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Dynamic Response of Sandwich Bottom Plate of Rigid Fluid Container Resting on Elastic Foundation

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Abstract

This study aims to investigate the dynamic behavior of the sandwich type bottom plate, which were not addressed before, attached to a rigid cylindrical fluid container resting on Pasternak type elastic foundation. Thin plate assumptions are incorporated with elastic foundation and plate/foundation interaction system is solved through a mixed finite element formulation. A boundary element procedure is also employed to determine the inertial effect of the involved fluid, which is considered to be ideal. The procedure is tested through the free vibration analysis of homogeneous circular bottom plates and some original results are presented regarding the properties of sandwich plates.

1. INTRODUCTION

Theories of structural elements are mainly based on their geometrical properties. As a two dimensional flat structural element, plates are frequently involved in many engineering disciplines, e.g. deck of bridges in civil engineering, tank bulkhead in marine engineering, splitter of engine entrance in aerospace engineering, etc. While reducing the formulation of structure from three dimensions to planar geometries; in order to achieve an adequate accuracy, many theories and solution procedures have been developed [1]. Due to their operating conditions plates commonly interact with external continuums, such as elastic foundation [2], fluid domain [3], or both of them [4] at the same time. Dynamic behavior of plates under such coupled states has been investigated extensively, with a special attention to the effect of interaction on the mechanical behavior. Early works based on the finite element analysis of the free vibration problem regarding Kirchhoff plate-Pasternak foundation interaction was initiated by Omurtag et al. [5] and Omurtag and Kadıoğlu [6]. A thorough review of the relatively early studies on the beam-foundation and plate-foundation interaction problems can be followed from [7]. Three dimensional elasticity equations of small deformation are involved in the free vibration analysis of thick circular plates resting on Pasternak foundation by Zhou et al. [8]. Akhavan et al. [9] reported an exact-closed form solution for the free vibration problem of rectangular Mindlin plates interacting with Winkler/Pasternak type elastic foundation and exposed to in-plane forces. Ferreira et al. [10] derived a collocation method employing radial basis functions in order to investigate the vibration behavior of shear deformable plates resting on Pasternak foundation. Dehghan and Baradaran [11] incorporated finite element and differential quadrature methods for the evaluation of buckling and free vibration characteristics of rectangular thick plates resting on Pasternak foundation, by means of the 3D elasticity theory. Considering the last two decades, studies concerning plate-fluid interaction problems can be reported as follows. Ergin and Uğurlu [12] proposed a boundary element solution for the free vibration problem of clamped rectangular plates partially immersed in fluid domain. Free vibration response of partially filled fluid storage tanks was evaluated by Ergin and Uğurlu [13] through the solution of a boundary integral equation representing the problem. Jeong [14] incorporated Fourier-Bessel series expansion and Rayleigh-Ritz method to present wet frequency

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parameters of two identical circular plates constraining a fluid domain laterally bounded by rigid walls. The same problem replacing circular plates by rectangular ones is solved by Jeong and Kim [15]. Askari et al. [16] performed analytical and experimental analyses of circular plates submerged in fluids regarding the free vibration response. Askari and Daneshmand [17] presented an analytical assessment of the dynamic parameters of a rigid cylindrical fluid storage tank's bottom plate while a rigid cylindrical body is submerged concentrically and partially into the fluid domain. Kwak and Yang [18] obtained the virtual added mass matrix representing the inertia of the fluid domain by implementing the Mathieu functions and analyzed the free vibration behavior of thin rectangular clamped plate partially immersed in fluid continuum. The free vibration analysis of elliptical bottom plate attached to a rigid fluid container was conducted by Hasheminejad and Tafani [19], where the container was partially filled with inviscid and incompressible fluid. In a recent study, Ugurlu [3] presented a dual reciprocity boundary element formulation for the free vibration analyses of Kirchhoff plates interacting with fluid.

Compared to the studies on plate-fluid and plate-foundation interaction problems, it can be easily noticed that the number of studies conducted on the dynamic problem of the plate-fluid-foundation was limited until recently. In a relatively early period, Chiba [20] presented the solution of axisymmetric vibration problem associated with a thin elastic bottom plate attached to a cylindrical fluid tank resting on Winkler foundation considering the free surface effect. Ugurlu et al. [4] presented a combined mixed finite element-boundary element solution procedure for determination of the free vibration parameters associated with a rectangular thin plate in contact with Pasternak foundation at one face and completely or partially coupled with an unconfined fluid domain on its other face. Following this work, Hashemi et al. [21] handled the same problem employing the Mindlin plate assumptions and adopting the Rayleigh-Ritz method through Timoshenko beam functions, where the fluid domain is assumed to be limited along depth and width, but infinite in its longitudinal direction. Then, Hashemi et al. [22] presented the buckling loads and free vibration characteristics of an elastic bottom attached to a rigid rectangular fluid container under linearly varying in-plane loads. Kutlu et al. [23] replaced the thin plate assumptions and isotropic foundation in Ugurlu et al. [4] by a first order shear deformable plate model and a newly proposed orthotropic three parameter foundation, respectively. Dynamic characteristics of symmetrically laminated cross-ply Mindlin plates leaning towards Pasternak foundation and vertically interacting with fluid were investigated by Shahbaztabar and Ranji [24]; they adopted the Rayleigh-Ritz method by employing the Chebyshev polynomials and considered also the influence of uniform in-plane load acting on the plate. Ugurlu [25] presented a higher order boundary element solution procedure to examine the free vibration behavior of elastic bottom plates of rigid fluid containers lying on Pasternak foundation. Hasheminejad and Mohammadi [26] improved the study of Ugurlu [25] by adopting active control applications through a semi-analytical approach, where the system is modeled to reflect 3D dynamic response characteristics. In a very recent study, Kutlu et al. [27] investigated the free vibration characteristics of a moderately thick bottom plate of rigid fluid storage tank by taking the free surface effect into account; they stated that the influence of the free surface on the natural frequencies of the bottom plate can be neglected.

The main purpose of this study is to display the free vibration characteristics of sandwich type bottom plates attached to rigid fluid storage tanks resting on elastic foundation. Sandwich structures are known for their lightweight architecture while presenting very high flexural stiffness compared to monocoque structures. Therefore, instead of using the same face material in monocoque construction, employing a core and increasing the distance between two face sheets is a more economical option that can provide lower lateral deformations, higher overall buckling loads, and higher natural frequencies [28]. A comprehensive review of the recent studies dealing with the free vibration behavior of sandwich plates can be found in [29]. In this study, the sandwich bottom plate is considered symmetric in stacking and modeled by Kirchhoff assumptions. The cylindrical rigid container rests on a Pasternak type foundation and it is filled by an ideal quiescent fluid. Bottom plate-foundation interaction is established through a mixed finite element formulation based on the Hellinger-Reissner principle. The fluid problem is solved by employing a boundary element scheme that expresses the fluid loading in terms of plate's lateral deflections and its effect on the dynamic behavior is considered by incorporating the associated added mass into the equation of motion of the overall system. A convergence and comparison study is presented to reveal the features of the proposed solution procedure first, and then in order to reveal the dynamic characteristics of the bottom plate-fluid-foundation system, some sensitivity analyses over numerical solutions are reported for key parameters of the system, such as stiffness ratio of core and face material, tank filling ratio, and foundation stiffness.

2. FORMULATION OF THE PROBLEM

2.1. Mixed Finite Element Formulation of the Plate-Foundation System

2.1.1. Equilibrium and Constitutive Equations

According to the Kirchhoff plate assumptions the displacement field of a thin plate is represented by a single transverse translation $\mathbf{U}(w)$ and stress resultants $\mathbf{M}(M_{xx}, M_{yy}, M_{xy})$, those are the integrals of the moments of the in-plane stresses $\boldsymbol{\sigma} = (\sigma_{xx} \quad \sigma_{yy} \quad \sigma_{xy})^T$ through the plate thickness. The positive directions of the field variables and orientation of the plate's global coordinates (x, y, z) are depicted in Fig. 1a. According to the kinematical description of the Kirchhoff plate, in-plane strains $\overline{\boldsymbol{\varepsilon}}(\varepsilon_{xx} \quad \varepsilon_{yy} \quad \varepsilon_{xy})^T$ at an arbitrary point z is obtained from $\overline{\boldsymbol{\varepsilon}} = z\boldsymbol{\varepsilon}$, where $\boldsymbol{\varepsilon} = (\kappa_{xx} \quad \kappa_{yy} \quad \kappa_{xy})$ collects the corresponding curvatures $\kappa_{xx} = -w_{,xx}$, $\kappa_{yy} = -w_{,yy}$, and $\kappa_{xy} = -w_{,xy}$ [30]. Here, $(...)_{,x}$ denotes the partial derivative with respect to the variable following the comma, e.g., $(...)_{,x} = \partial(...)/\partial x$. Fig. 1b shows how the sandwich plate section is decomposed into layers of core and face materials. In-plane stress developed in each layer is related to the associated in-plane stress by $\boldsymbol{\sigma} = \mathbf{Q}\boldsymbol{\varepsilon}$, where $\mathbf{Q}$ represents the material properties of each individual layer. For a layer of isotropic material, it is given as

$$\mathbf{Q} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{33} \end{bmatrix} \tag{1}$$

with $Q_{11} = Q_{22} = E/\left(1-\upsilon^2\right)$, $Q_{12} = Q_{21} = \upsilon E/\left(1-\upsilon^2\right)$, $Q_{33} = G_p$. Here, E , υ , and G_p are the elasticity modulus, Poisson's ratio, and shear modulus of the layer, respectively. Stress resultants through each layer (L_i) are obtained as

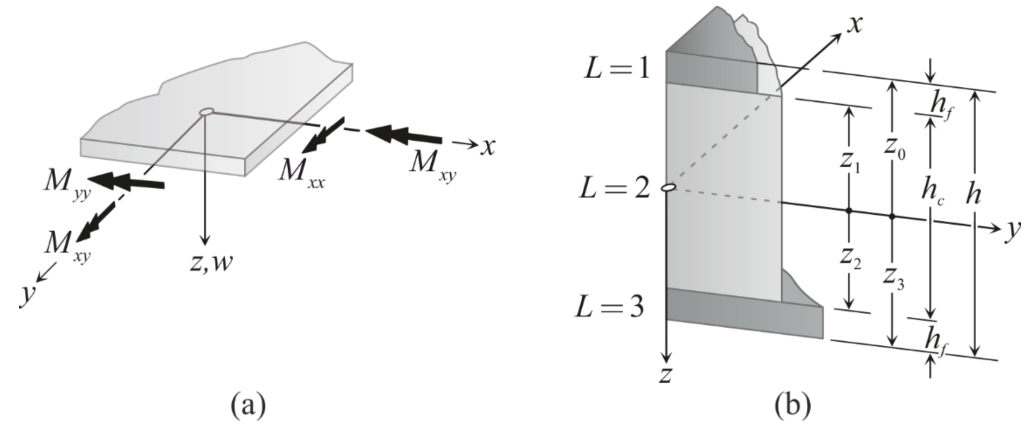


Figure 1. *Thin sandwich plate; a) Positive directions of stress resultants and transverse displacement b) Decomposition of the sandwich plate section:* h_f : *face layer thickness,* h_c : *core layer thickness,* h : *plate's thickness,* L : *number of layer*

$$\mathbf{M} = \int_{-h/2}^{h/2} \boldsymbol{\sigma}\, z\, dz = \sum_{L=1}^{3} \boldsymbol{\sigma}\, z\, dz \tag{2}$$

Replacing the stresses in Eq. (2) by the strains according to the constitutive equations, stress resultants can be represented in the form $\mathbf{M}=\mathbf{D}\boldsymbol{\varepsilon}$. Here, the elasticity matrix $\mathbf{D}$ of the entire section becomes $D_{ij}=\frac{1}{3}\sum_{L=1}^{N}(Q_{ij})_L(z_L^3-z_{L-1}^3)$. In order to express the midplane deformations of the plate in terms of the kinematic $(\boldsymbol{\varepsilon}^{\mathbf{u}})$ and constitutive $(\boldsymbol{\varepsilon}^{\boldsymbol{\sigma}})$ relations, the inverse of the matrix $\mathbf{D}$ is evaluated to express $\boldsymbol{\varepsilon}=\mathbf{D}^{-1}\mathbf{M}$ in the equality $\boldsymbol{\varepsilon}^{\mathbf{u}}=\boldsymbol{\varepsilon}^{\boldsymbol{\sigma}}$. Constitutive relations in explicit form are:

$$\left.\begin{aligned}\kappa_{xx}&=-w_{,xx}=D_{11}^{-1}M_{xx}+D_{12}^{-1}M_{yy}\\ \kappa_{yy}&=-w_{,yy}=D_{21}^{-1}M_{xx}+D_{22}^{-1}M_{yy}\\ \kappa_{xy}&=-w_{,xy}=D_{33}^{-1}M_{xy}\end{aligned}\right\} \tag{3}$$

The equilibrium equation of the Kirchhoff plate resting on Pasternak foundation is given by [4]

$$-M_{xx,xx}-M_{yy,yy}-2M_{xy,xy}+kw-G\left(w_{,xx}+w_{,yy}\right)=0 \tag{4}$$

where k and G are the Winkler foundation parameter and shear foundation parameter, respectively.

2.1.2. First Variation of the Energy Functional

According to the Hellinger-Reissner principle, the first variation of the energy functional is obtained as

$$\delta\Pi_{HR}=\int_V\left(\boldsymbol{\varepsilon}^{\mathbf{u}}-\boldsymbol{\varepsilon}^{\boldsymbol{\sigma}}\right)^T\delta\boldsymbol{\sigma}^{\boldsymbol{\sigma}}\mathrm{d}V+\int_V\left(\boldsymbol{\sigma}^{\boldsymbol{\sigma}}\right)^T\delta\boldsymbol{\varepsilon}^{\mathbf{u}}\mathrm{d}V=0, \tag{5}$$

where V denotes the problem domain or space. Employing constitutive equations (3) and equilibrium equation (4) in Eq. (5) yields the first variation of the functional associated with the plate-foundation interaction as

$$\begin{aligned}\delta\Pi_{HR}&=\int_\Omega\Big[w_{,x}\delta M_{xx,x}-\left(D_{11}^{-1}M_{xx}+D_{12}^{-1}M_{yy}\right)\delta M_{xx}\Big]\mathrm{d}\Omega\\ &+\int_\Omega\Big[w_{,y}\delta M_{yy,y}-\left(D_{21}^{-1}M_{xx}+D_{22}^{-1}M_{yy}\right)\delta M_{yy}\Big]\mathrm{d}\Omega\\ &+\int_\Omega\Big[w_{,x}\delta M_{xy,y}+w_{,y}\delta M_{xy,x}-D_{33}^{-1}M_{xy}\delta M_{xy}\Big]\mathrm{d}\Omega+\int_\Omega\big[kw\delta w\big]\mathrm{d}\Omega\\ &+\int_\Omega G\Big[w_{,x}\delta w_{,x}+w_{,y}\delta w_{,x}\Big]\mathrm{d}\Omega\\ &+\int_\Omega\Big[M_{xx,x}\delta w_{,x}+M_{xy,x}\delta w_{,y}\Big]\mathrm{d}\Omega+\int_\Omega\Big[M_{xy,y}\delta w_{,x}+M_{yy,y}\delta w_{,y}\Big]\mathrm{d}\Omega=0\end{aligned} \tag{6}$$

Here, Ω represents the plate midplane.

2.1.3. Finite Element Equations of the Plate-Foundation System

Numerically computing the integrals given in Equations (6) over four noded quadrilateral finite elements (Figure. 2a) by employing shape functions with C^0 continuity generates the system matrix $[\mathbf{K}]$ in the form:

$$[\mathbf{K}]=\begin{bmatrix}[\mathbf{k}_{ss}] & [\mathbf{k}_{su}]\\ [\mathbf{k}_{us}] & [\mathbf{k}_{uu}]\end{bmatrix} \tag{7}$$

Here, u and s corresponds displacement type and stress resultant type field variables, respectively, in the mixed finite element formulation.

2.1.4 Mass Matrix of the Plate

For the isoparametric element, consistent mass matrix associated with the transverse displacement w is obtained for the sandwich construction as

$$[\mathbf{m}]=\left[\left(2\bar{\rho}_f h_f+\bar{\rho}_c h_c\right)[\mathbf{k}_1]\right] \tag{8}$$

Here, $\bar{\rho}_f$ and $\bar{\rho}_c$ denote the mass densities of the face and core materials of the plate, respectively, and $[\mathbf{k}_1]=\int \hat{N}_i\hat{N}_j \mathrm{d}\Omega$, with $\hat{N}$ denoting the bilinear shape functions.

2.2. Fluid-Structure Coupling

Assuming that the fluid domain interacting with plate is ideal and its motion is irrotational, the fluid velocity field can be defined in terms of the velocity potential function Φ as $\mathbf{v}(\mathbf{x},t)=\nabla\Phi(\mathbf{x},t)$. Continuity condition assures Φ to be a harmonic function, yielding $\nabla^2\Phi=0$. Here, $\mathbf{x}=(x,y,z)^{\mathrm{T}}$ and t represent position and time, respectively. Linear form of the Bernoulli's equation defines the fluid pressure on the plate's wet surface by

$$p=-\rho_f\frac{\partial\Phi}{\partial t}, \tag{9}$$

where ρ_f is the fluid density. Kinematic boundary condition defined at the plate-fluid interface equalizes the normal velocity of the contacting fluid with the velocity of the plate:

$$\frac{\partial\Phi}{\partial n}=\frac{\partial w}{\partial t} \quad \text{at} \quad S_w \tag{10}$$

Here, $\mathbf{n}$ represents the normal on the plate surface S_w, directed from the fluid domain, and w denotes the deflection of the plate's mid-plane. Assuming that the plate vibrates in relatively high frequencies, the free surface effects are neglected by imposing the infinite frequency condition at the fluid free surface [27]:

$$\Phi=0 \tag{11}$$

The Laplace equation and boundary conditions specified in Eqs. (10) and (11) can be represented by a boundary integral equation defined at the plate surface as

$$c(\mathbf{X})\,\Phi(\mathbf{X})=\iint_{S_W}\left(\Phi^*(\mathbf{x},\mathbf{X})\,q(\mathbf{x})-q^*(\mathbf{x},\mathbf{X})\,\Phi(\mathbf{x})\right)\mathrm{d}S \tag{12}$$

Here, $\mathbf{X}=(X,Y,Z)$ and $\mathbf{x}$ are the source and field points, respectively, $\Phi^*(\mathbf{x},\mathbf{X})$ denotes the fundamental solution of the Laplace equation and $q=\partial\Phi/\partial n$ defines the flux. The free term $c(\mathbf{X})$ represents the fraction of $\Phi(\mathbf{X})$ involved in the domain of interest. In case of a three-dimensional homogeneous domain we have $\Phi^*(\mathbf{x},\mathbf{X})=1/(4\pi r)$ and $q^*(\mathbf{x},\mathbf{X})=-(\partial r/\partial n)/4\pi r^2$. Here, r identifies the distance between the points of field and source. Adopting the fundamental solution as $4\pi\Phi^*=1/r-1/r'$ satisfies the free surface condition in Eq. (11) identically so that the integral equation (12) can be defined only over the

plate surface. Here, $r' = [(x-X)^2 + (y-Y)^2 + (z+Z)^2]^{1/2}$ is the distance between the field point and free surface image of the source point.
For an arbitrary plate geometry, the plate surface over which Eq. (12) is defined can be discretized by surface elements and the variation of Φ and q can be expressed over each element in terms of nodal values:

$$\Phi^i = \sum_{j=1}^{n_i} N_j \Phi_{ij} \;\; , \;\; q^i = \sum_{j=1}^{n_i} N_j q_{ij} \tag{13}$$

Here, n_i stands for the number of nodes of a boundary element, Φ^i and q^i represent the potential and flux distribution over the *i*th element, respectively, Φ_{ij} and q_{ij} are the potential and flux values of the *j*th node of the element, respectively, and N_j denotes the associated shape functions. The presented solution employs four noded quadrilateral elements involving bilinear shape functions. Assigning all nodal points used in the discretization as the source point in Eq. (12), substituting the kinematic boundary condition Eq. (10), and accomplishing the approach in Eq. (13) generates the following algebraic set of equations for the potential function distribution over the plate surface:

$$c_k \Phi_k + \sum_{i=1}^{n_p} \sum_{j=1}^{n_i} (\Phi_{ij} \int_{S_i} N_j q^* dS) = i\omega \sum_{i=1}^{n_p} \sum_{j=1}^{n_i} (w_{ij} \int_{S_i} N_j \Phi^* dS) \qquad ; \quad k = 1,...,n_n \tag{14}$$

Here, n_n and n_p are the total numbers of nodes and boundary elements, respectively, S_i identifies the area of the *i*th element, Φ_k denotes the potential value at *k*th node, and w_{ij} represents the deflection at the *j*th node of the *i*th element. The product $i\omega$ on the right hand side of Eq. (14) is due to the harmonic time dependence and results from the boundary condition. By evaluating the integrals in Eq. (14) numerically, the algebraic set of equations becomes

$$\sum_{j=1}^{n_i} h_{ij} \Phi_j = i\omega \sum_{j=1}^{n_i} g_{ij} w_j \qquad ; \quad i = 1,...,n_n \qquad\qquad h_{ij} \tag{15}$$

In matrix notation, it can be expressed in the following form:

$$\mathbf{H}\boldsymbol{\Phi} = i\omega\, \mathbf{G}\,\mathbf{w} \tag{16}$$

Here, $\boldsymbol{\Phi}$ and $\mathbf{w}$ are the vectors of potential values and deflection, respectively. **G** and **H** matrices involve the double summed integrals containing the Green function and its flux, respectively, where g_{ij} and are the terms of these matrices.

Even though expression (12) is derived considering the interaction of the fluid with the vibrating plate, it can be arranged in the following form to be applicable in cases such as rigid walls confine the fluid domain:

$$\begin{bmatrix} \mathbf{H}^{pp} & \mathbf{H}^{pr} \\ \mathbf{H}^{rp} & \mathbf{H}^{rr} \end{bmatrix} \begin{Bmatrix} \boldsymbol{\Phi}^p \\ \boldsymbol{\Phi}^r \end{Bmatrix} = i\omega \begin{bmatrix} \mathbf{G}^{pp} & \mathbf{G}^{pr} \\ \mathbf{G}^{rp} & \mathbf{G}^{rr} \end{bmatrix} \begin{Bmatrix} \mathbf{w} \\ 0 \end{Bmatrix}. \tag{17}$$

Here, p and indices represent the plate surface and rigid walls, respectively, where $\partial\Phi/\partial n = 0$ condition is imposed instead of Eq. (10), respecti r vely. Coefficient matrices $\mathbf{H}$ and $\mathbf{G}$, and vectors $\boldsymbol{\Phi}$ and $\mathbf{w}$ are decomposed into sub segments associated with elastic and rigid surfaces. From the decomposed matrix representation, the potential distribution over the plate surface can be obtained as

$$\mathbf{\Phi}^{p} = i\omega \tilde{\mathbf{A}}\ \mathbf{w}, \tag{18}$$

Here, $\tilde{\mathbf{A}} = \left(\mathbf{H}^{pp} - \mathbf{H}^{pr}(\mathbf{H}^{rr})^{-1}\mathbf{H}^{rp}\right)^{-1}\left(\mathbf{G}^{pp} - \mathbf{H}^{pr}(\mathbf{H}^{rr})^{-1}\mathbf{G}^{rp}\right)$. The dynamic fluid pressure $\mathbf{p}$ exerted on the plate surface is determined by employing Eq. (18) in Eq. (9):

$$\mathbf{p} = \rho_f \omega^2 \tilde{\mathbf{A}}\ \mathbf{w} = -\mathbf{A}\ddot{\mathbf{w}} \tag{19}$$

$\mathbf{A} = \rho_f \tilde{\mathbf{A}}$ is the added mass matrix representing the inertial effect of the fluid on plate dynamic behavior.

2.3. Eigenvalue-Problem

The set of equations defining the harmonic motion of plate structure is established by the inclusion of the plate's consistent mass matrix $\mathbf{B}$ and the added mass matrix $\mathbf{A}$ in the global system matrix of the plate as follows:

$$\left(\begin{bmatrix} [\mathbf{k}_{ss}] & [\mathbf{k}_{su}] \\ [\mathbf{k}_{us}] & [\mathbf{k}_{uu}] \end{bmatrix} - \omega^2 \begin{bmatrix} [\mathbf{0}] & [\mathbf{0}] \\ [\mathbf{0}] & [\mathbf{B}+\mathbf{A}] \end{bmatrix}\right)\begin{Bmatrix} \{\mathbf{M}\}^T \\ \{\mathbf{U}\}^T \end{Bmatrix} = \begin{Bmatrix} \{\mathbf{0}\} \\ \{\mathbf{0}\} \end{Bmatrix} \tag{20}$$

Here, ω represents the natural frequencies of the plate interacting with the fluid and foundation. Due to the nature of the two field mixed finite element formulation, displacement and stress resultant type field variables are involved in Eq. (20). By eliminating stress resultant type variables from the expression, the condensed system matrix takes the form:

$$\left[\mathbf{K}^*\right] = [\mathbf{k}_{uu}] - [\mathbf{k}_{su}]^{\mathrm{T}}[\mathbf{k}_{ss}]^{-1}[\mathbf{k}_{su}] \tag{21}$$

$D = E_f\, h^3/12(1-\upsilon^2)$ Hence, the eigen-value equation of the problem is obtained in terms of displacements only:

$$\left(\left[\mathbf{K}^*\right] - \omega^2[\mathbf{B}+\mathbf{A}]\right)\{\mathbf{U}^T\} = \{\mathbf{0}\} \tag{22}$$

3. NUMERICAL RESULTS

The presented combined solution procedure is first verified by considering a homogeneous circular bottom plate of a cylindrical rigid container resting on a Winkler foundation where the convergence of the solution is also examined. Then, by employing a sandwich plate as the bottom of the cylindrical tank, some original results are presented and some sensitivity analysis are performed regarding the governing parameters of the system. For the generalization of the solutions, parameters and results are presented in nondimensional form. The nondimensional frequency parameter is given by $\Omega = \omega b^2\sqrt{\rho h/D}$, where and denotes the elasticity modulus of the face material. $\tilde{k} = kb^4/D$ and $\tilde{G} = Gb^2/D$ represent dimensionless Winkler foundation parameter and shear foundation parameter, respectively, where b stands for the radius of the circular plate. Unless stated otherwise, throughout the examples the Poisson's ratio is taken as $\upsilon = 0.3$, plate's face material-E_f fluid density ratio is set to $\bar{\rho}_f/\rho_f = 10$, and plate thickness ratio $h/b = 0.01$ is chosen with $h_f/h = 0.1$. Clamped boundary conditions are imposed for the bottom plate attached to the rigid container wall (Fig. 2b).

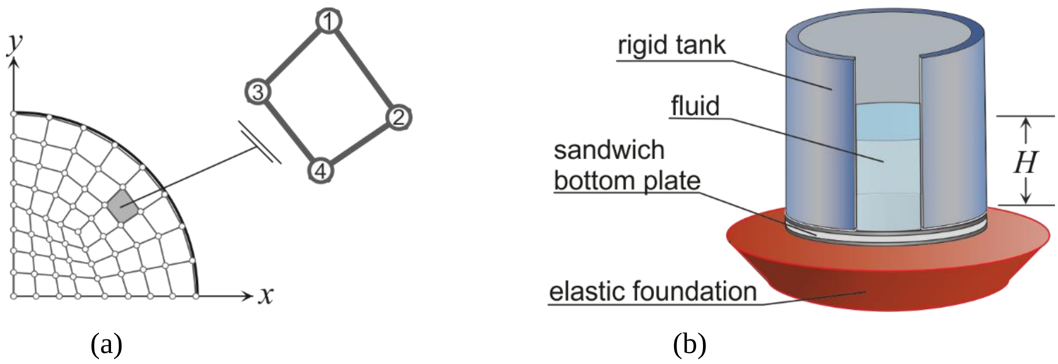


Figure 2. *Mesh and problem definitions; a) Discretization of plate geometry by four noded elements b) Physical problem, H: fluid height*

3.1. Homogeneous Bottom Plate Resting on Winkler Foundation: Convergence Behavior and Verification Through Axisymmetric Modes

In order to verify the proposed solution procedure, a comparison study is performed by considering homogeneous plate and replacing the foundation by Winkler model so that the same configuration with Chiba [20] is provided. Chiba [20] reported only the natural frequency values for the axisymmetric modes while taking the effect of in-plane forces due to the fluid static pressure into consideration. Three different finite element discretization (Fig. 2a) involving 108, 300, and 432 elements are employed. Choosing the Winkler foundation parameter as $\tilde{k}=100$, first two nondimensional frequency parameters (Ω_1 and Ω_2) associated with the axisymmetric modes are presented in Fig. 3 with respect to the filling ratio (H/b) of the tank. The plate surface discretizations are also employed for the boundary element solutions, where wetted lateral tank surface is covered by additional elements to impose rigid-wall boundary condition.

The results given in Fig. 3 show that the employed solution procedure ensures a consistent convergence behavior and produces quite compatible results with Chiba [20]. Although, the first mode frequencies convergence very quickly, for higher modes, a refined mesh configuration would be more suitable. Furthermore, it must be pointed out that the second axisymmetric mode correspond to the sixth mode in regular mode sequence.

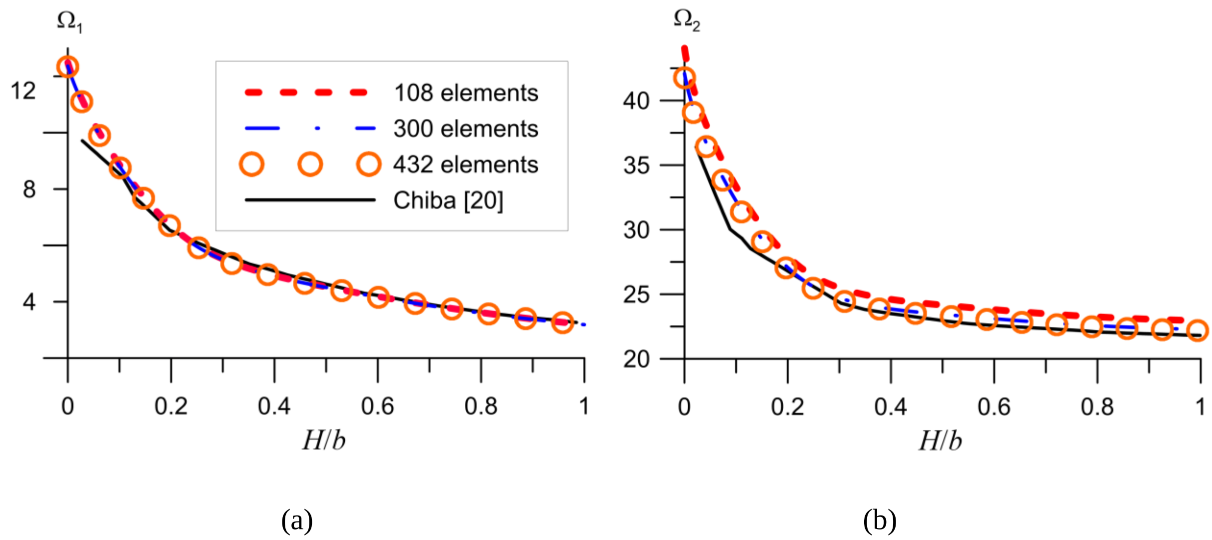
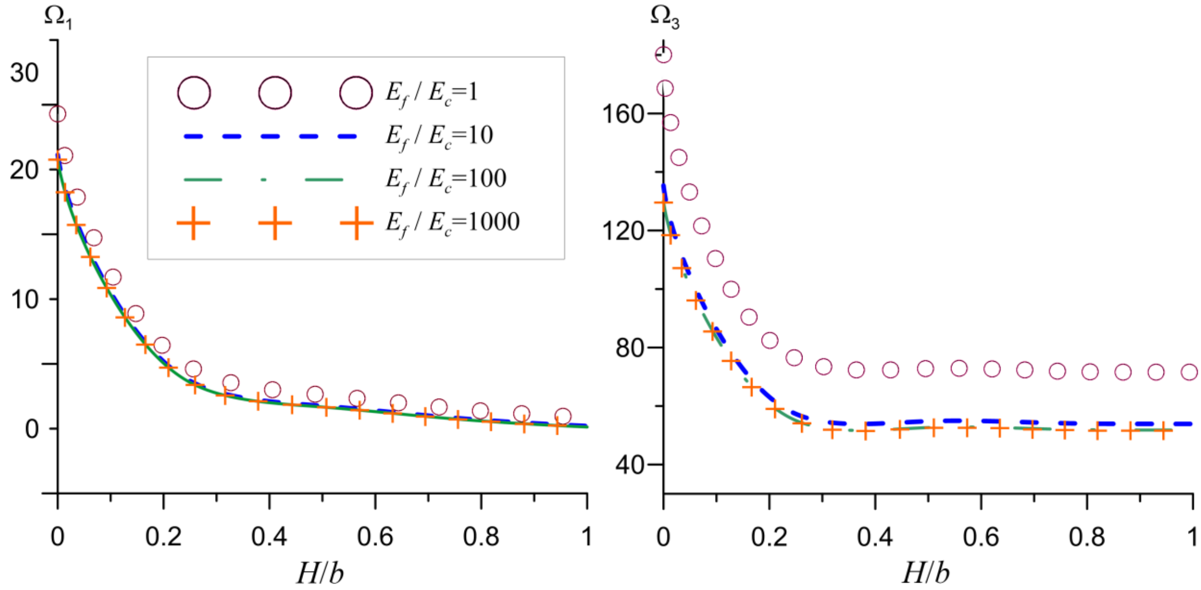


Figure 3. *Variation of the natural frequency for the axisymmetric modes of the homogenous plate on Winkler foundation with respect to fluid height; a) First mode b) Second mode*

3.2. Circular Sandwich Plate: The Effect of Elasticity Modulus Ratio of Face and Core Material (E_f/E_c) on the Free Vibration Behavior

In order to show the effect of sandwich plate configuration on the natural frequencies of the bottom plate, four different modulus ratios E_f/E_c are chosen as 1, 10, 100, and 1000, and the density of the core material is selected as the same of the fluid density, $\bar{\rho}_c = \rho_f$. 432 finite elements are employed over the plate domain due to the convergence analysis. Nondimensional foundation parameters are choosen as $(\tilde{k}, \tilde{G}) = (100, 5)$. Nondimensional frequencies corresponding to the first and third axisymmetric modes of the plate are given in Fig. 4a and Fig. 4b, respectively.



(a) (b)

Figure 4. *Variation of the natural frequency for the axisymmetric modes of the sandwich plate on Pasternak foundation with respect to fluid height; a) First mode b) Third mode*

It is well documented that the frequency parameters of elastic bottom plates decrease with increasing fluid height and approach to asymptotic values. This behavior is also observed here for sandwich plates, where as the E_f/E_c ratio increases the frequency parameters become less sensitive and it can be stated that after a certain value of E_f/E_c the parameters are not affected. The approach to the asymptote is quite rapid for the first symmetric mode, and as the corresponding mode gets higher a slower approach to the asymptotic values is observed. It can be perceived from the figure that the effect of E_f/E_c ratio on the change in frequency parameters with respect to a variation in fluid level is almost negligible, notwithstanding that for the first mode, as the fluid level increases the effect of E_f/E_c also increases but it is quite limited.

3.3. Circular Sandwich Plate: The Effect of Foundation Parameters and Density Ratio of Face and Core Material $(\bar{\rho}_f/\bar{\rho}_c)$ on the Free Vibration Behavior

This example investigates the effect of face and core material densities and Pasternak foundation parameters on the dynamic characteristics of sandwich bottom plates. Throughout the study $E_f/E_c = 10$ is selected. Three different set of parameters for both foundation $(\tilde{k}, \tilde{G}) =$ (100,5), (100,10), (200,5) and density ratios $(\bar{\rho}_f/\bar{\rho}_c = 5, 10, 20)$ are chosen. Nondimensional frequencies corresponding to the first three axisymmetric modes of the bottom plate are tabulated in Table 1. It is observed that the change in the frequency parameters due to the change in foundation parameters is independent of the $\bar{\rho}_f/\bar{\rho}_c$ ratio. In the case that the fluid is absent $(H/b=0)$, the effect of the ratio $\bar{\rho}_f/\bar{\rho}_c$ on natural frequencies is identical. This is due to the fact that the existence of foundation increases the plate stiffness, while the fluid related terms are only included in the mass matrix of the system. Table 1 also reveals that as the ratio $\bar{\rho}_f/\bar{\rho}_c$ increases the effect of a change in that ratio on the frequency parameters decreases, in other words for smaller values of $\bar{\rho}_f/\bar{\rho}_c$, frequency parameters are more sensitive to a variation in this ratio and it is more apparent in the higher modes.

Table 1. *First three axisymmetric mode frequencies of sandwich bottom plate for various* $\bar{\rho}_f/\bar{\rho}_c$ *ratios and foundation parameters* $(\tilde{k},\tilde{G})$

		Ω_1			Ω_2			Ω_3		
		$(\tilde{k},\tilde{G})$			$(\tilde{k},\tilde{G})$			$(\tilde{k},\tilde{G})$		
$\bar{\rho}_f/\bar{\rho}_c$	H/b	(100,5)	(100,10)	(200,5)	(100,5)	(100,10)	(200,5)	(100,5)	(100,10)	(200,5)
5	0.00	23.054	25.001	28.448	56.665	60.666	59.065	119.389	124.069	120.547
	0.25	8.550	9.264	10.549	23.713	25.378	24.718	55.902	58.105	56.448
	0.50	6.699	7.244	8.255	21.295	22.802	22.216	53.494	55.642	54.032
	0.75	5.782	6.243	7.113	20.554	22.027	21.472	52.882	55.027	53.425
	1.00	5.169	5.576	6.351	20.214	21.675	21.141	52.602	54.746	53.148
10	0.00	26.141	28.348	32.257	64.252	68.789	66.973	135.374	140.681	136.687
	0.25	8.684	9.408	10.714	24.188	25.886	25.213	57.327	59.587	57.887
	0.50	6.763	7.313	8.333	21.644	23.175	22.581	54.778	56.979	55.330
	0.75	5.822	6.286	7.162	20.879	22.374	21.812	54.147	56.345	54.704
	1.00	5.197	5.607	6.385	20.531	22.014	21.474	53.862	56.058	54.422
20	0.00	28.235	30.620	34.841	69.400	74.301	72.339	146.221	151.953	147.639
	0.25	8.753	9.483	10.799	24.437	26.152	25.473	58.082	60.371	58.650
	0.50	6.795	7.347	8.372	21.826	23.369	22.771	55.456	57.685	56.016
	0.75	5.842	6.308	7.187	21.047	22.554	21.989	54.816	57.041	55.380
	1.00	5.212	5.622	6.403	20.695	22.190	21.646	54.527	56.752	55.095

4. CONCLUSIONS

This study employs a numerical solution strategy incorporating a mixed finite element and a boundary element formulation to solve the free vibration problem of plate-fluid-foundation interaction. Using the proposed method, dynamic characteristics of a sandwich bottom plate attached to a rigid cylindrical fluid container resting on elastic foundation is investigated. Kirchhoff plate assumptions with Pasternak foundation model are employed in the Hellinger-Reissner variational principle based mixed finite element formulation and added mass matrix representing the inertia of the fluid is included in the eigenvalue problem through a boundary element formulation. Plate domain and plate surface are discretized by matching quadrilateral four noded finite and boundary elements, respectively. Stress resultant type field variables due to the mixed formulation are then eliminated from the set of eigenvalue equations by following a condensation process. Nondimensional frequency parameters of the sandwich bottom plate are presented after the convergence and verification of the methodology are demonstrated. The influence of the key parameters of sandwich plates is investigated through some parametric analyses, which were not addressed before. The effect of face and core materials rigidity and density is examined. The frequency parameters get lower as the ratio of elasticity moduli of face and core material increase and approaches to asymptotic values which happens quicker for lower modes compared to higher modes. It is also observed that as the densities of face and core materials get closer, frequency parameters become more sensitive to a change in their ratio.

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CONFLICTS OF INTEREST

No conflict of interest was declared by the authors.

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