



Fractal Dimension of Islamic Architecture: The case of the Mameluke Madrasas: Al-Sultan Hassan Madrasa

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Abstract

Islamic architecture represents a successful example in extracting the mathematical proportions and the fractal geometry of the natural organisms. The Mameluke architecture is considered a transitional stage to a more self-similar detailed geometry presented in a diverse scale range. That was the motive behind using the fractal geometry as a patterned grid in Mameluke designs. Therefore, this research objective is to reveal the hidden dimensions within the fractal geometry in Mameluke architecture with special emphasis on Al-sultan Hassan madrasa as a case study. Fractal geometry exists within its geometry in four levels; the internal spaces main subdivisions, floor patterns, al-muqarnas and ornaments. Thus, the research establishes an interactive parametric model, which has two reversible functions; First, to analyse by tracing the fractal geometry evolution of Al-sultan Hassan madrasa layout and secondly, to apply the fractal dimension as a design generator to more advanced fractal forms. Al-sultan Hassan madrasa represents the likelihood of analysing and generating further styles based on its fractal geometry. The process could be applied supplemented with the parameters and limitations change. Hence, an infinite number of design variations are generated based on the fractal geometry of a specific style.

1. INTRODUCTION

Fractal geometry dates back to centuries before the emergence of the fractal theory by Mandelbrot. The study of the Fractal geometry reveals that it has the potential to capture continuous and infinite processes, to metaphorize the universe by unity and subdivision [1, 2]. This distinctive feature is obviously seen in spiritual and sacred architecture. As a biomimicry approach, the sacred architecture had interpreted natural organisms' behaviour into geometries that possess the same characteristics. Hindu temples, Gothic cathedrals and Islamic mosques possess fractal properties [1-5]. Hindu temples [1, 3, 4] reflected it in 3-Dimensional form. Similarly, Gothic cathedrals has fractal characteristics in the external form and constructional elements at least in four iterations.

Islamic Architecture, as well, uses an abstract symbolized geometry that is extracted from the natural objects. The intricacy of this geometry increased gradually throughout the eras ranging from the simplest pure basic shapes to more complex ones. In the early Islamic period, architecture was in its simplest form using pure square shaped layout, gradually more geometrical transformations are applied such as; shapes addition and subtraction, subdivisions and branching, translation and rotation. Complexity reaches the maximum values in the late Islamic period, Mamluk and Ottoman architecture. It was a normal reflection to the diversity of the Islamic society at that time. As a result, rhythmic features, fractal subdivision and self-similar geometries are commonly applied to emphasize the function of the mosques and madrasas as shown in **Figure 1**.

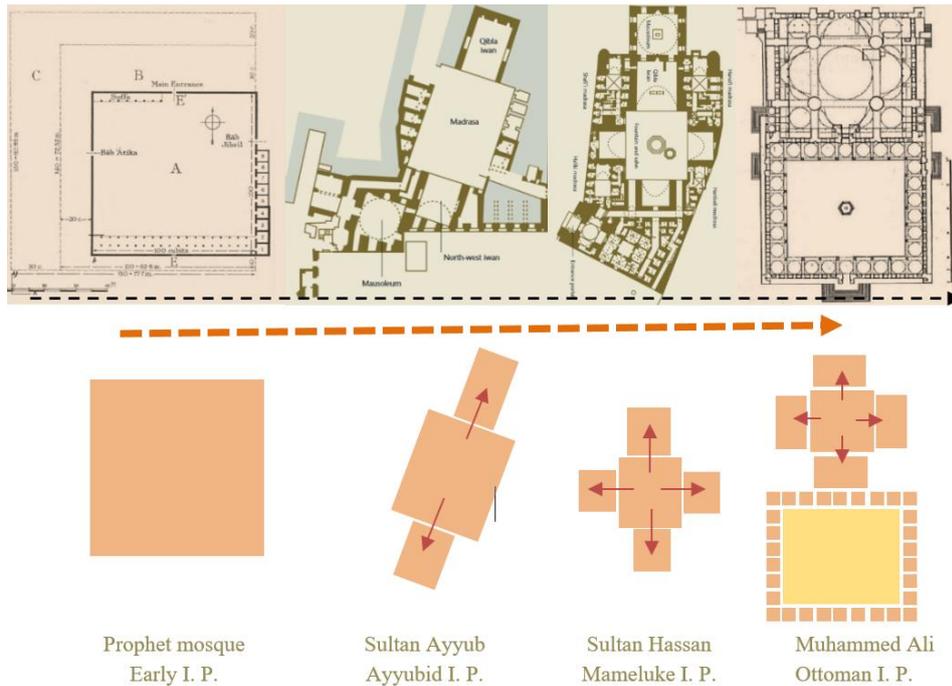


Figure 1. The evolution of the Mosque complexity by the time from simple squared mosque to more complex fractal forms in the late Islamic periods. Self- similar spaces, around the central court, increase by branching and subdivision

The Mameluke period nourishes the arts and geometry in the Islamic architecture. It is considered the practical beginning of using patterned grid which organize the internal and external features by fractal geometry. Despite of using pure and simple forms, fractal geometry is implemented among various scale in the layout subdivisions as well as the smallest scale of ornamental details. Fractals articulate the hierarchy of the internal spaces from a central courtyard to the aisles and consequently the surrounding spaces. The elevations are enriched by using self-similar geometries which are proficiently applied in the details of arches and openings. In addition to using fractals as a construction details under the domes and in the upper parts of the entrance arches named as muqarnas.

The major design concept of the mamluk madrasa is using the square as an origin shape on which transformations are applied; such as subdivisions and branching. In the meantime, the use of square root proportions and the golden ratio regulated systematically relations between the main square and the subdivided ones. Previous researches analyse the Islamic architecture as Euclidean geometry through the construction and ratios only. However, the design process undergoes a dynamic process of geometry evolutions. An embedded force formulates the spaces in a fractal form. It hovers from as unit state to subdivided state to articulate the design objectives as shown in the

Figure 2.

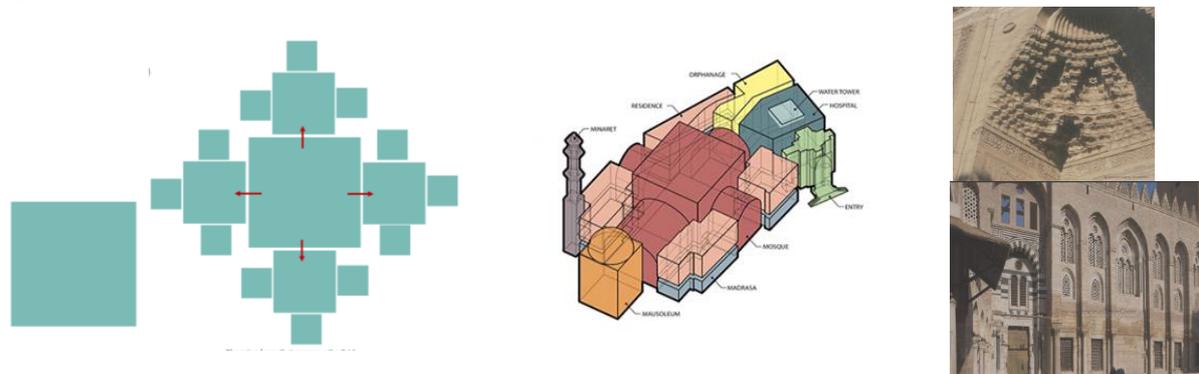


Figure 2. the self-similar forms in the mameluke architecture in Alsultan Hassan Mosque spaces, almuqarnas and Alsultan Qalawun Elevation

This paper studies the evolution process of the fractal geometry in Al-Sultan Hassan madrasa in Cairo as a well-known architectural masterpiece that symbolizes the diversity of Islamic philosophy and society during this era. The madrasa is an institutional building that signifies the Mameluke architecture. It is a multifunction building further than only being a prayer place. It is a complex centre composed of a prayer hall, a university, a social space, a dormitory and a mausoleum. Al-sultan Hassan madrasa magnificently presents the fractal systems in design concept. It is reflected on the plan subdivision, 3d forms subdivision and the architectural details. The patterns are originated from a fractal grid, which has self-similarity and self-replication characteristics. It exceeds the meaning of branching, repetition and modularity in the Islamic designs to new explorations of scaled self-similar pattern.

This study will lead to three benefits:

- 1) Revealing the ability of applying the evolution of natural forms in formulating the architectural style, particularly the fractal morphogenesis.
- 2) Deducing the fractal dimension of the mamluk architecture, as an archetype that could be applied on other comparable styles, by the fractal analysis using relevant software. The result is a value located on the fractal range scale between 1 and 2.
- 3) Introducing the researchers and professionals to the interactive process of exploring an architectural style starting from the analysis process, by tracing the fractal geometry changes, and the generative process by adding mathematical and geometrical proportions to the fractal dimension as a design generator.

The paper is divided into three main sections. First, Islamic architecture is literally reviewed in terms of geometry, proportions and patterns. Secondly, the fractal geometry, its properties, forces and benefits are examined. Thirdly, a parametric model is established to visualize the existing relationships into graphical frameworks. The established model compiles the major rules and proportions of the Al-Sultan Hassan madrasa supported with its fractal geometry. The fractal-based design process is set up by parametric platform, Rhino, grasshopper, and Python script language to perform the recursive process. History and theories of architecture researchers could easily test the model to investigate and trace the fractal geometry evolution of a specific style in addition to the self-similar pattern. Moreover, the fractal-based design provides an opportunity to acquire new methods of generating an infinite number of variations based on the fractal geometry of a specific style.

2. GEOMETRY IN ISLAMIC ARCHITECTURE

The system of geometric design in Islamic art and architecture basically starts from a square or a circle [5]. The patterns are generated from two main geometric theories; rotating a square or squaring the circle. Muslim mathematicians have presented studies in mathematics and geometry extracted from natural organisms. Makrizi and Al-Khawarizmi introduced algebra to modern mathematics through examples derived from Islamic inheritance laws [6]. Al-Buzjani [7] had presented in his treatise, “Those Geometric Constructions Which Are Necessary for a Craftsman”, calculation methods and geometric techniques to calculate simple and complex shapes in Islamic architecture such as domes, arches, tiles, as well as ornaments. He explained the main proportional systems in the Islamic patterns which reflect the natural systems and harmony. Euclidian rules are commonly used to present the Islamic patterns as shown in Figure 3&4 as well [5, 8]. These patterns possess a dynamic fractal feature as well as the Euclidean proportional system.

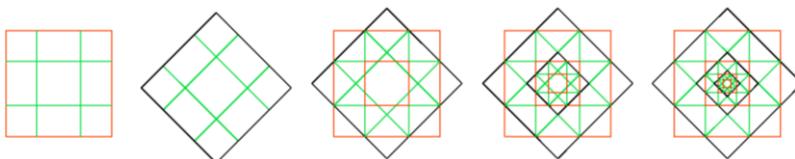


Figure 3. Islamic star formation by using the parameter of rotating the angle 45 degree [5]

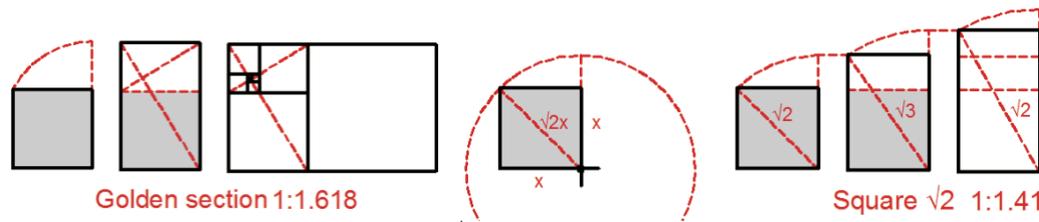


Figure 4. The difference between the $\sqrt{2}$ ratio, golden ratio applying the Pythagoras theory [9].

In most Mameluke madrasas, geometry and its subdivisions articulate design principles; orientation towards the qibla direction, simplicity inside complexity as an overall form, symmetry and balance in both sides of the madrasa and sometimes in the four quarters of the madrasa. Rhythm is apparent in the number of arcades “riwaqs” that define the madrasa patterned grid. The plan is either formed by addition around a central shape or division of a whole shape. Then a series of iterations occurs to formulate the entire form. The central square acts as a reference point or datum for all other plan components. The hierarchy is clear between the large central squared courtyard compared to the surrounding prayer Iwans. The spaces dimensions decrease towards the students’ cells gradually. This articulates the property of self-similar repeated squares either by branching or subdivision. The number of iterations is limited to the areas and the number of required spaces. The spaces of the same function are grouped together around the affiliated space. Although, this concept exists in the internal and external madrasa’s elevations as well, the research focus is on the plan layout of the Mameluke madrasas. The extraction will not be limited to the final form of the design but to the process in how it is formed. It is the deep study to the relationships between geometries and the binding rules between them. Tracing the formulation process opens a wide range of possibilities to change one parameter, accordingly the results will be easily changed.

This fractal approach aids the researchers and the professionals in studying Islamic architecture and geometry in more interactive way. it would probably provide a good reference and subliminal source of inspiration for the imitators of a historical style in a contemporary setting that might occurs in very limited designs. Such inspiration related to the process behind the design more than the imitation of its physical layout or features. The mosque of Jeddah in Figure 5, for example, is an example that shows direct and unsuccessful imitation for Al-sultan Hassan madrasa.

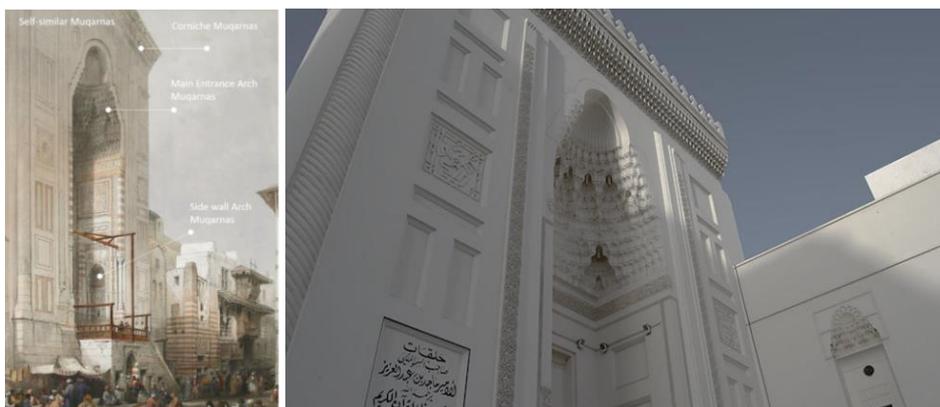


Figure 5. Jeddah’s Largest Mosque (right) resembles the entrance of Al-sultan Hassan (left) typically without advanced modifications [10]

3. THE FRACTAL SYSTEMS

Fractal Evolution in natural organisms is first mentioned in the pioneer book “on growth and form by D’Arcy Thompsons. As argued by D’Arcy, that “the form of an object is a ‘diagram of forces’ that from it we can judge of or deduce the forces that are acting or have acted upon it” [11, p. 33]. This reveals the reason for the difference between the final forms of natural organisms in the way they grow and evolve. Some organisms grow by branching and some by subdivisions to serve the function of its existence. The Fractal system represents one of the systems by which some organisms adopt to grow and subdivide. They

are commonly applicable to art and architecture as well. In 1977, Benoit released his fractal theory that applies the fractal geometry in the digital arts. He is a pioneer in calculating the Fractal dimension of irregular shapes. Later, architects such as Bovill [12] and Lorenz [3] explained the relation between the fractal geometry and architecture. They illustrate common characteristics, in the classical orders and modern architecture, such as: the presence of the initial shape on which rule is applied, self-similarity, self-affine, iteration process and the infinite complexity. Dependency occurs in the fractal geometry, that is why fractal is parametric. In some examples, transformations are applied by scaling or rotation or translation or altogether. Koch curve in Figure 6, for example, is a regular geometry that resemble the evolution of the snowflake in nature. According to the fractal geometry, it has fractal characteristics that signify it from the Euclidean geometry. It has also Fractal dimension that expresses how fast a fractal curve tends to infinity from one iteration to the next or how completely a fractal appears to fill space added to measuring the coefficient of self-similarity [3, 13, 14]. Ostwald and Vaughan deeply studied the architectural style in terms of their fractal dimension. They confirm the theory that any objects, including architecture, could have Fractal dimension but not all possess Fractal geometry.

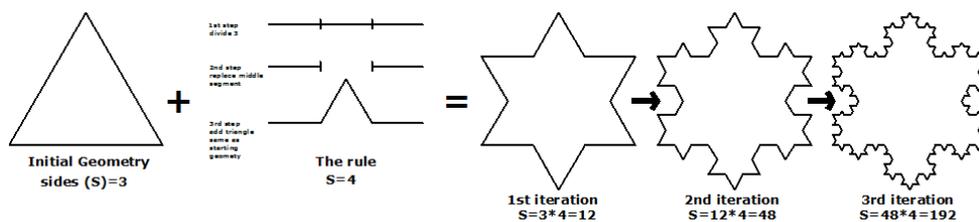


Figure 6. Koch curve in three iterations; a rule is applied to the middle third of each side of the equilateral triangle [12]

The Fractal Dimension D is a measure of the extent to which a structure exceeds its base dimension to fill the next dimension. It is the power relation between the number of pieces and the reduction factor [13]:

$$A = (1/s)^D \quad (1)$$

Fractal dimension D could thus be defined as:

$$D = \frac{\log(a)}{\log(1/s)} \quad (2)$$

where a = number of pieces and s = scaling factor [13]

4. THE FRACTAL GEOMETRY OF AL-SULTAN HASSAN, CAIRO

In Al-sultan Hassan madrasa, the square is the principle shape of the madrasa layout. The Ikhwan Al-Safa considered the square one of the stable shapes on which Islamic architecture depends [15]. The square represents the four main Islamic directions spiritually and articulate the qibla direction. The central square undergoes some transformation; stretching in the qibla direction. It is proportional to the adjacent squares by specific ratio either the square root ratio or the golden ratio. Hence, the addition or subtraction process of geometries is endless with an infinite number of iterations. The only limitations are the design guidelines and goals.

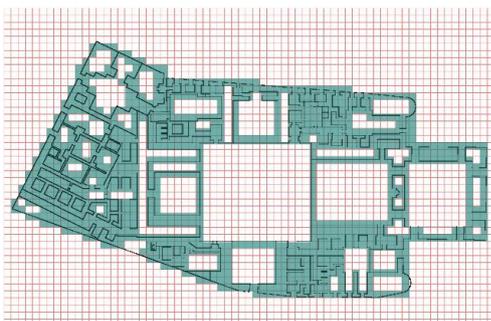


Figure 7. Al-sultan Hassan spatial subdivision [16]

The patterned grid is originated from the heart of a central square illustrating the courtyard. It is exposed to successive morphological transformations, translation and scaling. The first iteration results in forming the four Iwans representing the four Islamic curricula. In the meantime, it preserves the $1:\sqrt{2}$ proportional ratio [17] with the central courtyard side length. The second iteration generates smaller spaces surrounding each Iwan which represent the classrooms and the dormitory spaces. The third iteration is applied to subdivide the marble flooring patterns of the central courtyard. The floor is subdivided into nine squares and each square is fractally subdivided into proportional squares [18] approximately resembles the Sierpinski curve. If the evolution process extends to the next iteration, it will reveal more fractal proportions for the decorative wall ornaments. The generic diagram in Figure 8 assumes that the squares are equal in sides. Then, it is modified and stretched to articulate the qibla direction. Accordingly, one can say that Al-sultan Hassan madrasa is a living example of the evolution of the fractal geometry of the Mameluke architecture.

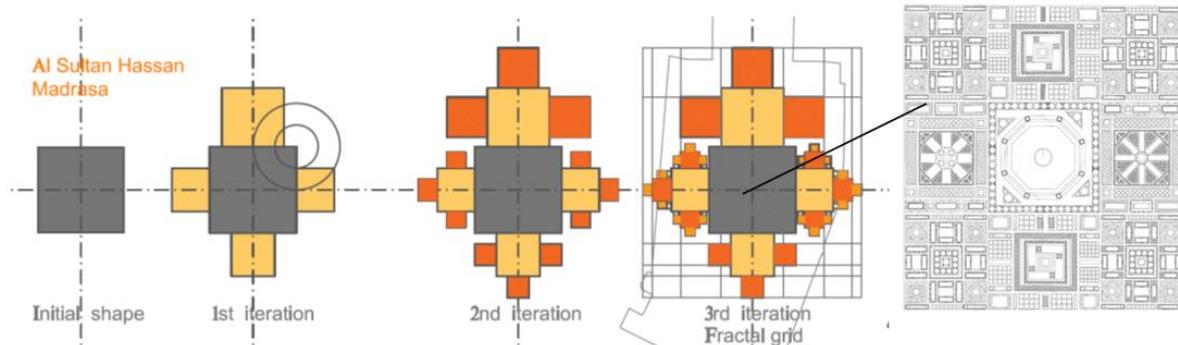


Figure 8. A generic diagram for Al-sultan Hassan fractal Geometry iteration process (the author), the floor pattern drawing [18]

5. THE FRACTAL-BASED DESIGN PROCESS

The process of generating a fractal-based design is mainly based on its fractal dimension as a principle design generator. In the case of Al-sultan Hassan madrasa, Firstly, the layout undergoes geometrical and proportional analysis of the central core, the prayer hall and the surrounding spaces. The second step is the compilation of the deduced rules and geometrical relationships along with the fractal dimension. While in the synthesis process, each space is drawn relatively to the adjacent spaces by number of parametric and proportional rules. The reference length is the courtyard side length. Finally, the exploration process creates a number of variations. Initially, the CAD drawings are optimized to single line drawing neglecting the wall carvings. The focus is on the court-Iwans relationship neglecting the secondary spaces between this zone and the site borders because it is the common feature among all the Mameluke madrasas. The plan is oriented such that the qibla direction represents the Y- Axis. The court and the Qibla Iwan are stretched in the Y-axis by a value equals to 0.1 or 0.2 the length of the court side. Next, the fractal dimension is calculated by substitution in equation 2. The Fractal dimension is the same as the quadratic Koch curve equivalent to 1.16 which is approximate value to the golden ratio. Then compiling this analysis in a parametric model to achieve the research goal as shown in Figure 9.

The fractal objects have a parametric property; such that all its parts are bonded by a relation in which the part is affected by the whole. The perpendicularity and the parallelism relationships of the Mameluke madrasa are converted into parametric rules¹ that conserve the relationships between geometries in a variable condition. Using a simple slider bar, changing one value leads to the change of all the dependencies accordingly. Therefore, Grasshopper facilitate a graphical establishment of the parametric model. Twelve parameters are derived to control the model representing the length and width of each spaces, parameters of the proportions and the fractal dimension.

¹ Parametric design is a rule-based design that associates objects in a set of relations that have variable values [20] to introduce a fully organized controllable geometry and adapt it to specific circumstances [21]. The parametric framework is an abstract form made of geometric components [21, 22, 23] to allow the lengths of lines, the angles between lines and other geometrical parameters to be instantly changed.

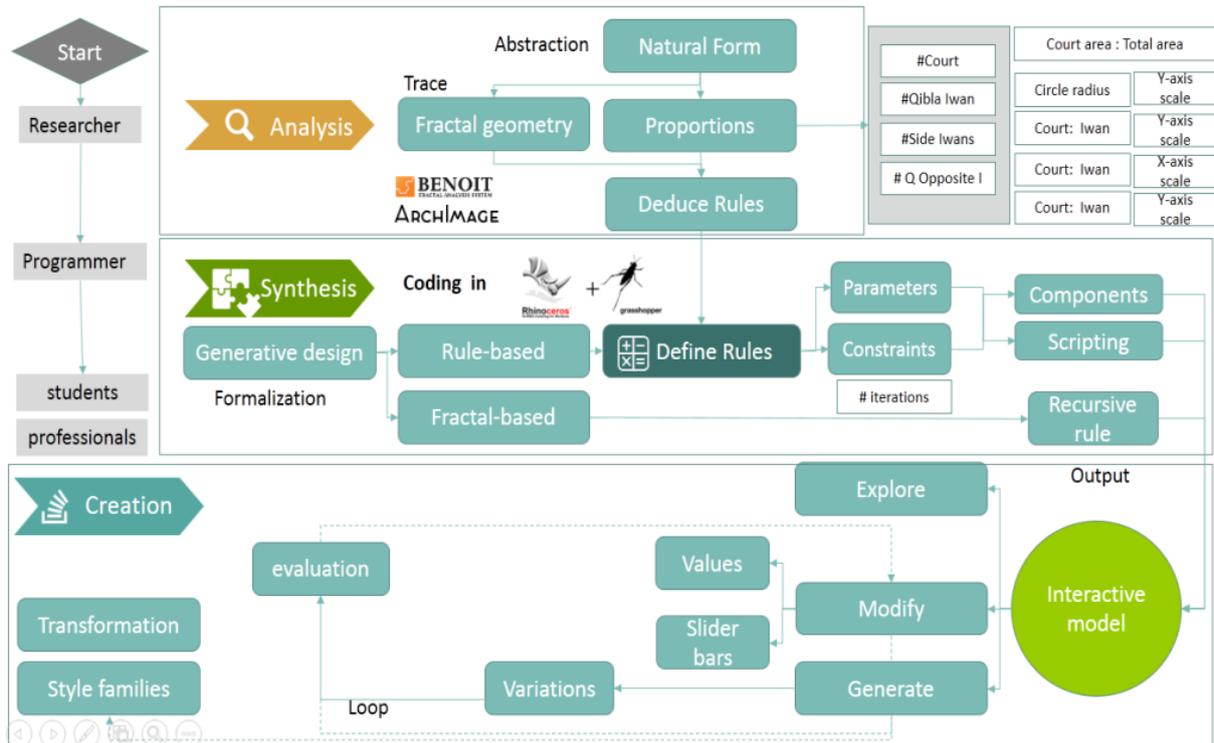


Figure 9. The fractal-based design process and used parameters (Author)

Fractal dimension represents the factor of self-similarity. It is limited to maximum and minimum values of the central square length. The simpler the geometric relations, the faster will be the calculations. Input values, as slider bars are connected to the rule components as well as the output values from both sides. Components could be a shape definition or mathematical equation. Connections illustrate the dependencies paths between the components. Slider bars provides instant change of the values by just moving the slider to the left and right. The recursive process is built up by Python language to represent the loop command of the fractal process.

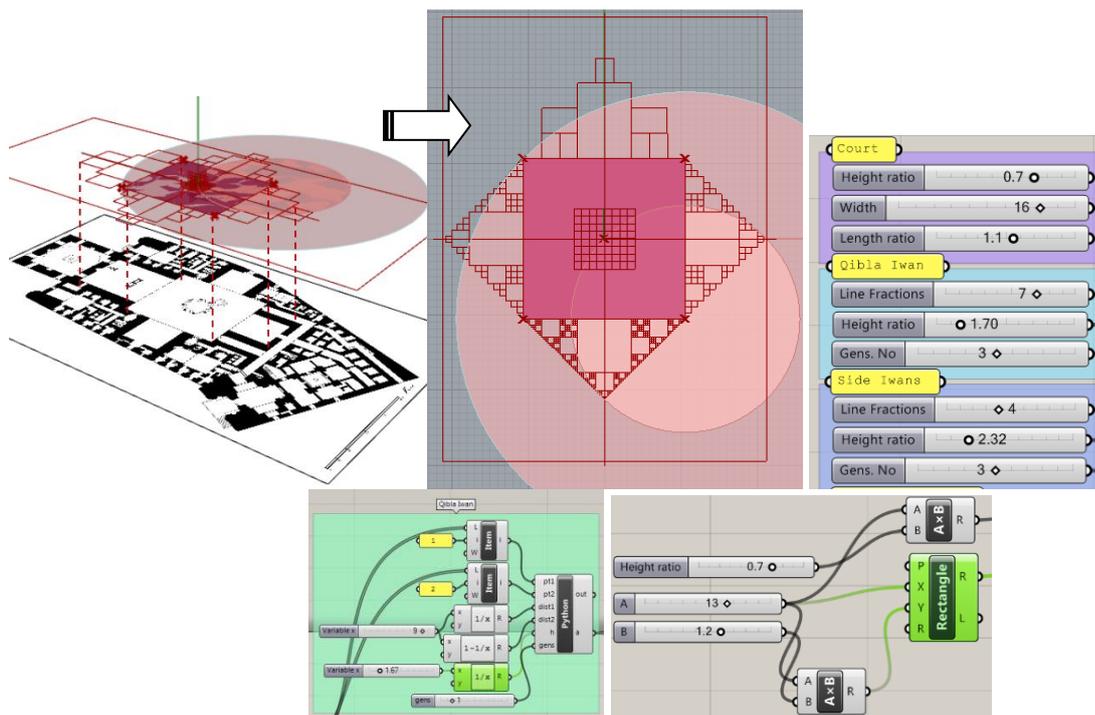
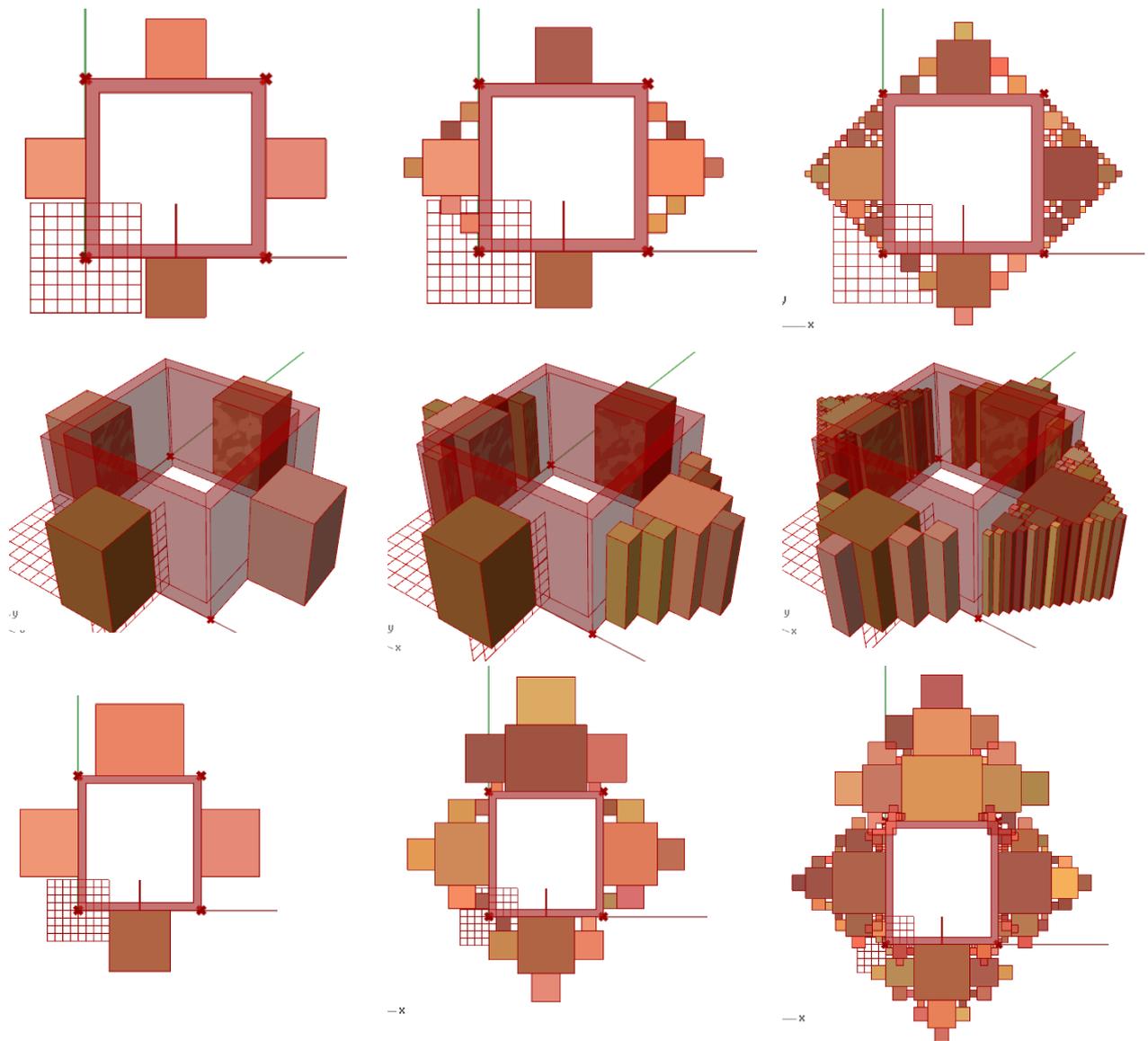


Figure 10. Overlaying and tracing the madrasa plan by the moving the values of the sliderbars in Grasshopper (Author)

Generally, the model is a user-friendly interface which enables the exploration of alternatives without programming background. Given these major parameters as inputs as well as fractal dimension and number of iterations, a group of relationships will increase the flexibility to change the options of the generated variations. Figure 10 is a screen shot, taken from the Grasshopper which shows the components, variables and the linking connections. The first step in interactive analysis process is overlaying the model on the architectural drawing and modifying the slider bars until reaching the approximate values. This process is not limited to the case study only, but it could be used to trace any other hypostyle building of comparable characteristics. The parametric framework is prepared to conjunct the analysis process of the Mamluk building and reinserting the data into synthesis process.

The final stage is the exploration process. The following Figure 11 shows the variations resulted by changing variables. The 3-dimensional form just to illustrate the concept but height is disintegrated with the 2d fractal grid of the plan. The heights of the court and the heights of the subdivided spaces could be varied according to designer vision. The first variation presents the generic case, it is equidistant equivalent square. Parameters values are as follows, Number of iterations=3, fractal dimension= 1.16, Fractions ratio = 1/3, Qibla and opposite Iwans have 2 recursions while side Iwan 3 recursions, the scaling factor in the qibla direction is zero. The second variation represents the data based on parameters of Al-sultan Hassan Madrasa Fractal Dimension= 1.16, scaling factor in the qibla direction is =1.1, with one iterative process in the qibla Iwan and two iterations in the side Iwans.



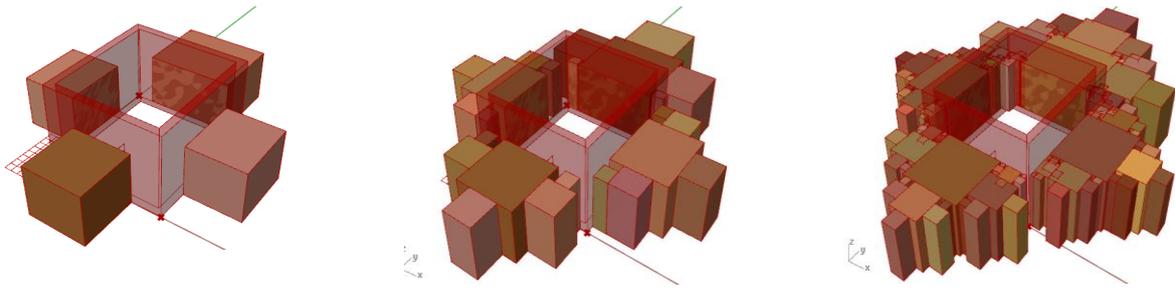


Figure 11. The exploration process of new variations based on AL sultan Hassan Madrasa (author)

6. CONCLUSION

The metaphor of Islamic forms is not limited to imitating the final forms of a plan or an element of the elevation design. The Mamluk madrasa has undergone several iterations to reach its final. The tracing of the fractal morphogenesis of the Mamluk architecture reveals the beauty of the organized and ordered complexity of the mamluk madrasa design. Therefore, it acts as a good example of architectural design based on the fractal geometry. It reflects the philosophy of the universe unity and its self-subdivision. The applied parametric model provides two reversible functions; the analysis of existing mamluk madrasa, and the generation of advanced infinite variations based on its fractal characteristics. This research presents formal analysis method of the Al- sultan Hassan madrasa which can be applied on any other building or style. Fractal geometry and Euclidean geometry are merged in one parametric model. It emphasizes a number of the fractal characteristics such as the presence of an initial geometry from which the design emerges. The self- similar branched shapes, rhythm and hierarchy of scaled spaces are all fractal attributes.

This approach provides four main benefits concerning generating a style as follows:

- Style analysis, which will facilitate the study of architectural history and theories.
- Style preservation, for example designing a building which lies within a sensitive architectural context that carries a specific style order.
- Style generation, which provides new families of one style based on its fractal dimension, origin and self-similarity. This means the application of the style morphogenesis, not only the imitation of the final features.
- Style transformation, which is an advanced process after generation. It is resulted from the modification of specific parameters, using modifiers, slider bars and transformation operations in the architectural style to create contemporary concepts.

By adopting fractal geometry as a formal design generator, architects manipulate mathematical fractals to produce shapes, layouts or patterns. This commences with a starting shape and a generating rule that is recursively applied to a shape. This process can be used to create a plan, elevation or three-dimensional form. However, there could be more complex relationships that express the evolution process of the design until it appears in its final complete form. Transformations enable the design to be responsive to changes in design style requirements. Moreover, the interactive parametric model will enhance the teaching process of the historical buildings by tracing its formal evolution.

The exploration of new variations widens the possibility of creating form by applying advanced transformation rules on the resulted form such as angle rotation, skew or inclination. model is an interesting tool in two simple steps of analysis. It opens a wide range of innovations that based on the evolution process. The interactive slider bars enable the contribution of the normal users, researchers in architecture history or designers, without programming background to carry on the tracing process. It will help in the education process in the creative studios; academic or professional to present the complex characteristics of a style.

For further researches, the generation of fractal geometry could be applied on 3-D forms of buildings, elevation vocabulary, such as the fractal subdivision of arches and windows. Another application, the fourth and fifth iterations of the basic form could be possibly used in the floor patterns or the opening scale or ornaments scale. This is not applied exactly, but similarly the whole building begins with a cube which

undergoes several subdivisions for surrounding spaces to the main central space. This is why it is important to represent the building as one unit as a flower in its morphogenesis process. Meanwhile, the design is treated as one subdivided unit which follows the same morphogenesis, in a parametric way, to be generate in the large architectural elements and the small artistic ornaments.

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