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A Residual Thermodynamic Analysis of Inert Wear and Attrition, Part 1: Theory

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Abstract

An open-system irreversible thermodynamic analysis on inert wear and attrition is presented. The aim is to derive a theory which may be implemented in computational fluid dynamics flow solvers. In order to conduct analysis on the differential scale, it is shown that traditional macroscopic-scale concepts, earlier well-established in the literature, cannot be immediately applied. Hence, new differential concepts are introduced. It is argued that the overall analysis can be split up into sub-processes, where different types of specific sub-processes of wear and attrition may be extracted, and directly connected with the corresponding breakage or deformation of either ductile- or brittle-type target materials. Applying the residual thermodynamics framework, the new concepts of *wear work* and *attrition work* (at adiabatic conditions) can be defined, which typically only represent a small – often negligible – fraction of the total work.

Keywords: *Differential; residual; wear work; attrition work.*

1. Introduction

1.1 Scope of Present Paper

Traditional “simple” wear models [1] work excellently for developing new wear-resistant materials and analyzing specific engineering wear problems (of fixed geometry).

The traditional approach aims at analyzing apparent net wear in terms of experimental operating parameters as model input. For instance, Finnie’s single-particle ductile erosion model [2] and Archard’s empirical model [3] applied for ductile abrasion, are generally considered to represent well-proven models. In most situations, one may connect an empirical- or general black-box model to experiments.

Since the end of 1980’s, Eulerian Computational Fluid Dynamics (CFD) analysis on wear is attempted. The merits of a CFD analysis is discussed by *e.g.* [4-14].

The utilization or implementation of a traditional wear model in a CFD flow solver, is not straightforward. The problem at hand is that a CFD solver models a number of variables on the grid-cell scale, providing local forces, mass-flows, and momentum- and energy balances. If applying a traditional wear model, locally in a grid cell, and reversing the analysis, it can be shown that the traditional wear model input variables (which are different than the variables simulated by a CFD flow solver), combined with knowledge on the experimental conditions at which the traditional wear model was originally developed, would translate into a different set of Eulerian local forces, mass-flows, and momentum- and energy balances, as the set computed by the CFD flow solver. Hence, at the same grid cell position, a double set of the same Eulerian variables exists, which hence, creates serious problems of physical inconsistency. Several cases of erratic implementations of traditional wear models in CFD solvers can be found in the literature – and is discussed below.

The means of hitherto validating a CFD model is typically by comparing simulation results and/or behavioral trends with (supposedly) corresponding experiments. Remarkably often, test cases used for validation are comparatively complex, such as *e.g.* a transient-bubbling fluidized bed with internals subject to wear. A matter of concern is that while such a CFD validation test case in published literature often arrives at arguably fair or good correlating agreement – in terms of simulated accumulated wear *vs.* experimental accumulated wear – the application of the same CFD wear model in situations resembling laboratory wear test conditions (simple flows, steady-state conditions) can at the same time arrive at physically inconsistent results.

The present paper, asks if it is possible to develop an Eulerian differential theory which at the most simple outset (inert wear and attrition), and beyond the full accounting of force balances-, mass balances-, and momentum balances- (as simulated by a CFD flow solver), can also give a full accounting of changes in energy and entropy which may occur in connection with some basic modes of inert wear and attrition processes? Irreversible thermodynamics is utilized in this paper in order to investigate this.

1.2 Length Scales

The issue of length scales, relating to the matter of classifying and defining the processes, is important to address.

Consider the macroscopic scale – often the scale of the experimental apparatus – which is used for the description of wear behavior in terms of macroscopic flow structures (*e.g.* bubble diameter) or macroscopic conditions (*e.g.* sliding distance, total work, jet velocity, jet impact angle, etc.). Indeed, various classifications of wear modes in the traditional wear literature follow apparent differences on the experimental scale. Arguably, however, such categorizations cannot be directly translated to the differential scale.

Regarding the matter of defining the processes, the wear literature often presumes the possibility to extrapolate the operating conditions of an erosion experiment, far from the surface, to the vicinity of the target surface, with little error. To illustrate errors that follow from this for an Eulerian differential model, *cf.* Example 1. The widely-adopted belief that impact angles and impact velocity always represent key elements in an erosion process – regardless of length scales – has also had an influence on earlier tendered differential models, *cf.* Example 2.

These two examples illustrate that an impact angle cannot, and should not, be extracted in the vicinity of a target surface, if a steady-state particle *flow* – not to be confused with individual particle *motion* – is considered. Arguably, the established concept of ductile erosion is not well-defined on the differential scale.

EXAMPLE 1: According to the sciences of multiphase flow, in the vicinity of a non-worn solid target surface, the influence of the conservation of mass principle results in a locally spatially-time-averaged particle-flow velocity profile, which can be depicted as in Figure 1, where a boundary condition:

$$\mathbf{U}_{\mathrm{p,t}} = L \frac{\partial \mathbf{U}_{\mathrm{p}}}{\partial \mathbf{n}} \bigg|_{\text{target surface}} \qquad (1)$$

provides a slip flow velocity $\mathbf{U}_{\mathrm{p,t}}$ along the target surface. Here, L represents a slip-flow coefficient, and $\mathbf{n}$ represents the unit normal vector (pointing from the target surface into the flow field). The normal-direction flow velocity at the surface is zero, for a non-worn surface, since there is no particle flow through the surface. This boundary condition, Eq. (1), is used in Eulerian CFD flow solvers of multiphase flows, and is discussed further in [15].

If one would attempt to extract the erosion input parameters impact angle and impact velocity directly from a particle-flow field (in an Eulerian CFD flow field), for instance obtained in a computational grid cell in contact with the target surface (typically in the center position of the grid cell), will result in inconsistencies: A result when mesh refining is that the computed impact angle in the center-position of the grid cell will converge towards zero, which in turn results in computed erosion converging towards zero (for any flows). [It can in this context be noted that the literature suggests applying the Finnie model through following particle trajectories in CFD flow simulations. This may arguably work, if the particle phase is diluted, and the fluid phase has a negligible influence on the particles during the ductile cutting process (*i.e.* if the fluid phase represents vacuum or a low-pressure gas, *cf. e.g.* simulations of erosion of compressor blades in gas turbines by random dust particles [16-17]). However, although not within the scope of the present paper, one may find examples in the published literature where inconsistencies are obtained when attempting the same approach of following trajectories also for dense-flow situations, *e.g.* dense fluidized beds [18]. Clearly, Finnie's assumptions [2] on individual particles cutting interaction with the target surface to occur undisturbed by the surroundings, arguably does not hold.]

EXAMPLE 2: In the Eulerian granular flow erosion model [7] the particle-phase averaged flow vector in the vicinity of the target surface is modelled with a partial-slip boundary condition, in full accordance with Figure 1. Unique for the granular flow theory of multiphase flows, is that the particle-phase viscosity is modelled following an extension of the kinetic theory of gases applied to discrete particles. Hence, according to this theory, there exists a self-vibrating motion for the individual particles in the flow, assumed to occur in all angular directions. The vibrating-motion amplitude depends on several parameters possible to model in an Eulerian CFD computation, such as particle concentration, particle shape, the slip velocity and locally-time-space-averaged shear stress of the particle phase. Vibration motion of particles may also occur in the vicinity of the target surface, and hence incorporate "impact angles".

One may question the utilization of local particles fluctuations motion as model input for the Finnie model (only through instantaneous particle fluctuations – offset from the time-averaged flow field depicted in Figure 1 – are impact angles possible). It is clear that Ding & Lyczkowski [7] wish to combine a fully consistent flow modelling, *i.e.* following the averaged flow field depicted in Figure 1, with the traditional wear theory statements on the necessity of using impact angles and impact velocities in order to be able to model ductile erosion.

Consider, for instance, the application of this model to a diluted stream of impacting particles impacting a ductile target surface, with an experimental-scale impact angle and impact velocity. According to the multiphase flow literature, no particle fluctuations occur in dilute flows. Hence, with no fluctuations present, the granular-flow erosion model will predict zero erosion. Hence, the granular flow erosion model suffers from the inconsistency that despite being derived from the Finnie-erosion model, it is not capable of reproducing Finnie-erosion curves, when Finnie-type flow conditions are applied.

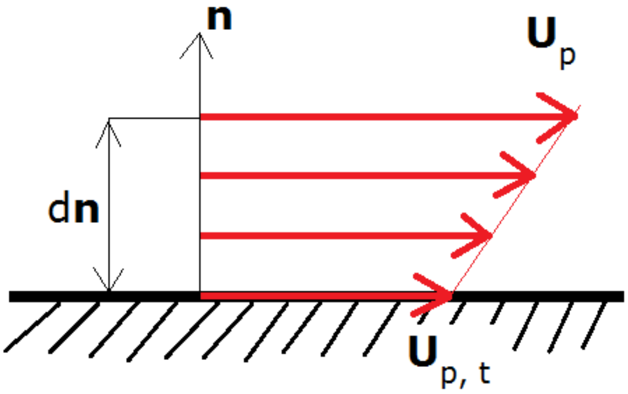

Figure 1. Particle-phase averaged flow vector in the vicinity of the target surface, modelled with a partial-slip boundary condition.

1.3 Fundamental Nature of the Processes

A good illustrative starting point on a discussion on the nature of processes involved is to consider a dense jet flow of particles impacting a target surface, below – and above – the threshold for onset of erosion.

Consider a comparable flow, well-known and well-described in fluid dynamics: a regular laminar jet flow of water impacting a target surface, without resulting in erosion, with nominal jet impact velocity and impact angle at a position far from the target surface. In fluid dynamics, the conditions at the target surface (which does not erode) are very well known: Since the fluid has a viscosity, the flow velocity is zero along the entire wall for all continuum length scales (on the molecular length scale, there is however a tangential slip-flow). No work-transfer to the wall occurs, unless the wall is moving, or possibly allows for a flow through (if the wall is permeable). However, a normal force can be identified acting along the wall surface, and also a tangential force acting on the wall surface can be identified, as a result of the fluid jet flow. Within the fluid flow, fluid flow gradients along the surface, as well as normal forces and shear stresses can be identified, *cf.* fluid dynamics literature.

If replacing the jet flow of water with a non-erosive dense jet flow of particles in a low-pressure gaseous environment, or vacuum, impacting the surface in a similar way, the particle flow – assumed to behave in a "fluidized manner" [19-20] – is principally (but not identically) similar to the impacting water flow. One distinction is that the corresponding "particle-phase viscosity" is much different from the viscosity of water. Some examples of such non-erosive flow simulations are visualized in [10], assuming sand-type particles. In case no erosion occurs, *i.e.* below the erosion onset threshold, no work transfer from the flow to the target surface occurs. While the normal-direction particle flow velocity along the wall surface is practically zero, a non-zero tangential slip-flow of particles is present along the wall, *cf.* Figure 1, very possibly incorporating fluctuations of the particles. Also, particle-phase flow gradients and particle-phase shear stresses can be observed.

A key question is what differences become evident in the dense particle jet, including the reflective flow, just above the threshold condition, when the onset of erosive ductile wear occurs?

Comparing the inflow and reflective flow, at both sub-threshold- and above-threshold conditions, where dense-jet erosion onsets, it is fairly obvious from experiments that the reflective flow does not change dramatically, in any way which would allow the experimentalist to clearly identify the flow behavior at which the reflective flow represents a sub-threshold erosive condition, or an above-threshold erosive condition. (This, however, might be the case in a Finnie experiment, for single particle impacts.) No such categorization of below- or above-threshold behavior has to date been reported in the literature, to the author's knowledge, for a dense particle jet flow.

The above experiment is interesting as regards describing the nature of a wear process:

Arguably, since wear is caused by an irreversible work transfer, from the flow into the target surface, the magnitude of this specific wear work is of interest to speculate on.

The lack of dramatic change in reflective flow behavior of a dense impacting jet, at the onset threshold of erosion, implies that only a small fraction of work from the total jet work rate can physically be assigned to connect with the erosion process.

In this context, it is interesting to note that in CFD simulations of erosive wear processes, the work transfer into the wall surface is typically set to zero. (It is practically difficult to remove work energy in a CFD flow solver at a target surface.) This is a viable approach, if the overall flow behavior appears almost indifferent to whether or not erosion occurs, and if it is possible to estimate alternative components in the flow which can be shown to connect with the specific wear process of interest, *cf. e.g.* [10].

These findings differ significantly from empirical wear models, *e.g.* Archard's model or Finnie's model, in which typically 100% of total work rate in the experiment is utilized in the model and modelling, *cf.* Example 3.

It can be contended that wear and attrition processes are non-linear irreversible processes, occurring at far-from-equilibrium conditions. The proposed nature of a wear process is that it represents a sub-process, one of several parallel sub-processes, which may be occurring simultaneously, and possibly also interacting with other sub-processes (*e.g.* friction). A specific wear work can be outlined, as a fraction of the total work, employing the residual thermodynamics framework [21]. In the approach proposed below, the thermodynamic energy of breakage or deformation is included in the thermodynamic analysis and plays a significant role. (If not incorporating this thermodynamic energy of breakage or deformation in the analysis, several problems follow – *cf. e.g.* Example 4.) Adopting the concept of wear work – which arguably directly connects with wear – appears to explain several earlier unexplained phenomena, such as the particle-size dependency on ductile wear, further discussed in [15].

EXAMPLE 3: Consider a traditional model which can be reformulated into the expression $\text{wear rate} = \text{constant} \times \left(\delta W/\mathrm{d}t\right)$ (applied for a closed-system analysis). The advantage with this approach is that the total work rate $\delta W/\mathrm{d}t$ is fairly straightforward to estimate. This approach provides practical advantages, in that by connecting net wear rate with total work rate there is no requirement to estimate the component $\mathrm{d}E_{\text{wear}}/\mathrm{d}t$, or consider sub-processes. (It is noteworthy that in most wear models, the component $\mathrm{d}E_{\text{wear}}/\mathrm{d}t$ is not modelled or considered at all.) Empirical analysis can be successfully performed with this model. The drawback, however, is that this simplified formulation does not allow refined understanding of the processes.

EXAMPLE 4: When studying some reports on thermodynamic analysis of wear reported in the literature, typically a closed-system approach is employed, resulting in considerations on total heat (or calculated entropy flow) and total work (or work dissipation). In case "degradation" is concerned, often the "entropy increase" of the entire machine component is considered. In earlier thermodynamics analysis in the literature, it is clear that there is no objective in estimating the wear work, or the component $\mathrm{d}E_{\text{wear}}/\mathrm{d}t$. (Typically the assumption $\Delta E_{\text{system}} = 0$ is made.) This unfortunately has a drawback in withholding a fundamental understanding of the processes involved.

To illustrate, the 1st law of thermodynamics states that for a closed system, if assuming $\Delta E_{\text{system}} = 0$ and steady-state conditions, $\delta Q/\mathrm{d}t = \delta W/\mathrm{d}t$. [This connection is for wear and attrition applications typically valid in most wear experiments, approximately, even when accounting for $\Delta E_{\text{wear}} \neq 0$, since typically $\left|\mathrm{d}E_{\text{wear}}/\mathrm{d}t\right| << \left|\delta W/\mathrm{d}t\right|$.] Hence, from discussion in Example 3 and in [15], one may formulate the Archard relation as:

$$\text{wear rate} = -\left(K^{\text{weak}}/\chi H\right)\left(\delta W/\mathrm{d}t\right) \tag{2a}$$

or

$$\text{wear rate} = -\left(K^{\text{weak}}/\chi H\right)\left(\delta Q/\mathrm{d}t\right) \tag{2b}$$

These expressions incorporate parameters from the original Archard relation.

If one would choose to look at accumulated wear in a steady-state experiment, one can re-express the latter Archard relation as:

$$\text{accumulated wear}\left(\text{time position}\, t_N\right) = -\left(K^{\text{weak}}/\chi H\right) \times \int_{t=0}^{t=t_N} \delta Q \tag{3a}$$

Assuming the experimental temperature at steady-state conditions to be nearly constant throughout the experiment, the latter Archard relation can be expressed as:

$$\text{accumulated wear} \cong -\left(K^{\text{weak}} T_{\text{av}}/\chi \text{H}\right) \int_{t=0}^{t=t_N} \frac{\delta Q}{T} \cong -\left(K^{\text{weak}} T_{\text{av}}/\chi H\right) \underbrace{\sum_N \frac{\Delta Q_N}{T_N}}_{\text{entropy flow}} \tag{3b}$$

The last relation is identical with the one used by [22] (and referred to in follow-up papers, *e.g.* [23-25]), in order to claim a connection between wear and entropy flow.

Six series of experiments at various operating conditions (total 60 minutes each, removing data from first 5 minutes) were reported in [22], for an abrasion experiment. The heat Q_N was indirectly calculated (through the total work), and the temperature T_N was recorded. Looking at the Doelling *et al.* experiments, it is clear that for all six tests in the time interval 5 minutes to 60 minutes, the temperature points in any of the six experiments did not deviate more than 2.3% from the average temperature $T_{\text{av}} \cong 329\,\text{K}$. When plotting the accumulated wear versus the indirectly-computed entropy flow in these experiments, a correlation was claimed at 2.5% margin of error [22].

The above accumulated wear expression represents a variable-transformed version of the original Archard relation, where the temperature reading has no other net effect than introducing some additional noise in the apparent correlation (approximately introducing noise at a level of 2.3%) between entropy flow and accumulated wear. It should, hence, not come as a surprise that the experiments made in [22] reproduced wear coefficients K^{weak}, in apparently close agreement to wear coefficients reported in the relevant literature, for similar abrasion experiments on similar materials.

The proposed connection between wear and entropy as presented by [22] can be questioned if assuming $\Delta E = 0$ during the wear process, *cf.* Example 6. This concerns also the "Degradation-Entropy Generation (DEG) Theorem" applied to wear, *cf. e.g.* [24, 26].

2. Theory

2.1 Preamble

Applying the Reynolds Transport Theorem [27] to the 1st law of thermodynamics [28], the energy equation for a fluid element (an open thermodynamic system) is obtained as follows:

$$\frac{\delta Q}{\mathrm{d}t} - \frac{\delta W}{\mathrm{d}t} = \frac{\mathrm{d}E}{\mathrm{d}t} = \frac{\partial}{\partial t}\left(\int_{\text{CV}} e\rho \mathrm{d}V\right) + \int_{\text{CS}} e\rho\left(\mathbf{U}\cdot\mathbf{n}\right)\mathrm{d}A \tag{4}$$

where e is the energy per unit mass of the fluid, ρ is density, $\mathbf{U}$ is the flow velocity vector, $\mathbf{n}$ is the unit normal vector, with either $\mathrm{d}V$ volume integration across the fluid element control volume CV, or surface integration $\mathrm{d}A$ across a fluid element control volume surface CS. [In engineering thermodynamics textbooks, the "pressure work" (one component of the general "flow work") in the left-hand side of Eq. (4) is carried over to the right-hand side to form enthalpy (*i.e.* the sum of internal energy and pressure work), which gives a different presentation of the energy equation.]

Equation (4) is in the following considered for both cases: open system encompassing a target surface material (for the thermodynamic analysis of wear), or an open system encompassing a particle flow (for the thermodynamic analysis of attrition). However, for wear, an exterior open system encompassing a particle flow, which is in direct contact with the target surface open system, is considered, when analyzing the work transfer to the target surface.

For an open system encompassing particles, or other phases, it should be observed that each component or phase – whether in gaseous, liquid or solid state – is considered separately. (The solid particles are modelled as a "fluidized" phase [20]).

The energy per unit mass (for each individual phase), e, may be subdivided into energy of several different types:

$$e = e_{\text{internal}} + e_{\text{kinetic}} + e_{\text{potential}} + e_{\text{surfaces}} + e_{\text{deformation}} + e_{\text{other}} \tag{5}$$

where e_{surfaces} represents the energy of surfaces, and $e_{\text{deformation}}$ represents the energy of deformation. The term e_{internal} represents the internal energy, e_{kinetic} represents the kinetic energy, $e_{\text{potential}}$ represents the potential energy, and e_{other} represents other types of energy (*e.g.* nuclear, chemical etc.) that are not considered to any detail in the following. For wear and attrition processes it is necessary to account for changes in e_{surfaces} and $e_{\text{deformation}}$.

The work rate exerted by the fluid element, may be subdivided into:

$$\frac{\delta W}{\mathrm{d}t} = \dot{W} = \dot{W}_{\text{pressure}} + \dot{W}_{\text{viscous stresses}} + \dot{W}_{\text{interfacial drag}} + \dot{W}_{\text{body,other}} \tag{6}$$

where the terms $\dot{W}_{\text{pressure}}$ (representing the pressure work) and $\dot{W}_{\text{viscous stresses}}$ (representing the viscous work) are found in both single-phase- and multiphase flows. For multiphase flows incorporating particles (or solids phase), one often needs to consider the work due to interfacial drag force $\dot{W}_{\text{interfacial drag}}$ [29] – a drag force which represents an interpenetrating *body force* resulting from the local velocity differences between the internal phases, such as liquid *vs.* solids, or gas *vs.* solids etc. Occasionally, for multiphase flows, additional types of body forces may be found as a result of interaction between the different phases, *e.g.* added mass force [30], history force [31], etc. – which may render other types of work, here grouped in $\dot{W}_{\text{body,other}}$. Although $\dot{W}_{\text{interfacial drag}}$ and in other cases $\dot{W}_{\text{body,other}}$ often have a dominant impact on generating specific flow structures (such as transient gas bubbles in fluidized beds) – which may yield apparent correlations between time-averaged wear rates and such flow structures [9, 32] – these terms evidently have no *direct* relationship with the basic mechanisms of wear and attrition. Eulerian differential-scale models for wear or attrition based on work of body forces violate the 1st law of thermodynamics. [For example, an erosion model based on the interfacial drag between a particle- and a surrounding gas phase (which has been proposed in the multiphase flow literature as a simple model of wear, *cf.* p. 38 in [33]), fails if applied in vacuum gas pressure conditions. The violation of the 1st law of thermodynamics is obvious, since this model would always predict zero erosion for all particle flows in vacuum.]

Arguably, wear and attrition mechanisms should be searched for within irreversible residual terms of the so-called total (flow) work of surface forces of a fluid element:

$$\dot{W}_{\text{total}}=\dot{W}_{\text{pressure}}+\dot{W}_{\text{viscousstresses}}=-\int_{\text{CS}}\left(-P\overset{=}{I}+\overset{=}{\tau}\right)\cdot\mathbf{U}\cdot\mathbf{n}\mathrm{d}A \tag{7}$$

where P is the pressure (of the relevant fluidized phase), $\overset{=}{\tau}$ is the Cartesian shear stress sensor (of the relevant fluidized phase), and $\overset{=}{I}$ is the Cartesian identity matrix.

2.2 Constitutive Relations

When applying the divergence theorem on Eq. (7), the differential-scale subdivision can be split up into a sum of 4 types [9, 19, 27] of flow work of surface forces, referred to as type (a), -(b), -(c) and -(d), *cf.* Eq. (4) in [9]. (Present paper presents these with opposite mathematical sign, in accordance with thermodynamic conventions.)

Arguably, some basic inert wear mechanisms can be accounted for through residual thermodynamic irreversible work expressions of terms (a) and (c), presented in Eq. (9) in [21]. Also, some basic inert attrition mechanisms for incompressible particles can be accounted for through residual thermodynamic irreversible work expressions of terms (b) and (d), presented in Eq. (9) in [21]. One simple illustration of arriving at this proposition is to consider an adjacent flow passing a small control surface CS with a mass flow rate $\dot{m}$:

$$\dot{m}\,\overline{w}=\dot{W}_{\text{total}}\Big|_{\text{CS}}\cong-\overline{\left(-P\overset{=}{I}+\overset{=}{\tau}\right)}\cdot\underbrace{\int_{\text{CS}}\mathbf{U}\mathrm{d}A}_{\overline{\dot{V}}}$$

$$=\left[\dot{m}=\alpha\rho\dot{V}=\right]=-\overline{\left(-P+\tau\right)}\frac{\dot{m}}{\overline{\alpha\rho}} \tag{8}$$

Here, w represents the work per unit mass, and $\dot{V}$ represents the volume flow rate. Next, applying the averaging locally, one obtains:

$$w=-\left(-P+\tau\right)\underbrace{\frac{1}{\alpha\rho}}_{v}=\left(P-\tau\right)v \tag{9}$$

which after differentiation provides a simplified representation of Eq. (9) in [21]:

$$\delta w=\underbrace{P(\mathrm{d}v)}_{\text{(a)}}+\underbrace{v(\mathrm{d}P)}_{\text{(b)}}-\underbrace{\tau(\mathrm{d}v)}_{\text{(c)}}-\underbrace{v(\mathrm{d}\tau)}_{\text{(d)}} \tag{10}$$

On the one hand, the phenomenon of wear is an irreversible process associated with a work transfer at a solid surface connected with an irreversible volume change in the target surface, *i.e.* $\left|(\mathrm{d}v)_{\text{wear}}\right|\geq 0$ is a constitutive relation. Volume change as a result of fracture [influencing e_{surfaces} in Eq. (5)] or deformation [influencing $e_{\text{defomation}}$ in Eq. (5)] may occur for wear processes. Comparing Eq. (9) in [21] with Eq. (10), this constitutive relation implies that term (b) and term (d) do not relate to wear phenomena. On the other hand, the phenomenon of attrition of incompressible ductile or brittle particles is an irreversible process not associated with any volume change of the particle phase undergoing attrition, *i.e.* $\left(\mathrm{d}v_{\text{p}}\right)_{\text{attrition}}=0$ is a constitutive relation. Comparing Eq. (9) in [21] with Eq. (10), this constitutive relation implies that terms (a) and (c) do not relate to attrition phenomena.

2.3 Residual Thermodynamic Analysis

Consider the 1st-law residual irreversible work expression Eq. (9) in [21], where *the thermodynamic flow* [21, 28, 34-35] *is set equal to* <u>*an invariant representation*</u> *of the local, instantaneous fluid flow, i.e.* $\dot{\mathbf{Y}}=\mathbf{U}_{\text{flow,invariant}}$. In order to extract relevant residual work mechanisms from Eq. (9) in [21] for the basic mechanisms of wear and attrition analyzed below, a careful selection of real- and ideal process is required [21], *cf.* Section 3.

It can be noted that the thermodynamic flow $\dot{\mathbf{Y}}=\mathbf{U}_{\text{proc}}\equiv\mathbf{U}_{\text{noproc}}\equiv\mathbf{U}$ of the real- and ideal process is *different* from a corresponding computational fluid dynamics flow field simulation (utilizing Navier-Stokes equations or similar) resulting in $\mathbf{U}$, for a situation in which wear or attrition does not occur.

For the case of inert attrition of incompressible particles, one may – for sake of simplicity – start with considering the *stationary* residual sub-process of attrition occurring in an open system at adiabatic conditions.

Applying Eq. (4) for the particle phase, and only for the residual sub-process, provides the following 1st law result:

$$-\delta W_{\text{res, attrition}} / \mathrm{d}t = \mathrm{d}E_{\text{attrition}} / \mathrm{d}t > 0 \tag{11}$$

At stationary conditions:

$$\frac{\partial}{\partial t} \int_{\text{CV}} e_{\text{attrition}} \rho \mathrm{d}V = 0 \tag{12}$$

which gives:

$$\mathrm{d}E_{\text{attrition}} / \mathrm{d}t = \int_{\text{CS}} e_{\text{attrition}} \rho (\mathbf{U} \cdot \mathbf{n}) \mathrm{d}A \tag{13}$$

i.e. the exiting particles have greater energy than incoming ones – due to the attrition process. For an entropy balance relation, *cf.* Example 5.

For the case of inert wear of an incompressible target surface, one may – for sake of simplicity – start with considering the *stationary* residual sub-process of wear occurring in an open thermodynamic system, encompassing only the target surface region. A stationary wear process can be analyzed assuming an open thermodynamic system, at adiabatic conditions, in which exiting mass flow represents debris of the target material (as a result of the wear process), assume to occur evenly across the target surface, which is balanced by an incoming mass flow of target material on the opposite-side open system boundary by a continual shift of the adjacent system boundary, in order to maintain a strictly time-independent total mass of this open system. Applying Eq. (4) for this open system, and only for the residual sub-process, provides the following 1st law result:

$$-\delta W_{\text{res, wear}} / \mathrm{d}t = \mathrm{d}E_{\text{wear}} / \mathrm{d}t > 0 \tag{14}$$

It is important to note that the residual work transfer is exerted by an exterior fluid element, *i.e.* outside this target surface open system, by discrete interaction. At stationary conditions, for cutting- and brittle wear (but *not* for deformation wear, *cf.* note below):

$$\frac{\partial}{\partial t} \int_{\text{CV}} e_{\text{wear}} \rho \mathrm{d}V = 0 \tag{15}$$

which gives:

$$\mathrm{d}E_{\text{wear}} / \mathrm{d}t = \int_{\text{CS}} e_{\text{wear}} \rho (\mathbf{U} \cdot \mathbf{n}) \mathrm{d}A \tag{16}$$

i.e. the exiting target material debris has greater energy than incoming target material – due to the wear process. For an entropy balance relation, *cf.* Example 5.

Note: In case deformation wear occurs, the target surface material is not incompressible. A stationary process is not possible to model, since the hammering effect compresses the target surface material, and continually adjusts the material properties. (Typically, the hardness is gradually increased, hence the material-scientist's descriptive phrase "work-hardening" of materials by deformation wear.) In any case, applying Eq. (4) for the non-stationary case, for an open system boundary in which the total mass of the open system is unchanged, since no debris is created, there is no influx of target material in this hypothetical open system. Hence:

$$\int_{\text{CS}} e_{\text{deformation wear}} \rho (\mathbf{U} \cdot \mathbf{n}) \mathrm{d}A = 0 \tag{17}$$

for this case, and applying Eq. (4) gives:

$$\begin{aligned} -\delta W_{\text{res, deformation wear}} / \mathrm{d}t &= \mathrm{d}E_{\text{deformation wear}} / \mathrm{d}t \\ &= \frac{\partial}{\partial t} \int_{\text{CV}} e_{\text{deformation wear}} \rho \mathrm{d}V > 0 \end{aligned} \tag{18}$$

where the latter inequality confirming the deformation wear process is not modelled as a stationary process. In any case, also for the case of deformation wear, it is possible to associate a residual work exerted by an exterior fluid element, resulting in an irreversible increase in energy of the target surface open system.

Hence, Eq. (9) in [21] incorporates different types of residual work, potentially available in the total flow work of surfaces forces, which may directly connect with different forms of breakage (ductile and brittle), *i.e.* resulting in a residual work transfer to the target material, balanced by a residual change in ΔE (and corresponding increases in entropy).

In earlier thermodynamics studies, the discussion on entropy – or entropy generation – was vague. Arguably, increases can be claimed, but which entropy is referred to? Example 6 illustrates this problem, when consistently assuming the energy change ΔE to be zero.

EXAMPLE 5: For real wear and attrition processes, assuming the effective thermodynamic forces [21] to behave according to the right-hand-side residual process illustrated in Figure 1 in [21], one may assume that at high thermodynamic flow rates, *i.e.* considerably above the threshold level at which wear or attrition commences:

$$(\mathbf{F}_{\text{proc}} - \mathbf{F}_{\text{noproc}}) \underbrace{\propto}_{\text{approximately}} \overset{\bullet}{\mathbf{Y}} = \mathbf{U}_{\text{flow}} \tag{19}$$

Hence, with the above selection of thermodynamic flow $\overset{\bullet}{\mathbf{Y}} = \mathbf{U}_{\text{flow}}$ gives:

$$\left(\frac{\mathrm{d_i} S_{\text{res}}}{\mathrm{d}t} \right)_{CV} \underbrace{\propto}_{\text{approximately}} |\mathbf{U}_{\text{flow}}|^2 \tag{20}$$

according to Eq. (7a) in [21].

For the stationary process conditions as described in this Section (excluding deformation wear), the entropy balance for respective open system is:

$$0 = \left(\frac{\mathrm{d}S_{\text{res}}}{\mathrm{d}t} \right)_{CV} = \left(\frac{\mathrm{d_e} S_{\text{res}}}{\mathrm{d}t} \right)_{CS} + \left(\frac{\mathrm{d_i} S_{\text{res}}}{\mathrm{d}t} \right)_{CV} \tag{21}$$

which gives:

$$\left(\frac{\mathrm{d_i} S_{\text{res}}}{\mathrm{d}t} \right)_{CV} = -\left(\frac{\mathrm{d_e} S_{\text{res}}}{\mathrm{d}t} \right)_{CS} \tag{22}$$

Considering the entropy change in the “universe”, *i.e.* open system + its surroundings, as a result of the irreversible residual sub-process, gives:

$$\frac{\mathrm{d}S_{\mathrm{res}}}{\mathrm{d}t} = \left(\frac{\mathrm{d}S_{\mathrm{res}}}{\mathrm{d}t}\right)_{CV} + \left(\frac{\mathrm{d}S_{\mathrm{res}}}{\mathrm{d}t}\right)_{\mathrm{surroundings}}$$

$$= \left(\frac{\mathrm{d}S_{\mathrm{res}}}{\mathrm{d}t}\right)_{\mathrm{surroundings}} > 0 \qquad (23)$$

[For deformation wear, corresponding expressions can be formulated.]

Arguably:

$$\frac{\mathrm{d}S_{\mathrm{res}}}{\mathrm{d}t} = \left|\dot{W}_{\mathrm{res}}\right| \Big/ T > 0 \qquad (24)$$

Hence, when:

$$\mathrm{d}s_{\mathrm{wear}} / \mathrm{d}t = \left|\dot{w}_{\mathrm{wear}}\right| \Big/ T \qquad (25)$$

and

$$\mathrm{d}s_{\mathrm{attrition}} / \mathrm{d}t = \left|\dot{w}_{\mathrm{attrition}}\right| \Big/ T \qquad (26)$$

one obtains:

$$\dot{w}_{\mathrm{wear}} \underbrace{\propto}_{\mathrm{approximately}} \left|\mathbf{U}_{\mathrm{flow}}\right|^2 \qquad (27)$$

and

$$\dot{w}_{\mathrm{attrition}} \underbrace{\propto}_{\mathrm{approximately}} \left|\mathbf{U}_{\mathrm{flow}}\right|^2 \qquad (28)$$

The quadratic relationship between wear rates and thermodynamic flow correlates well with extensive experiments (at high relative velocities) for ductile materials, *cf.* comparisons [15] with Finnie-erosion- [2] and Archard [3] models.

EXAMPLE 6: Earlier approaches to connect thermodynamics to wear (for closed systems) typically proposed a connection between entropy and “degradation” of the target object. Of multiple problems with earlier treatment of the thermodynamics, one key question to pose is what entropy is referred to, in earlier work? The question is relevant, since earlier work consistently proves to assume $\Delta E = 0$ throughout the wear process.

To illustrate a fundamental problem with analyzing the wear in terms of entropy, while simultaneously assuming $\Delta E = 0$, is that for inert, incompressible samples, *e.g.* polishing/machining an object without introducing any accumulated change in internal material structure of the final object, the analysis will necessarily become identical to the thermodynamic treatment of a so-called “pure substance” in thermodynamics. This means that when studying the process by providing a system boundary to incorporate the original object, and throughout the process allowing the system boundary to incorporate the worn-down object together with the removed material from the original object, so that total mass studied is unchanged during the process, the entropy of the system only changes with temperature.

In any case, if the entropy change cannot be expressed in other terms than $s = s(T)$ throughout the wear process, how do we connect entropy with wear? And, can we really study entropy flows as a means to obtain the entropy change, *cf. e.g.* [22], which in turn would give us a measure of wear?

3. Interpretations

3.1 Wear Work

The irreversible work transfer to the target surface resulting in wear is balanced by corresponding residual 1st-law expressions of the flow work of surface forces acting in the vicinity of the target surface. A work residual based on the following comparison is proposed for the analysis of wear:

$$\left.\frac{\delta W}{\mathrm{d}t}\right|_{\mathrm{wall}} = \dot{W} = \dot{W}_{\mathrm{moving\,boundary,rev}} + \dot{W}_{\mathrm{shaft,rev}} \qquad (29a)$$

$$\text{(no-process conditions)}$$

$$\left.\frac{\delta W}{\mathrm{d}t}\right|_{\mathrm{wall}} = \dot{W} = \dot{W}_{\mathrm{moving\,boundary,rev}} + \dot{W}_{\mathrm{shaft,rev}} + \dot{W}_{\mathrm{surface,irr}}$$

$$\text{(process conditions)} \qquad (29b)$$

In the traditional fluid dynamics literature, a mechanical work transfer between a fluid- or fluidized flow and a solid wall occurs only when the wall is moving. A tangential-direction movement as a result of a so-called “shaft work” $\dot{W}_{\mathrm{shaft,\,rev}}$, is balanced by the action of viscous stress work (of surface forces) [27]. A normal-direction movement as a result of a so-called “moving boundary work”, $\dot{W}_{\mathrm{moving\,boundary,rev}}$, is in turn balanced by the action of pressure work (of surface forces). These two modes of basic work can be considered *reversible* processes as long as no local deformation or local fracture of the solid wall occurs. (Note: In the case of a shock wave work transfer, a temporary local deformation occurs when the translational wave passes the solid wall. A temporary local deformation of a solid wall is generally *not* a reversible process – since such a deformation is in a real material always associated with internal friction.)

The *irreversible* work transfer $\dot{W}_{\mathrm{surface,\,irr}}$ incorporates the actual wear work, along with other irreversible work occurring simultaneously, such as a shock wave transfer/reflection work $\dot{W}_{\mathrm{transfer/reflection}}$. [An example of a reflected and/or transferred irreversible work loss residual associated with impact wear (drop erosion, cavitation erosion) is sound. Sound is an irreversible work transfer passing through the target surface and/or an irreversible work transfer reflected back into the fluid (if not vacuum), hence included in $\dot{W}_{\mathrm{transfer/reflection}}$.] In addition, in connection with wear processes there may be other types of irreversible work, $\dot{W}_{\mathrm{etc.}}$, not identified in this paper.

[Sound may also be generated in connection with term (c)-type wear, here included in $\overset{\bullet}{W}_{\mathrm{etc.}}$.]

Regarding the issue of friction, it is important to stress that an unknown portion of the apparent friction in experiments occur within the exterior fluid flow field, here labelled as “external” friction. This external friction relates to the irreversible conversion of work into internal energy within the exterior fluid element. For both the no-process- and process conditions for wear, it is clear that the external friction in the exterior fluid element does not result in any work transfer to the target surface, and hence does not appear in Eqs. (29a)-(29b).

On the other hand, considering the process conditions, *i.e.* when wear in the target surface occurs, it is reasonable to assume that in connection with the local deformation or local breakage in the target surface, a certain amount of the irreversible work loss may occur in the target surface, resulting in irreversible conversion of work into internal energy (in the vicinity of the breakage or deformation zone). Hence, for the process conditions, a certain fraction of the net irreversible work transfer $\overset{\bullet}{W}_{\mathrm{surface,\,irr}}$ can be associated with a sub-process occurring within the target surface which results in irreversible conversion of work into internal energy. This resembles a different friction process occurring within the target surface, occurring in conjunction with the wear process. Hence, a certain fraction of work incorporated in $\overset{\bullet}{W}_{\mathrm{surface,\,irr}}$ must exist, here referred to as wall friction work $\overset{\bullet}{W}_{\mathrm{friction}}$, which accounts for this latter “internal friction” sub-process, occurring in the target surface.

It should be noted that this wall friction work $\overset{\bullet}{W}_{\mathrm{friction}}$ is weakly connected to the total work, and weakly- or not at all connected with the external friction. The reason for this follows the unknown split-up of respective external- and internal friction, both of which contribute to the apparent friction (which is recorded in experiments).

The wear literature states that the apparent friction does not necessarily correlate with wear – generally speaking – while some studies in the wear literature implies such correlations may exist, at least locally [1]. However, on the other hand, below it is argued that the wall friction work $\overset{\bullet}{W}_{\mathrm{friction}}$ is locally proportional to the net irreversible work transfer $\overset{\bullet}{W}_{\mathrm{surface,\,irr}}$ – an assumption that is validated in [15].

The residual irreversible work exerted by the exterior fluid element, transferred to the target surface, can be expanded by Eq. (9) in [21] and Eqs. (29a)-(29b) into:

$$
\begin{aligned}
\overset{\bullet}{W}_{\mathrm{surface,\,irr}} &= \overset{\bullet}{W}_{\mathrm{wear}} + \underbrace{\overset{\bullet}{W}_{\mathrm{transfer/reflection}} + \overset{\bullet}{W}_{\mathrm{friction}} + \overset{\bullet}{W}_{\mathrm{etc.}}}_{\overset{\bullet}{W}_{\mathrm{other,irr}}} \\
&= -\int_{\mathrm{CS,\,wall}} \alpha\left(\overset{=}{T}_{\mathrm{proc}} - \overset{=}{T}_{\mathrm{noproc}}\right)\cdot \mathbf{U}\cdot \mathbf{n}\,\mathrm{d}A \\
&= \underbrace{\int_{\mathrm{CS,\,wall}} \alpha\left(P_{\mathrm{proc}} - P_{\mathrm{noproc}}\right)\left(-U_2\right)\mathrm{d}A}_{\propto\ \text{impact or cavitation wear/erosion (term (a))}} \\
&\quad + \underbrace{\int_{\mathrm{CS,\,wall}} \alpha\left(\tau_{\mathrm{proc,21}} - \tau_{\mathrm{noproc,21}}\right)\mathrm{U}_1\,\mathrm{d}A}_{\propto\ \text{ductile erosion or abrasion (term (c))}} \\
&\quad + \underbrace{\int_{\mathrm{CS,\,wall}} \alpha\left(\tau_{\mathrm{proc,22}} - \tau_{\mathrm{noproc,22}}\right)U_2\,\mathrm{d}A}_{\propto\ \text{brittle erosion or work-hardening (term (c))}}
\end{aligned}
\qquad (30)
$$

for the 2D case, where the outward-pointing unit normal direction vector at the lower side of the fluid element in virtual contact with the solid wall is $\mathbf{n} = (0,-1)$ for the co-ordinate system defined by direction 1 in the wall plane, and 2 in the normal direction from the wall surface (direction into the fluid). Terms (b) and (d) of Eq. (9) in [21] – associated with attrition processes – are not included in the irreversible work transfer of Eq. (30). (Corresponding expressions may be presented for the general 3D case.)

The specific selection of work residual in Eqs. (29a)-(29b) has practical reasons. Arguably, it is fair to assume a – more or less – locally-valid direct correlation between $\overset{\bullet}{W}_{\mathrm{wear,\,irr}}$ and $\overset{\bullet}{W}_{\mathrm{other,irr}}$, *i.e.* allowing one to assume an apparent linear correlation $\overset{\bullet}{W}_{\mathrm{wear,\,irr}} \propto \overset{\bullet}{W}_{\mathrm{other,irr}}$, which in turn gives $\overset{\bullet}{W}_{\mathrm{wear,\,irr}} \propto \overset{\bullet}{W}_{\mathrm{surface,\,irr}}$. Since the residual work $\overset{\bullet}{W}_{\mathrm{surface,\,irr}}$ is more easily estimated than its individual components, a grouping according to Eqs. (29a)-(29b) is employed here.

Hence, through apparent linear correlations it is possible to associate different types of wear with their corresponding irreversible residual work. For instance – as is also indicated in Eq. (30) – along the surface one may find a shear-work transfer associated with term (c) due to a slip flow, which results in *ductile wear* [15], *e.g.* ductile erosion (Figure 2) or ductile abrasion (Figure 3). Alternatively, a normal-component dissipative work transfer associated with term (c) may occur, from which momentum exchange between particles and the target surface may generate brittle erosion or deformation wear/work hardening (Figure 4).

Also, as indicated in Eq. (30), a pressure-work indentation associated with term (a) (and a thermodynamic flow represented by the indentation velocity $\overset{\bullet}{\mathbf{Y}} = (0,-U_2)$ at the solid surface) results in impact wear (Figure 5).

To conclude, there are two basic Eulerian differential work mechanisms that generate inert wear:

$$
\overset{\bullet}{w}_{\mathrm{pressure,surface,\,irr}} = \left\{\alpha\left(P_{\mathrm{proc}} - P_{\mathrm{noproc}}\right)\nabla\cdot\mathbf{U}\right\}\Big|_{\mathrm{target\ surface}} \qquad \text{(term (a)-type)} \qquad (31)
$$

$$
\overset{\bullet}{w}_{\mathrm{viscous,surface,\,irr}} = \left\{-\alpha\left(\overset{=}{\tau}_{\mathrm{proc}} - \overset{=}{\tau}_{\mathrm{noproc}}\right):\nabla\mathbf{U}\right\}\Bigg|_{\mathrm{target\ surface}} \qquad \text{(term (c)-type)} \qquad (32)
$$

which summed together represent $\overset{\bullet}{w}_{\mathrm{surface,\,irr}}$.

Also, $\overset{\bullet}{w}_{\mathrm{term\ (a)\ wear}} \propto \overset{\bullet}{w}_{\mathrm{pressure,surface,\,irr}}$ and $\overset{\bullet}{w}_{\mathrm{term\ (c)\ wear}} \propto \overset{\bullet}{w}_{\mathrm{viscous,surface,\,irr}}$ can be assumed, locally.

The operator $(\bullet)\big|_{\text{target surface}}$ indicates mechanisms obtained in the vicinity of the target surface.

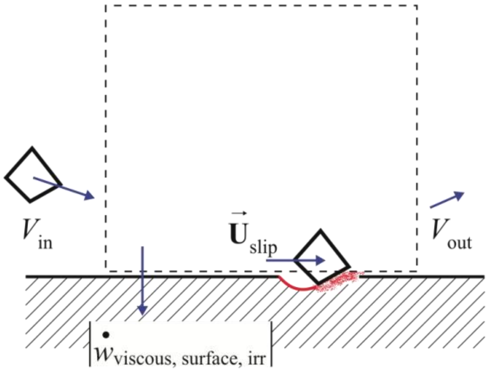

Figure 2. Wear of a target surface by a net shearing viscous work transfer, referred to in the literature as a ductile erosion process. A corresponding tangential slip flow, for any impact angle, can be assigned [15]. [In all figures in this paper, the hashed line indicates the control volume (or system boundary) following thermodynamic conventions.]

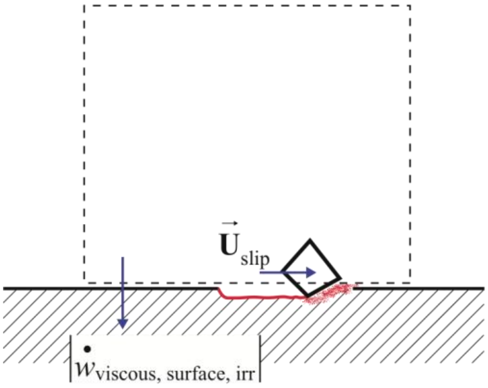

Figure 3. Wear of a target surface by a shearing viscous work transfer, referred to in the literature as abrasion. It should be noted that the slip flow is tangential to the surface.

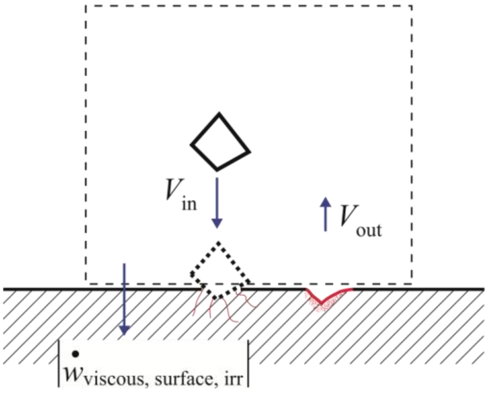

Figure 4. Erosion of a target surface by a normal viscous work transfer, resulting in brittle erosion (irreversible surface fracture of brittle-type materials) or deformation wear/work hardening (irreversible indentation of ductile-type materials). It should be noted that there exists no slip flow for this mode of erosion at the target surface.

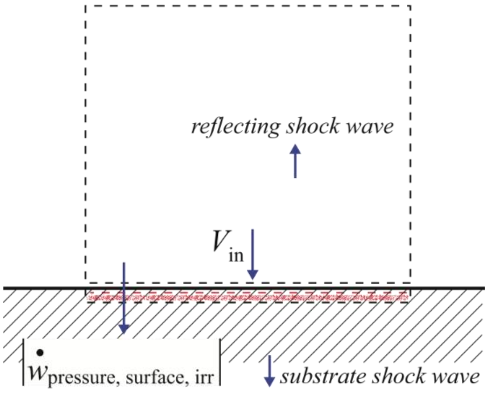

Figure 5. Impact wear of a target surface by pressure work (irreversible boundary deformation). It should be noted that there exists no slip flow for this mode of wear at the target surface. The wear model based on local pressure-work indentation may be used to model phenomena such as shock impact erosion, hammering effects, rain impact, and cavitation erosion.

3.2 Attrition Work

The specific selection of thermodynamic flow $\dot{\mathbf{Y}} = \mathbf{U}_{\text{flow, invariant}}$ leads one – following corresponding steps of analysis – to state effective thermodynamic forces of inert attrition as $\nabla\left(\alpha\left(P_{\text{proc}} - P_{\text{noproc}}\right)\right)/T$ and $\nabla\cdot\left(\alpha\left(\overset{=}{\tau}_{\text{proc}} - \overset{=}{\tau}_{\text{noproc}}\right)\right)\Big/T$, where T is the absolute temperature in Kelvin.

Hence, one can identify two basic irreversible work mechanisms that generate inert attrition:

$$\dot{w}_{\text{brittle attrition, irr}} = \mathbf{U}\cdot\nabla\left(\alpha\left(P_{\text{proc}} - P_{\text{noproc}}\right)\right)$$

(term (b)-type) (33)

$$\dot{w}_{\text{ductile attrition, irr}} = -\mathbf{U}\cdot\left(\nabla\cdot\left(\alpha\left(\overset{=}{\tau}_{\text{proc}} - \overset{=}{\tau}_{\text{noproc}}\right)\right)\right)$$

(term (d)-type) (34)

The sum of these two mechanisms equals the net attrition work. It is hereby proposed that the first term be defined as the *mechanism of brittle attrition (for inert, adiabatic flows)*, and the second term as the *mechanism of ductile attrition (for inert, adiabatic flows)*, of incompressible particles. Their definitions follow a separation of mechanisms resulting in attrition due to residual pressure work (Figure 6), or attrition due to residual viscous shearing work (Figure 7). [The author has not found any earlier attempts in the literature to correlate attrition processes to Eulerian fluid dynamic mechanisms.]

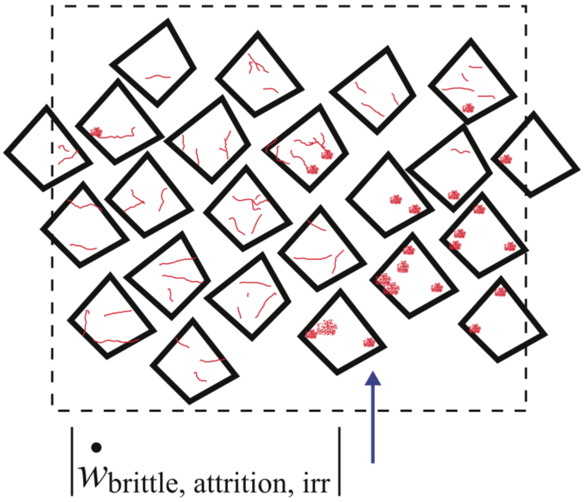

Figure 6. Attrition by surface forces pressure work, resulting in an irreversible increase of surface energy by fracturing of a brittle-type particle phase medium.

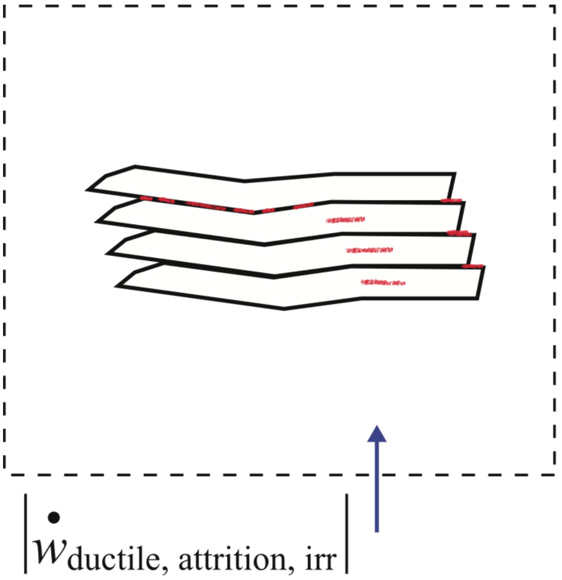

Figure 7. Attrition by surface forces viscous work, resulting in an irreversible increase of surface energy by irreversibly elongating of a ductile particle phase medium.

4. Conclusions

In the literature there exists no widely-approved general set of differential equations, possible to utilize in an Eulerian CFD flow solver, which connects the computational variables flow gradients, density gradients, pressure gradients, slip flow at boundaries, etc., within a single fluid element, to compute inert wear or inert attrition.

The relevant wear literature has earlier stated [1] that "A challenge facing the research community is the production of a sound theoretical framework to underpin [the] design aids [of analytical or computational models for describing the involved processes]".

The present paper approaches the analysis of inert wear and attrition utilizing irreversible thermodynamics as such a theoretical framework. The aim is to develop a physically consistent differential theory based on the computational variables simulated in a multiphase CFD flow solver. The present theory connects the thermodynamic flow with the physical flow (which is simulated by the CFD solver). The target material behavior, models wear and attrition in terms of a change in energy. (In the present paper, the target material is broadly categorized as being either ductile or brittle.)

By performing a thermodynamic process comparison, the new concepts "wear work" and "attrition work" (at adiabatic conditions) can be defined. These may be connected with different residual components of various fractions of irreversible total work of surface forces, of the multiphase flow. A discussion is provided on the different available residual work mechanisms and their potential interaction with-, and influence on the target material. In addition, the new differential concepts of ductile and brittle attrition have been defined, as well as the new concept of ductile wear. Also, a differential mechanism for impact wear has been developed.

The application of residual thermodynamics allows for the study of far-from-equilibrium, non-linear, and non-continuum processes. The present derivation does not utilize linear phenomenological equations, between thermodynamic flows and thermodynamic forces. The treatment of the thermodynamic flow, *e.g.* the space-time-averaged particle flow as a continuum flow, does not imply that the thermodynamic processes modelled are continuum processes. From a thermodynamic point of view, the derived differential work-loss mechanisms represent discrete interactions, incorporating non-linear threshold behavior.

It is believed that the present theory may be developed further. One possible direction of further development would be to consider other traditional classifications of wear and attrition, not covered in present paper, for differential-scale analysis. The possibilities to implement chemical reactions, and more complex target material behavior, in a differential-scale analysis, may also be investigated.

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Nomenclature

A	area (m^2)
CV	fluid element control volume (-)
CS	fluid element control volume surface (-)
E	energy (J)
e	energy, per unit mass ($J\,kg^{-1}$)
$e_{\text{deformation}}$	energy of deformation, per unit mass ($J\,kg^{-1}$)
e_{internal}	internal energy, per unit mass ($J\,kg^{-1}$)
e_{kinetic}	kinetic energy, per unit mass ($J\,kg^{-1}$)
e_{other}	other types of energy, per unit mass ($J\,kg^{-1}$)
$e_{\text{potential}}$	potential energy, per unit mass ($J\,kg^{-1}$)
e_{surfaces}	energy of surfaces, per unit mass ($J\,kg^{-1}$)
$\mathbf{F}$	intensive thermodynamic force

H	hardness ($\mathrm{N\,m^{-2}}$)
$\overline{\overline{I}}$	Cartesian identity matrix (-)
K^{weak}	wear coefficient used in relation between total work and wear rate (-), *cf.* [15]
L	slip flow coefficient (m)
$\dot{m}$	mass flow rate ($\mathrm{kg\,s^{-1}}$)
$\mathbf{n}$	surface unit normal vector (-)
N	index (-)
P	pressure of the relevant fluidized phase ($\mathrm{N\,m^{-2}}$)
Q	heat (J)
S	entropy ($\mathrm{J\,K^{-1}}$)
s	entropy per unit mass ($\mathrm{J\,kg^{-1}\,K^{-1}}$)
$\frac{\mathrm{d}S}{\mathrm{d}t}$	entropy change rate ($\mathrm{J\,K^{-1}\,s^{-1}}$)
$\frac{\mathrm{d}s}{\mathrm{d}t}$	entropy change rate per unit mass ($\mathrm{J\,kg^{-1}\,K^{-1}\,s^{-1}}$)
T	temperature (K)
$\overline{\overline{T}}_k$	Cartesian stress sensor of phase k ($\mathrm{N\,m^{-2}}$)
t	time (s)
U_i	component of mean velocity vector ($\mathrm{m\,s^{-1}}$)
$\mathbf{U}$	mean velocity vector ($\mathrm{m\,s^{-1}}$)
$\mathbf{U}_k$	mean velocity vector of phase k ($\mathrm{m\,s^{-1}}$)
$\mathbf{U}_{k,\mathrm{t}}$	mean (slip) velocity vector of phase k at the surface ($\mathrm{m\,s^{-1}}$)
V	volume ($\mathrm{m^3}$)
v	specific volume ($\mathrm{m^3\,kg^{-1}}$)
$\dot{V}$	volume flow rate ($\mathrm{m^3\,s^{-1}}$)
W	work (J)
w	work per unit mass ($\mathrm{J\,kg^{-1}}$), or work per unit volume ($\mathrm{J\,m^{-3}}$)
$\dot{W}_{\mathrm{body,other}}$	work rate due to other types of body forces ($\mathrm{J\,s^{-1}}$)
$\dot{W}_{\mathrm{interfacial\,drag}}$	work rate due to interfacial drag force ($\mathrm{J\,s^{-1}}$)
$\dot{W}_{\mathrm{moving boundary,rev}}$	moving boundary work rate ($\mathrm{J\,s^{-1}}$)
$\dot{W}_{\mathrm{pressure}}$	pressure work rate ($\mathrm{J\,s^{-1}}$)
$\dot{W}_{\mathrm{shaft,rev}}$	shaft work rate ($\mathrm{J\,s^{-1}}$)
$\dot{W}_{\mathrm{surface,irr}}$	irreversible work (transfer) rate (to target surface) ($\mathrm{J\,s^{-1}}$)
$\dot{W}_{\mathrm{total}}$	total work rate of surface forces ($\mathrm{J\,s^{-1}}$)
$\dot{W}_{\mathrm{viscous stresses}}$	viscous work rate ($\mathrm{J\,s^{-1}}$)
$\mathbf{Y}$	extensive thermodynamic flow

Greek letters

α_k	average occurrence of phase k (-)
χ	ratio of tangential to normal force acting on gross contacting area (-)
ρ	density ($\mathrm{kg\,m^{-3}}$)
τ	shear stress of the relevant fluidized phase ($\mathrm{N\,m^{-2}}$)
$\overline{\overline{\tau}}$	Cartesian shear stress sensor of fluidized phase k ($\mathrm{N\,m^{-2}}$)

Subscripts

1, 2	co-ordinate directions in Cartesian co-ordinate system
attrition	for process attrition
av	average
k	phase index
N	index
no proc	excluding specific sub-process of interest (*e.g.* no wear, no attrition)
p	particulate phase
proc	including specific sub-process of interest (*e.g.* wear, attrition)
res	for residual sub-process
wear	for process wear

Special notations

$(\cdot)\vert_{\mathrm{target\,surface}}$	conditions in vicinity of target surface
$\delta(\cdot)$	inexact (or imperfect) differential
$\mathrm{d}(\cdot)$	differential
$\Delta(\cdot)$	difference
∇	nabla operator
$(\cdot)\vert_{\mathrm{wear}}$	sub-process wear
$(\cdot)\vert_{\mathrm{attrition}}$	sub-process attrition
$\overline{(\cdot)}$	space-time averaging operator
$\dot{(\cdot)}$	rate
$\frac{\mathrm{d}}{\mathrm{d}t}, \frac{\delta}{\mathrm{d}t}$	time derivative ($\mathrm{s^{-1}}$)

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