogeneration unit is lower than the total electric loads and consequently there is no excess electricity to be sold to the grid.

Thermal loads include space heating and supply of hot water. It is noted that the constant cooling load appearing in periods that the external temperature is low (such as in January) is due to rooms with equipment that produce heat and there is need to keep these rooms at a certain temperature.

[1] The symbols are explained in the Nomenclature.

It is clarified that in each time interval the load is considered constant (Table 1), but the cooling power output of the absorption chiller may change with time following the Gompertz function (as explained in Sub-section 3.2), depending on whether the unit is turned on or off, as dictated by the optimizer. Any missing cooling power is supplemented by the compression chillers.

Certain specifications of the main components of the system at the nominal load (design point) are given in Table 2, while the values of the economic parameters considered are given in Table 3.

3. Simulation and Optimization Approach

3.1 Objective Function and Constraints

The minimization of the total annual cost for covering the energy needs of the building is selected as the objective function, which consists of the annualized capital cost, and the costs for personnel, maintenance, fuel and electricity purchased from the network:

$$\min C_{tot} = C_{ac} + C_p + C_m + C_f + C_e \tag{1}$$

The annualized capital cost is given by the equation

$$C_{ac} = \sum_i c_{ci} \cdot P_i = \sum_i \frac{C_{ci} \cdot CRF_i}{\dot{P}_{Di} \cdot t_{ai,\max}} \cdot P_i \tag{2}$$

where

$$CRF_i = \frac{d\left(1+d\right)^{N_i}}{\left(1+d\right)^{N_i} - 1} \tag{3}$$

Equation (2) is based on the consideration that the capital (investment cost) of component *i* is "consumed" by the end of its technical life, which corresponds to a total number of units of useful product and, if the unit is not used at a certain time period, there will be "life" left to be used in another period, more profitable. Thus, each unit of product bears with it a portion of the investment cost. It is often written in the literature that the capital cost can be considered sunk for operation optimization. At least in the case studied here, this is not correct and may lead to wrong decisions regarding the mode of operation of a system. For example if the capital cost is ignored, then the marginal cost of electricity produced would be much lower and the unit might be put in operation even in periods with very low cost of electricity purchased from the network.

The cost of personnel is due only to overtime that the operation of the trigeneration system may require, while the operation during the normal working hours is covered by the technical team that works in the complex anyway.

The maintenance cost is proportional to the annual production of a component, the cost of fuel is proportional to the fuel consumption and the cost of electricity is proportional to the electric energy purchased from the grid, with the proportionality factors given in Table 3.

The optimization is subject to certain equality constraints such as the following.

Since there is no storage of electric energy, the electric energy produced by the cogeneration system plus the energy purchased from the network, must be equal to the total electric load, consisting of the power absorbed by the compression chillers and the power for any other load, as given in Table 1:

$$\dot{W}_{cog} + \dot{W}_b = \dot{W}_{cons} + \dot{W}_{chel} \tag{4}$$

Energy balance in the hot water storage tank:

$$m_{hwt} c_p \frac{dT_{hwt}}{dt} = \dot{Q}_{cog} + \dot{Q}_B - \dot{Q}_{abs} - \dot{Q}_{cons} - \left(UA\right)_{hwt}\left(T_{hwt} - T_r\right) \tag{5}$$

Energy balance in the cold water storage tank:

$$m_{cwt} c_p \frac{dT_{cwt}}{dt} = \dot{\Psi}_{cons} - \dot{\Psi}_{abs} - \dot{\Psi}_{chel} - \left(UA\right)_{cwt}\left(T_{cwt} - T_r\right) \tag{6}$$

The left hand side of Eq. (5) represents the change in the thermal energy stored in the tank, which is due only to temperature change, because, as it is mentioned in Section 2, the mass of water in the storage tanks remains constant. The right hand side contains the inflow of thermal energy coming from the cogeneration unit and the additional burner and the outflow of thermal energy towards the absorption unit, the various thermal loads and to the environment (thermal losses). A similar explanation can be writer for Eq. (6).

There are also inequality constraints such as the following:

$$0.3 \cdot \dot{W}_{Dcog} \le \dot{W}_{cog} \le \dot{W}_{Dcog} \quad \text{or} \quad \dot{W}_{cog} = 0 \tag{7}$$

$$0.1\dot{\Psi}_{Dabs} \le \dot{\Psi}_{abs} \le \dot{\Psi}_{Dabs} \quad \text{or} \quad \dot{\Psi}_{abs} = 0 \tag{8}$$

$$80°C \le T_{hwt} \le 95°C \tag{9}$$

$$7°C \le T_{cwt} \le 12°C \tag{10}$$

Equations (7) and (8) indicate that there is a technical lower limit on the operating power of the cogeneration unit and the absorption chiller. If a value lower than the lower limit is derived during the optimization procedure, then the power output is set automatically equal to zero. The limits on the hot and cold water temperatures, Eqs. (9) and (10), are specified by the effective operation of the central heating and cooling systems, respectively.

Additional equality and inequality constraints are derived with the simulation of the system.

3.2 Simulation of the system

The main components of the system are simulated by taking into consideration external performance characteristics obtained either from the literature or from information given by the manufacturers.

The partial load performance of the cogeneration unit is described by the equations for calculation of the energy flow rate of the fuel consumed and of the useful heat produced as functions of the electric power output:

$$\dot{H}_{fcog} = \sum_{k=0}^{2} c_{fcog,k} \left(\dot{W}_{cog}\right)^k \tag{11}$$

$$\dot{Q}_{cog} = \sum_{k=0}^{2} c_{Q_{cog},k} \left(\dot{W}_{cog}\right)^k \tag{12}$$

For the boilers and the auxiliary burner of the hot water storage tank, a constant efficiency is assumed (Table 2). The coefficient of performance of the compression chillers at nominal load is calculated as a function of the ambient temperature

$$COP_{chel} = \sum_{k=0}^{2} c_{chel,k} \left(T_{amb}\right)^k , \qquad T_{amb} \text{ in °C} \tag{13}$$

while the simplifying assumption is made that it remains constant at partial load. A more elaborate simulation is performed for the absorption chiller, as follows.

The heat flow rate required for the operation of the absorption chiller at nominal load (design point) is given by the equation

$$\dot{Q}_{Dabs} = \dot{\Psi}_{Dabs} \big/ COP_{Dabs} \tag{14}$$

At partial load, the following equation is applicable:

$$\dot{Q}_{abs} = \dot{Q}_{Dabs} \sum_{k=0}^{2} c_{Q_{abs},k} \left(\frac{\dot{\Psi}_{abs}}{\dot{\Psi}_{Dabs}}\right)^k \tag{15}$$

The transient behavior of the absorption chiller is represented by the Gompertz function (*S*-curve), which takes two forms, one for load increase from zero to nominal load (Figure 2)

$$G_{incr}(t) = ab^{c^t} - ab \tag{16}$$

and one for load decrease from nominal to zero load (Figure 3)

$$G_{decr}(t) = 1 - ab^{c^{1.75t}} + ab \tag{17}$$

where t is elapsed time in minutes and

$a = 1.228455, \quad b = 0.000128, \quad c = 0.810818.$

Thus, it is:

$$\dot{\Psi}_{abs}(t) = G(t) \cdot \dot{\Psi}_{Dabs} \tag{18}$$

The heat flow rate required by the absorption chiller for operation during transients is given by the equation

$$\dot{Q}_{abs}(t) = \dot{\Psi}_{abs}(t) \big/ COP_{abs}(t) \tag{19}$$

where

$$COP_{abs}(t) = COP_{Dabs} \sum_{k=0}^{6} c_{COP_{abs},k} \left(\frac{\dot{\Psi}_{abs}(t)}{\dot{\Psi}_{Dabs}}\right)^k \tag{20}$$

It is interesting to note that the thermal power required for the operation of the absorption chiller at a certain cooling output under steady state, Eq. (15), is different from the thermal power required at the same cooling output in transients, Eq. (19).

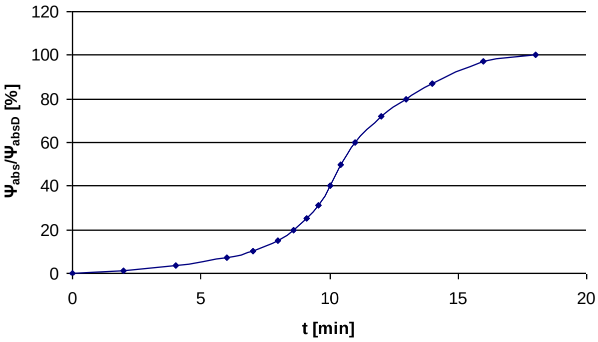

Figure 2. Gompertz function for load increase.

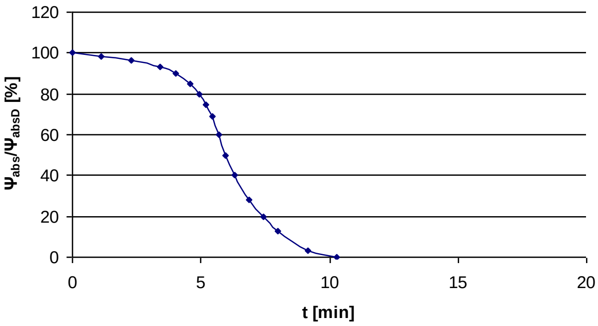

Figure 3. Gompertz function for load decrease.

In the preceding, the most important parts of the simulation have been presented, while further details are not given due to space limitations.

3.3 Independent Variables of the Optimization Problem

The objective function, the equality constraints derived by the analysis of the system and the number of parameters with externally imposed values leave the optimization problem with four degrees of freedom. The solution of the problem, as described in the following section, is facilitated if the electric power output of the cogeneration unit, $\dot{W}_{cog}$, the thermal power output of the boilers and the burner of the hot water tank, $\dot{Q}_B$, the cooling power of the absorption chiller, $\dot{\Psi}_{abs}$, and the cooling power of the compression chillers, $\dot{\Psi}_{chel}$, at each instant of time are the independent variables (decision variables). Thus:

$$\mathbf{x} = \left(\dot{W}_{cog}, \dot{Q}_B, \dot{\Psi}_{abs}, \dot{\Psi}_{chel}\right) \tag{21}$$

It is noted that $\dot{Q}_B$ and $\dot{\Psi}_{chel}$ are treated as independent variables and consequently the temperatures in the storage tanks (T_{hwt} and T_{cwt}) are dependent variables (state variables). If there were no storage tanks, $\dot{Q}_B$ and $\dot{\Psi}_{chel}$ would be dependent variables.

3.4 Solution Method

As it is explained in the preceding, there is interdependency between the time intervals of a typical weekday. On the other hand, during the weekend the building complex is not in operation, but the existence of storage tanks bridges the gap between Friday and Monday. Under these conditions, optimization for each time interval individually is not correct. Instead, the time intervals of a whole week are treated simultaneously. For this purpose, instead of four, Eq. (21), the number of independent variables is

$$n = 4\frac{\text{variables}}{\text{interval}}\times 9\frac{\text{intervals}}{\text{day}}\times 7\frac{\text{days}}{\text{week}} = 252\,\text{variables} \qquad (22)$$

Thus, the initial point for the optimization consists of the values of all these variables plus the initial conditions required for the numerical solution of the differential equations, Eq. (5) and (6).

For the numerical solution of the optimization problem, the SNOPT software has been used (Gill, Murray & Saunders, 2002), which is based on the Sequential Quadratic Programming algorithm. The software is supplemented with subroutines for the calculation of the dependent variables and the objective function. Numerical differentiation and integration is performed wherever is needed.

With SNOP, the optimum point reached can be a local and not the global optimum. Therefore, various techniques, such as starting from different initial points, are used in order to increase the chances for reaching the global optimum.

Of course, SNOP is not designed for dynamic optimization. It can be used successfully, however, only because of the discretization of the optimization problem, as it is explained in the preceding, in connection with Eq. (22).

4. Optimization Results and Sensitivity Analysis

4.1 Results for the Nominal Set of Data

The results for the nominal set of data, i.e. for the values of parameters given in Tables 1, 2 and 3, are given in Table 4 (for two months only, January and July, in order to save space). In addition to the optimum values of the independent variables, Eq. (21), Table 4 gives also the optimum values of important dependent variables.

It is noted that during the cold period (January), the thermal energy produced by the cogeneration unit is used to cover thermal loads and it is not economical to use thermal energy from the boilers to drive the absorption chiller; therefore, it is $\dot{\Psi}_{abs} = 0$ and the cooling load is covered by the compression chiller. The temperature in the hot water storage tank is kept at the lower limit (80°C), an indication that it is not economically justified to store thermal energy above a certain limit.

During the hot period (July), the absorption chiller operates at its capacity. It is interesting to note that in certain time intervals the temperature in the cold water storage tank drops to the lower limit (7°C), thus preparing the system to cover the cooling load at the peak of the day.

Figures 4 to 8 supplement the information given in Table 4 with the change of the independent variables and the temperature in the hot and cold water storage tanks during a typical day or week in January and July, as examples. Even though it is not evident in Figure 7, due to the scale of the horizontal axis, the transient behavior of the absorption chiller follows indeed the pattern presented in Figures 2 and 3.

In order to check the effectiveness of the optimization procedure, the total cost for covering the energy needs obtained with optimization for the typical day of each month (case A) is compared with the cost that results with typical predetermined operation modes, namely, heat-match operation of the cogeneration unit (case B), electricity-match operation of the cogeneration unit (case C) and no operation of the trigeneration system (case D). In cases B

Table 4. Optimization results for the nominal set of data (Tables 1, 2 and 3). (The two columns under each one of T_{hwt}, T_{cwt}, and $\dot{\Psi}_{abs}$ correspond to the beginning and the end of each time period).

Time period	$\dot{W}_{cog}$ (kW)	$\dot{Q}_{cog}$ (kW)	$\dot{Q}_{cog,us}$ (kW)	$\dot{Q}_B$ (kW)	T_{hwt} (°C)		T_{cwt} (°C)		$\dot{\Psi}_{abs}$ (kW)		$\dot{Q}_{abs}$ (kW)	$\dot{\Psi}_{chel}$ (kW)
January												
00-06	0	0	0	0	80.0	79.5	12.0	12.3	0	0	0	0
06-08	540	759	759	625.7	79.5	80.0	12.3	12.0	0	0	0	701.0
08-10	540	759	759	932.2	80.0	80.0	12.0	12.0	0	0	0	700.2
10-12	540	759	759	772.2	80.0	80.0	12.0	12.0	0	0	0	700.2
12-14	540	759	759	602.2	80.0	80.0	12.0	12.0	0	0	0	700.2
14-16	540	759	759	562.2	80.0	80.0	12.0	12.0	0	0	0	700.2
16-18	540	759	759	662.2	80.0	80.0	12.0	12.0	0	0	0	700.2
18-20	540	759	759	452.2	80.0	80.0	12.0	12.0	0	0	0	700.2
20-24	0	0	0	0	80.0	79.7	12.0	12.2	0	0	0	0
July												
00-06	0	0	0	0	80.0	79.6	12.0	12.4	0	0	0	0
06-08	540	759	759	0	79.6	80.9	12.4	7.0	0	189.080	283.6	973.4
08-10	540	759	759	0	80.9	80.7	7.0	7.0	189.1	206	293.9	1444.9
10-12	540	759	759	0	80.7	80.5	7.0	7.0	206	206	294.2	1854.6
12-14	540	759	759	0	80.5	80.4	7.0	12.0	206	206	294.2	2210.0
14-16	540	759	759	0	80.4	80.2	12.0	12.0	206	206	294.2	2414.4
16-18	540	759	759	0	80.2	80.0	12.0	12.0	206	206	294.2	2284.4
18-20	540	759	609.8	0	80.0	82.2	12.0	12.0	206	80.517	127.9	1985.3
20-24	0	0	0	0	82.2	80.0	12.0	12.0	80.5	0	6.8	0

Table 5. Total cost for covering the energy needs in one typical day of each month and cost increase with no optimization. A: optimization results, B: heat-match operation of the cogeneration unit, C: electricity-match operation of the cogeneration unit, D: no operation of the trigeneration system.

Month	Cost (Euros per day)				Cost increase (%)		
	A	B	C	D	100(B-A)/A	100(C-A)/A	100(D-A)/A
January	1652,86	1652,86	1671,63	1989,82	0,00	1,14	20,39
February	1570,20	1570,20	1588,94	1907,15	0,00	1,19	21,46
March	1535,30	1535,30	1554,12	1872,30	0,00	1,22	21,95
April	1530,90	1530,90	1549,73	1867,91	0,00	1,23	22,01
May	1502,89	1578,04	1512,73	1758,04	5,00	0,65	16,98
June	1767,11	1858,08	1772,35	2012,82	5,15	0,30	13,90
July	1910,03	2003,10	1915,37	2158,06	4,87	0,28	12,99
August	1823,43	1916,03	1828,74	2070,78	5,08	0,29	13,56
September	1572,46	1657,64	1577,95	1818,14	5,42	0,35	15,62
October	1317,78	1330,21	1330,75	1637,63	0,94	0,98	24,27
November	1553,54	1553,54	1572,32	1890,50	0,00	1,21	21,69
December	1677,10	1677,10	1695,85	2014,04	0,00	1,12	20,09

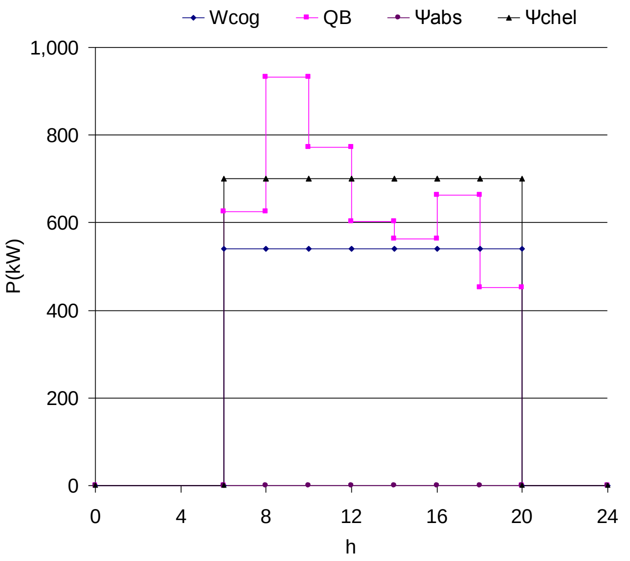

Figure 4. Change of the independent variables during a typical day in January.

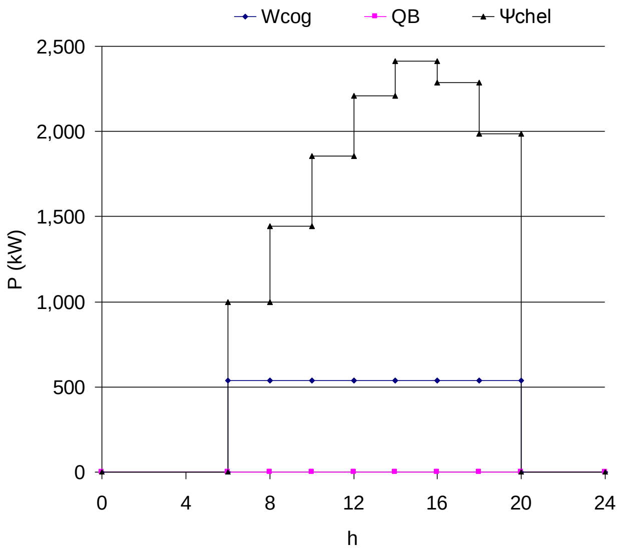

Figure 6. Change of the operating power of the cogeneration unit, the boilers and the compression chillers during a typical day in July.

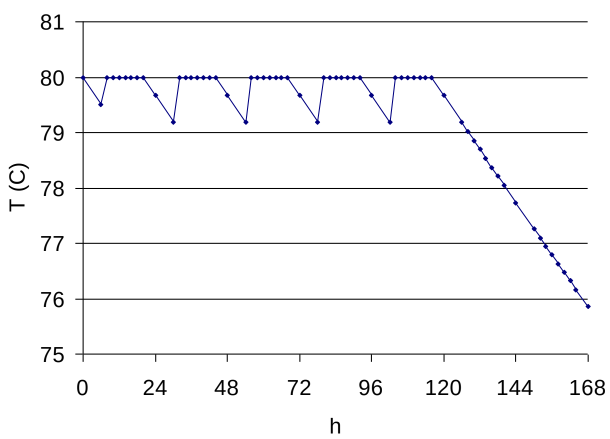

Figure 5. Change of the temperature in the hot water storage tank during a typical week in January.

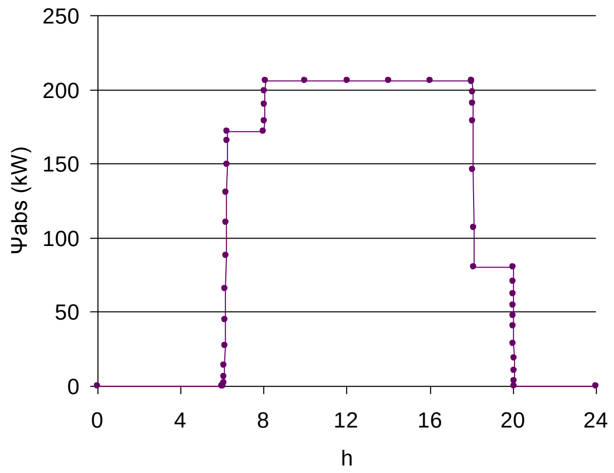

Figure 7. Change of the cooling power of the absorption chiller during a typical day in July.

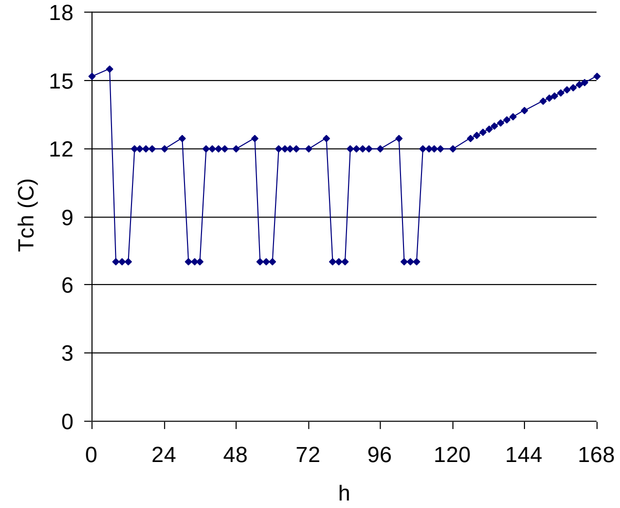

Figure 8. Change of the temperature in the cold water storage tank during a typical week in July.

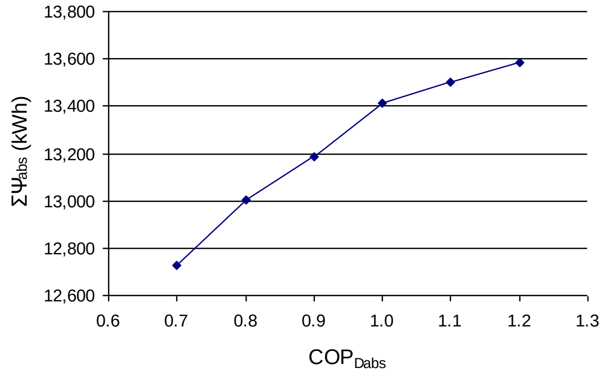

Figure 9. Effect of the nominal value of the absorption chiller COP on the total cooling energy supplied by the chiller.

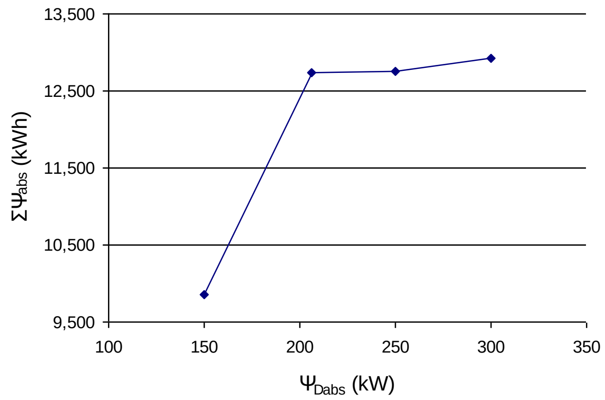

Figure 10 Effect of the nominal cooling power of the absorption chiller on the total cooling energy supplied by the chiller.

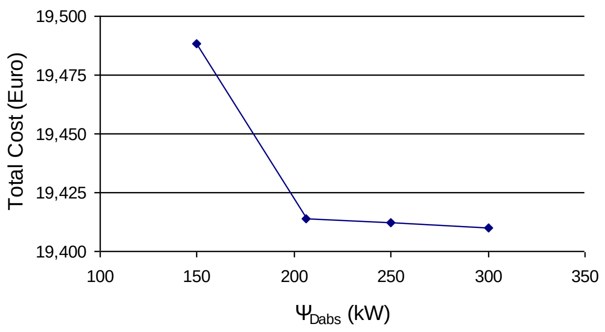

Figure 11. Effect of the nominal cooling power of the absorption chiller on the total cost for covering the energy needs.

and C the absorption chiller operates at its nominal load. The results appearing in Table 5 demonstrate the effectiveness of the optimization: if, instead of the optimal operation, heat-match is selected, the cost increase in certain days reaches about 5%; with electricity match the increase reaches about 1.2%, and if the trigeneration system does not operate at all, the cost increase reaches about 24%.

4.2 Indicative Results of a Parametric Study

The effect of the values of certain parameters on the optimization results is illustrated in Figures 9-16. The term "nominal value" means the value given in Table 2 or 3. For simplicity, the Total Cost, the summation of the electricity generated by the cogeneration unit, $\sum W_{cog}$, and the summation of the cooling energy supplied by the absorption chiller, $\sum \Psi_{abs}$, are obtained by summation of the related quantities over the 12 typical days of the year.

Figure 9 shows the effect that an increase of the nominal value of the absorption chiller COP from 0.7 to 1.2, due to technological improvement, has on the total cooling energy supplied by the chiller, with the same size of the cogeneration unit (consequently, the same quantity of available heat) and of the absorption chiller. Of course, a higher value of the COP allows for a higher capacity of the absorption chiller (for the same size of the cogeneration unit), but in this parametric study everything else is considered constant, except of the COP.

The cooling capacity of the absorption chiller, 206 kW_c (Table 2) has been specified so that the useful heat provided by the cogeneration unit (759 kW_{th}) covers first the thermal needs in summer period (465 kW_{th}) and the remaining is fed to the absorption chiller:

$$\dot{\Psi}_{Dabs} = (759\text{-}465) \times 0.7 = 206\,\mathrm{kW_e} \tag{23}$$

Thus, a lower value of $\dot{\Psi}_{Dabs}$ results in abrupt decrease of the total cooling energy produced, while a higher value of $\dot{\Psi}_{Dabs}$ does not offer any advantage on the cooling energy or the total cost, because there is no more available heat from the cogeneration unit and it is proved non-economical to feed the absorption chiller with heat from the boilers (Figures 10 and 11).

Figures 12 and 13 show that increase of the nominal power of the cogeneration unit from 540 to 700 kW_e (29.6%) results in appreciable increase of the cooling energy produced by the absorption chiller (21.57%) and non-negligible reduction of the total cost (3.86%).

It can be calculated analytically (on the basis of marginal costs) that there exists a critical value of the fuel cost for the cogeneration unit:

$$c_f^{crit} = 1.41 c_{fN} \tag{24}$$

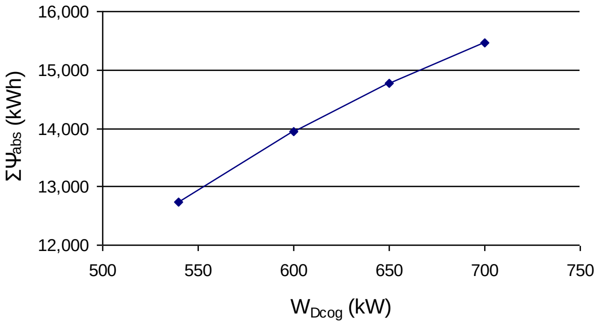

Figure 12. Effect of the nominal electric power of the cogeneration unit on the total cooling energy supplied by the chiller.

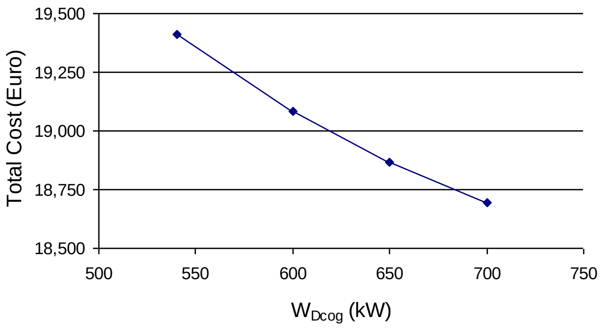

Figure 13. Effect of the nominal electric power of the cogeneration unit on the total cost for covering the energy needs.

For fuel cost higher than this value, the installation and operation of an absorption chiller is not justified economically. Figure 14 presents verification: values of the fuel cost higher than the critical one lead to zero cooling energy supplied by the absorption chiller. As a consequence during the summer period, when the thermal load is lower than the nominal thermal output of the cogeneration unit, the optimization leads to a power output lower than the nominal one, so that there is no heat wasted. This is the reason for the abrupt drop in the total electric energy produced by the cogeneration unit for values of fuel cost higher than the critical one (Figure 15).

In a similar way, it can be calculated analytically that there exists a critical value of the electricity cost purchased from the network:

$$c_e^{crit} = 0.78 c_{eN} \tag{25}$$

For electricity cost lower than this value, it is more economical to cover all the cooling load with compression chillers; the installation and operation of an absorption chiller is not justified economically and the operating point of the cogeneration unit is adjusted accordingly during the summer period (Figure 16).

5. Conclusions

The operation optimization problem of a trigeneration system taking into consideration not only variation of loads with time but also the transient behavior of the storage tanks and the absorption chiller has been effectively solved. Even though it was not the objective of this work, the economic analysis supported by a parametric study revealed the optimal size of the absorption chiller, as well as critical values for the cost of fuel or electricity, above or below of

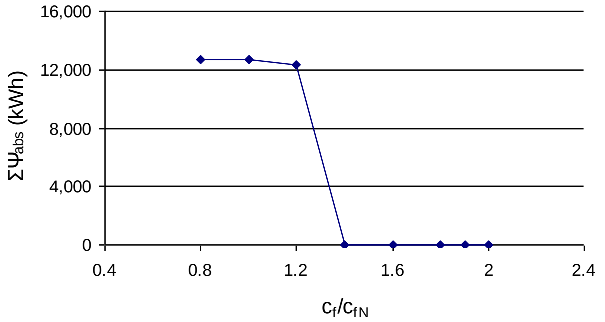

Figure 14. Effect of the cost of fuel on the total cooling energy supplied by the chiller.

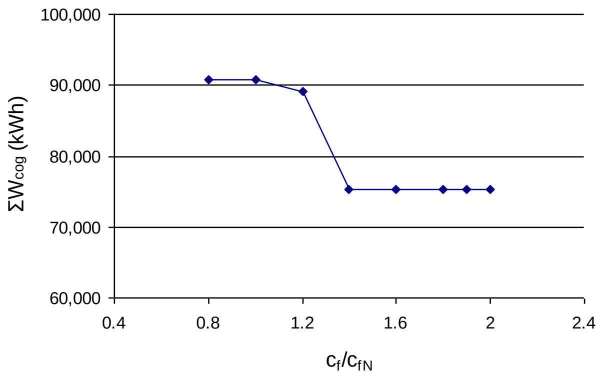

Figure 15. Effect of the cost of fuel on the total electric energy supplied by the cogeneration unit.

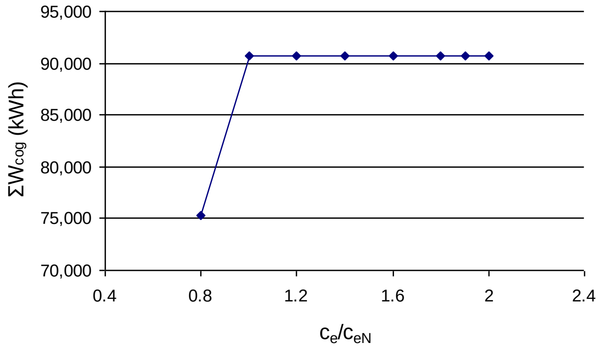

Figure 16. Effect of the cost of electricity on the total electric energy supplied by the cogeneration unit.

which, respectively, the operation of the absorption chiller is not economical.

The dynamic optimization problem has been solved by the SNOPT software, which is not initially developed for this type of problems. The difficulty encountered was that with a high number of time intervals, the computational time becomes very long. Therefore, a more efficient method and algorithm, appropriate for dynamic optimization is needed. Also it is worth investigating whether a formal application of thermoeconomic analysis could aid the solution of the problem. The revealing of the critical values of fuel and electricity costs, as mentioned in the preceding section, may give a hint towards this direction.

Nomenclature

A	external surface area of a water storage tank (m^2)
C_{ac}	annualized capital cost (Euros)
C_{ci}	investment cost of component i (Euros)

C_e annual cost of electricity purchased from the grid (Euros)
C_f annual cost of fuel (Euros)
C_m annual maintenance cost (Euros)
C_p annual personnel cost (Euros)
C_{tot} total annual cost (Euros)
COP coefficient of performance
CRF capital recovery factor
c_{ci} specific investment cost of component *i*, i.e. investment cost per unit of product, as defined by Eq. (2) (e.g. Euros/kWh)
c_p specific heat capacity (kJ/kg·K)
$c_{ji,k}$ constant parameters
d market interest rate (called also market discount rate)
G Gompertz function (*S*-curve)
$\dot{H}_{fcog}$ energy flow rate of the fuel consumed by the cogeneration unit (kW)
m mass (kg)
N_i technical life of component *i* (years)
$\dot{P}_{Di}$ production rate of component *i* at the design point (e.g., electric power output of the cogeneration unit, $\dot{W}_{Dcog}$, in kW)
P_i annual useful product of component *i* (e.g., annual electricity production of the cogeneration unit, in kWh)
$\dot{Q}_{abs}$ thermal power required by the absorption chiller (kW_{th})
$\dot{Q}_B$ thermal power of the boilers and the burner of the hot water tank (kW_{th})
$\dot{Q}_{cog}$ useful thermal power of the cogeneration unit (kW_{th})
$\dot{Q}_{cog,us}$ utilized thermal power of the cogeneration unit (kW_{th}) (the difference $\dot{Q}_{cog} - \dot{Q}_{cog,us}$ is wasted)
$\dot{Q}_{cons}$ heat flow rate supplied to the building complex (kW_{th})
T temperature (°C)
t time (hours)
$t_{ai,max}$ maximum expected period of operation of component *i*, taking into consideration reliability and availability (hours)
U overall heat transfer coefficient (kW/m^2·K)
$\dot{W}_b$ electric power bought from the grid (kW_e)
$\dot{W}_{chel}$ electric power consumed by the compression chillers (kW_e)
$\dot{W}_{cog}$ electric power of the cogeneration unit (kW_e)
$\dot{W}_{cons}$ electric power consumed by the building complex, excluding the power consumed by the compression chillers (kW_e)
x set of independent variables

Greek letters
$\dot{\Psi}$ cooling power (kW_c)

Subscripts
B boiler
b electricity bought from the grid
abs absorption chiller
amb ambient
chel compression chillers, electrically driven
cog cogeneration unit
cons consumed by the building complex
cwt cold water storage tank
D design point
hwt hot water storage tank
N nominal value
r room

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