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Quasilinear Evolution Integrodifferential Equations in Banach Spaces

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Abstract – Existence and uniqueness of local classical solutions of the quasilinear evolution integrodifferential equation in Banach spaces are studied. The results are demonstrated by employing the fixed point technique on C_0-semigroup of bounded linear operator. At last, we deal an example to interpret the theory.

***Keywords* –** Quasilinear evolution integrodifferential equation, local classical solution, C_0-semigroups, fixed point theorem.

1 Introduction

In this work, we examine the quasilinear evolution equation of the following form

$$\frac{du(t)}{dt} + A(t,u)\,u(t) = H(u)(t) + f\big(t,u(t),G(u)(t)\big), \tag{1}$$

$$u(0) = u_0, t \in [0,T] = J \tag{2}$$

where $A(t,u)$ is the infinitesimal generator of a C_0-semigroup in a Banach space X. $u_0 \in X$, $f : J \times X \times X \to X$ are functions and H and G are the nonlinear Volterra operators

$$H(u)(t) = \int_0^t k(t-s)\,h(s,u(s))\,ds \text{ and } G(u)(t) = \int_0^t a(t-s)\,g(s,u(s))\,ds$$

where $a,k : J \to J$ are real valued continuous functions and $h,g : J \times X \to X$ are functions.

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A lot of researchers have investigated the existence of solutions of various types of abstract quasilinear evolution equations in Banach space [2, 3, 9, 14]. Pazy [11] considered the quasilinear equation of the form

$$u'(t) + A(t, u)u(t) = 0,\ 0 < t \le T,\ u(0) = u_0$$

and studied the mild and classical solutions by applying fixed point theorem. Abbas et.al [1] considered a class of quasilinear functional differential equations in which the author investigated the existence of solutions for the same system by employing the thought of C_0-semigroup of bounded linear operator. Results on the existence and uniqueness of solutions for problems of quasilinear differential equation with deviating arguments can be found in [8].

Quasilinear integrodifferential systems in abstract form have got more notice because such equations appear in different domain of science e.g. mathematical physics, population dynamics etc. Different kinds of quasilinear integrodifferential equation in Banach space have been investigated by numerous authors [4- 7, 10, 12, 13].

The remaining work is ordered as follows. In segment 2, we state some prelude. In segment 3, we give main result. Finally a concrete example is given in last segment 4 to show the relevance of abstract theory.

2 Preliminaries

Let X and Y be two Banach spaces such that Y is densely and continuously embedded in X. The norm in any Banach space Z is expressed by $\|.\|$ or $\|.\|_Z$. Consider $B(X,Y)$ be the set of all bounded linear operators from a Banach space X to a Banach space Y. We write $B(X,X)$ by $B(X)$.

Let $B \subset X$ and let $A(t,b)$ be the infinitesimal generator of a C_0-semigroup $S_{t,b}(s), s \ge 0,$ on X. $\{A(t,b)\},(t,b) \in J \times B$ is the family of operators which is stable if there exist constants $M \ge 1$ and ω such that

$$\rho(A(t,b)) \supset]\omega,\infty[\qquad \text{for } (t,b) \in J \times B,$$

where $\rho(A(t,b))$ is the resolvent set of $A(t,b)$ and

$$\left\| \prod_{j=1}^{k} R\left(\lambda : A(t_j,b_j)\right) \right\| \le M\,(\lambda - \omega)^{-k}$$

for $\lambda > \omega$ and every finite sequence $0 \le t_1 \le t_2 \ldots\ldots \le t_k \le T, b_j \in B, 1 \le j \le k.$

The stability of $\{A(t,b)\},(t,b) \in J \times B$ implies that

$$\left\|\prod_{j=1}^{k} S_{t_j,b_j(s_j)}\right\| \le M \exp\left\{\omega \sum_{j=1}^{k} s_j\right\}, s_j \ge 0$$

and any finite sequence $0 \le t_1 \le t_2 \ldots\ldots \le t_k \le T, b_j \in B, 1 \le j \le k.$

Suppose a linear operator S in X and let a subspace Y of X. The operator $\tilde{S}$ with domain $D(\tilde{S}) = \{x \in D(S) \cap Y : Sx \in Y\}$ and $\tilde{S}x = Sx$ for $x \in D(\tilde{S})$ is remarked to be the part of S in Y.

Let $S_{t,b}(s), s \ge 0$ be the C_0-semigroup generated by $\{A(t,b)\}, (t,b) \in J \times B.$ A subspace Y of X is called $A(t,b)$-admissible if Y is an invariant subspace of operator $S_{t,b}(s), s \ge 0$ and the restriction of $S_{t,b}(s)$ to Y is a C_0-semigroup in $Y.$

For deep information of the above noticed notions, we refer the work of Pazy [11] in chapters 5 and 6. On the family of operators $\{A(t,b) : (t,b) \in J \times B\}$, we perform the same hypothesis $(H_1) - (H_4)$ given in section 6.6.4 in Pazy [11] for the homogenous quasilinear evolution equation, as recall below.

(H_1) The family $\{A(t,b) : (t,b) \in J \times B\}$ is stable.

(H_2) Y is $A(t,b)$-admissible for $(t,b) \in J \times B$ and the family $\{A(t,b)\}, (t,b) \in J \times B$ of parts of $A(t,b)$ of $A(t,b)$ in Y, is stable in $Y.$

(H_3) For $(t,b) \in J \times B, D(A(t,b)) \supset Y,$ $A(t,b)$ is a bounded linear operator from Y to X and the map $t \mapsto A(t,b)$ is continuous in the $B(Y,X)$ norm $\|.\|_{Y \to X}$ for every $b \in B.$

(H_4) There is a constant L such that

$$\|A(t,b_1) - A(t,b_2)\|_{Y \to X} \le L\|b_1 - b_2\|_X$$

holds for every $b_1, b_2 \in B$ and $0 \le t \le T.$

Definition 2.1: A two parameter family of bounded linear operators $U(t,s), 0 \le s \le t \le T,$ on X is called an evolution system if the following two conditions are satisfied:

(i) $U(s,s) = I$, $U(t,r)U(r,s) = U(t,s)$ for $0 \le s \le r \le t \le T.$

(ii) $(t,s) \to U(t,s)$ is strongly continuous for $0 \le s \le t \le T.$

Moreover, let $B \subset X$ and let $\{A(t,b)\}$, $(t,b) \in J \times B$ be a family of operators fulfilling the above stated hypothesis $(H_1)-(H_4)$. If $u \in C(J,X)$ has values in B then there is a unique evolution system $U_u(t,s), 0 \le s \le t \le T$, in X satisfying

(i) $$\|U_u(t,s)\| \le M \exp \omega(t-s) \tag{3}$$

for $0 \le s \le t \le T$, where M and ω are stability constants;

(ii) $$\frac{\partial^+}{\partial t} U_u(t,s) w\Big|_{t=s} = A(s,u(s))\, w \tag{4}$$

for $w \in Y$, and $0 \le s \le t \le T$;

(iii) $$\frac{\partial}{\partial s} U(t,s;u)\, w = -U_u(t,s)\, A(s,u(s))\, w \tag{5}$$

for $w \in Y$, and $0 \le s \le t \le T$.

Again, there is a constant C_1 such that for every $u,v \in C(J,X)$ with values in B and for every $w \in Y$, we have

$$\|U_u(t,s)\, w - U_v(t,s)\, w\| \le C_1 \|w\|_Y \int_s^t \|u(\tau) - v(\tau)\| \, d\tau. \tag{6}$$

To find the above noticed outcomes in details, the Theorem 6.4.3 and Lemma 6.4.4 is given in Pazy [11].

Further we consider that

(H_5) For every $u \in C(J,X)$ satisfying $u(t) \in B$ for $t \in J$, we have

$$U_u(t,s)Y \subset Y, 0 \le s \le t \le T$$

where $U(t,s)$ is strongly continuous in Y for $t,s \in J$ and $s \le t$.

(H_6) Every closed convex and bounded subset of Y is also closed in X.

(H_7) The real-valued function a and b are continuous on I and there exist positive constants k_T and a_T such that $|k(t)| \le k_T$ and $|a(t)| \le a_T$ for $t \in J$.

(H_8) $h : J \times X \to X$ is continuous and there exist constants $H_L > 0$ and $H_0 > 0$ such that

$$\int_0^t \|h(s,x) - h(s,y)\| ds \le H_L \left[\|x(t) - y(t)\| \right]$$

and

$$H_0 = \max \int_0^t \|h(s,0)\| ds.$$

For the conditions (H_9) and (H_{10}), Z be taken as both X and Y.

(H_9) $f : J \times Z \times Z \to Z$ is continuous and there exist constants $F_L > 0$ and $F_0 > 0$ such that

$$\| f(t,u_1,v_1) - f(t,u_2,v_2)\|_Z \le F_1 \left(\|u_1 - u_2\|_Z + \|v_1 - v_2\|_Z\right)$$

and

$$F_0 = \max_{t\in J} \| f(t,0,0)\|_Z .$$

(H_{10}) $g : J \times Z \to Z$ is continuous and there exist constants $G_L > 0$ and $G_0 > 0$ such that

$$\int_0^t \|g(s,u_1) - g(s,u_2)\|_Z \, ds \le G_L \left(\|u_1(t) - u_2(t)\|_Z\right)$$

and

$$G_0 = \max\left\{\int_0^t \|k(s,0)\| ds\right\}.$$

Let $M = \max\left\{\|U_u(t,s)\|_{B(Z)}, 0 \le s \le t \le T, u \in B\right\}$.

(H_{11}) $M_0 \left\{\|u_0\|_Y + k_T rTH_L + k_T TH_0 + F_L rT + a_T F_L G_L rT + a_T F_L G_0 T + F_0 T\right\} \le r$

and

$$\Gamma = \begin{bmatrix} C_1 T\|u_0\|_Y + C_1 T^2 \left\{k_T \left(H_L r + H_0\right) + F_L\left(r + a_T G_L r + a_T G_0\right) + F_0\right\} \\ + MTk_T H_L + MF_L T + MF_L G_L a_T T \end{bmatrix} < 1.$$

We mentioned that condition (H_6) is always satisfied if X and Y are reflexive Banach space. Next we prove the existence of local classical solution of the quasilinear problem (1)–(2). By a mild solution to (1) – (2) on $J = [0,T]$, we signify a function $u \in C(J,X)$ with values in B satisfying the integral equation

$$u(t) = U_u(t,0)u_0 + \int_0^t U_u(t,s)\left[Hu(s) + f\left(s,u(s),\int_0^s a(s-\tau)\, g(\tau,u(\tau))\, d\tau\right)\right] ds. \quad (7)$$

A function $u \in C(J,X)$ such that $u(t) \in Y \cap B$ for $t \in (0,T]$ and $u \in C^1((0,T],X)$ satisfying the equation (1) – (2) in X is called a classical solution of (1) – (2) on J, where $C^1(J,X)$ space of all continuously differentiable functions from J to X.

3 Existence Result

Theorem 3.1: Let $u_0 \in Y$ and let $B = \{u \in X : \|u\|_Y \le r\}, r > 0.$ If the hypothesis $(H_1)-(H_{10})$ are satisfied, then (1)–(2) has a unique classical solution $u \in C([0,T]:Y) \cap C^1((0,T]:X).$

Proof: Let S be the nonempty closed subset of $C([0,T],X)$ defined by

$$S = \{u : u \in C([0,T],X), \|u(t)\|_Y \le r \text{ for t} \in \text{J}\}.$$

Suppose a mapping F on S defined by

$$(Fu)(t) = U_u(t,0)u_0 + \int_0^t U_u(t,s)\left[H(u)(s) + f\left(s,u(s),\int_0^s a(s-\tau)g(\tau,u(\tau))d\tau\right)\right]ds.$$

We state that $F : S \to S$. For $u \in S,$ we have

$$\|Fu(t)\|_Y = \left\| U_u(t,0)u_0 + \int_0^t U_u(t,s)\left[\int_0^s k(s-\tau)h(\tau,u(\tau))d\tau + f\left(s,u(s),\int_0^s a(s-\tau)g(\tau,u(\tau))d\tau\right)\right]\right\| ds$$

$$\le \|U_u(t,0)u_0\| + \int_0^t \|U_u(t,s)\| \left[\begin{array}{l} \int_0^s \|k(s-\tau)\| \{\|h(\tau,u(\tau)) - h(\tau,0)\| + \|h(\tau,0)\|\} d\tau \\ + \left\| f\left(s,u(s),\int_0^s a(s-\tau)g(\tau,u(\tau))d\tau\right) - f(s,0,0)\right\| + \|f(s,0,0)\| \end{array}\right] ds$$

using the hypothesis

$$\le M\|u_0\|_Y + M\left[\begin{array}{l} \int_0^t \left\{\int_0^s \|k(s-\tau)\|(\|h(s,u(\tau)) - h(s,0)\| + \|h(s,0)\|)d\tau\right\} ds \\ + \int_0^t \left\{\left\| f\left(s,u(s),\int_0^s a(s-\tau)g(\tau,u(\tau))d\tau\right) - f(s,0,0)\right\| + \|f(s,0,0)\|\right\} ds \end{array}\right]$$

$$\le M\left[\begin{array}{l} \|u_0\|_Y + k_T\int_0^t H_L\|u(s)\|ds + k_T H_0 T \\ + \int_0^t F_L\left\{\|u(s)\| + \int_0^s \|a(s-\tau)\|(\|g(\tau,u(\tau)) - g(\tau,0)\| + \|g(\tau,0)\|)d\tau\right\} ds + F_0 T \end{array}\right]$$

$$\le M\left[\|u_0\|_Y + k_T rTH_L + k_T TH_0 + F_L rT + a_T rTF_L G_L + a_T TF_L G_0 + F_0 T\right]$$

By using hypothesis $(H_{11}),$ we get $\|Fu(t)\|_Y \le r.$ Therefore F maps S into itself. Moreover, for $u, v \in S,$ we have

$$
\begin{aligned}
\left\|Fu(t)-Fv(t)\right\| \le{}& \left\|U_u(t,0)u_0-U_v(t,0)u_0\right\| \\
&+\int_0^t \left\| U_u(t,s)\left\{H(u)(s)+f\left(s,u(s),\int_0^s a(s-\tau)g(\tau,u(\tau))d\tau\right)\right\}\right. \\
&\left. -U_v(t,s)\left\{H(v)(s)+f\left(s,v(s),\int_0^s a(s-\tau)g(\tau,v(\tau))d\tau\right)\right\}\right\| ds \\
\le{}& \left\|U_u(t,0)u_0-U_v(t,0)u_0\right\| \\
&+\int_0^t \left[\left\| U_u(t,s)\left\{H(u)(s)+f\left(s,u(s),\int_0^s a(s-\tau)g(\tau,u(\tau))d\tau\right)\right\}\right.\right. \\
&\left. -U_v(t,s)\left\{H(u)(s)+f\left(s,u(s),\int_0^s a(s-\tau)g(\tau,u(\tau))d\tau\right)\right\}\right\| \\
&+\left\| U_v(t,s)\left\{H(u)(s)+f\left(s,u(s),\int_0^s a(s-\tau)g(\tau,u(\tau))d\tau\right)\right\}\right. \\
&\left.\left. -U_v(t,s)\left\{H(v)(s)+f\left(s,v(s),\int_0^s a(s-\tau)g(\tau,v(\tau))d\tau\right)\right\}\right\|\right] ds
\end{aligned}
$$

Using our hypothesis, we get

$$
\begin{aligned}
\left\|Fu(t)-Fv(t)\right\| \le{}& C_1\left\|u_0\right\|_Y T\max_{\tau\in J}\left\|u(\tau)-v(\tau)\right\| \\
&+\int_0^t \left\|U_u(t,s)-U_v(t,s)\right\|\left\{\left\|H(u)(s)\right\|+\left\|f\left(s,u(s),\int_0^s a(s-\tau)g(\tau,u(\tau))d\tau\right)\right\|\right\} ds \\
&+\int_0^t \left\|U_v(t,s)\right\|\left\{\left\|H(u)(s)-H(v)(s)\right\|+\left\|\begin{matrix} f\left(s,u(s),\int_0^s a(s-\tau)g(\tau,u(\tau))d\tau\right) \\ -f\left(s,v(s),\int_0^s a(s-\tau)g(\tau,v(\tau))d\tau\right)\end{matrix}\right\|\right\} ds \\
\le{}& C_1\left\|u_0\right\|_Y T\max_{\tau\in J}\left\|u(\tau)-v(\tau)\right\| + C_1 T\max_{\tau\in J}\left\|u(\tau)-v(\tau)\right\| \times \\
&\int_0^t \left\{\begin{matrix} \int_0^s \left\|k(s-\tau)\right\|\left(\left\|h(\tau,u(\tau))-h(\tau,0)\right\|+\left\|h(\tau,0)\right\|\right)d\tau \\ +\left\|f\left(s,u(s),\int_0^s a(s-\tau)g(\tau,u(\tau))d\tau\right)-f(s,0,0)\right\|+\left\|f(s,0,0)\right\| \end{matrix}\right\} ds
\end{aligned}
$$

$$+M\int_0^t \left\{ \begin{array}{l} \left\| \int_0^s k(s-\tau)\, h(\tau,u(\tau))\, d\tau - \int_0^s k(s-\tau)\, h(\tau,v(\tau))\, d\tau \right\| \\ + \left\| f\left(s,u(s),\int_0^s a(s-\tau)\, g(\tau,u(\tau))\, d\tau\right) - f\left(s,v(s),\int_0^s a(s-\tau)\, g(\tau,v(\tau))\, d\tau\right)\right\| \end{array} \right\} ds$$

$$\le C_1\|u_0\|_Y T\max_{\tau\in J}\|u(\tau)-v(\tau)\| + C_1T\max_{\tau\in J}\|u(\tau)-v(\tau)\| \int_0^t \left[k_T(H_Lr+H_0)+F_L\{r+(a_TG_Lr+a_TG_0)\}+F_0\right]ds$$

$$+M\max_{\tau\in J}\|u(\tau)-v(\tau)\|\int_0^t\{k_TH_L+F_L+a_TF_LG_L\}\,ds$$

$$\le C_1\|u_0\|_Y T\max_{\tau\in J}\|u(\tau)-v(\tau)\| + C_1T^2\left[k_T(H_Lr+H_0)+F_L\{r+(a_TG_Lr+a_TG_0)\}+F_0\right]\max_{\tau\in J}\|u(\tau)-v(\tau)\|$$

$$+MT\{k_TH_L+F_L+a_TF_LG_L\}\max_{\tau\in J}\|u(\tau)-v(\tau)\|$$

$$\le \left[\begin{array}{l} C_1\|u_0\|_Y T + C_1T^2\{k_T(H_Lr+H_0)+F_L(r+(a_TG_Lr+a_TG_0))+F_0\} + \\ MT\{k_TH_L+F_L+a_TF_LG_L\} \end{array}\right]\max_{\tau\in J}\|u(\tau)-v(\tau)\|$$

This gives

$$\|Fu(t)-Fv(t)\| \le \Gamma\max_{\tau\in J}\|u(\tau)-v(\tau)\|, \text{ by hypothesis } (H_{11})$$

where $0<\Gamma<1$. Thus F is a contraction from S to S. By the contraction mapping theorem F has a unique fixed point $u\in S$ which is the mild solution of (1)–(2) on J. From (H_6), it leads that $u(t)$ is in $C(J,Y)$ (see [10] Lemma 7.4). Indeed, $u(t)$ is weakly continuous as a Y-valued function. This means that $u(t)$ is separably valued in Y, hence it is strongly measurable. Then, $\|u(t)\|_Y$ is bounded and measurable function in t. Therefore, $u(t)$ is Bochner integrable (see e.g. [15], Chapter-V). Applying the relation $u(t)=Fu(t)$, we conclude that $u(t)$ is in $C(J,Y)$.

Now, consider the following evolution equation

$$\frac{du(t)}{dt}+A(t,u)u(t)=H(u)(t)+f(t,u(t),G(u)(t)),$$

$$u(0)=u_0, t\in[0,T]=J,$$

The above equation can be noted as

$$v'(t)+A(t)v(t)=h(t), t\in J \tag{8}$$

$$v(0)=u_0 \tag{9}$$

where $A(t)=A(t,u(t))$ and $h(t)=H(u)(t)+f(t,u(t),G(u)(t))$, $t\in J$ and u is the unique fixed point of F in S. We note that $A(t)$ satisfies $(H_1)-(H_3)$ of [11] (Section 5.5.3) and $h(t)\in C(J,Y)$. By using theorem 5.5.2 in Pazy [11] we summarize that unique function $v\in C(J,Y)$ exists such that $v\in C^1((0,T],X)$ satisfying (8)–(9) in X and hence v is given by

$$v(t)=U_u(t,0)u_0+\int_0^t U_u(t,s)\big[H(u)(s)+f(s,u(s),G(u)(s))\big]ds, t\in J,$$

where $U_u(t,s), 0\le s\le t\le T$ is the evolution system generated by the family $\{A(t,u(t))\}$, $t\in J$, of linear operator in X. The uniqueness of v implies that $v\equiv u$ on J and hence u is a unique classical solution of (1)–(2) and $u\in C([0,a]:Y)\cap C^1((0,a]:X)$. This completes the proof.

4 Example

Consider $\Omega\subset\mathbb{R}^n$ be a bounded domain with smooth boundary $\partial\Omega$. Let the differential operator

$$A(t,x,u;D)\,\omega=-\sum_{i,j=1}^{n}\frac{\partial}{\partial x_i}\left(a_{ij}(t,x,u(t,x))\frac{\partial\omega}{\partial x_j}\right)+c(t,x,u(t,x))\,\omega,$$

where $a_{ij}(t,x,u(t,x))$ and $c(t,x,u(t,x))$ are valued functions described on $J\times\overline{\Omega}\times\mathbb{R}$ and $J=[0,T]$, $0<T<\infty$. Let us suppose that $a_{ij}\in C[J\times\overline{\Omega}\times W,\mathbb{R}]$, where $W=C^{2l+1}(J\times\overline{\Omega},\mathbb{R})$ with $\frac{1}{2}<l<1$, $a_{ij}=a_{ji}, (1\le i,j\le n)$ and there exists some $c>0$ such that

$$\sum_{i,j=1}^{n}a_{ij}(t,x,u(t,x))q_iq_j\ge c|q|^2, q=(q_1,q_2,\ldots,q_n)\in\mathbb{R}$$

holds for each $(t,x,u(t,x))\in J\times\overline{\Omega}\times\mathbb{R}$.

Consider the partial integrodifferential equation

$$\frac{\partial u(t,x)}{\partial t}+A(t,x,u;D)u(t,x)=G(u)(t,x)+f(t,x,u(t,x),K(u)(t,x)),(t,x)\in(0,T]\times\Omega, \quad (10)$$

with the boundary condition

$$u(t,x)=0 \text{ for } (t,x)\in(0,T]\times\partial\Omega$$

and initial condition

$$u(0,x)=u_0(x) \text{ for } x\in\Omega,$$

where $G(u)(t,x)=\int_0^t a(t-x)\,h(s,x,u(s,x),\nabla u(s,x))\,ds$

and $K(u)(t,x)=\int_0^t k(t-x)\,g(s,x,u(s,x),\nabla u(s,x))\,ds$

$$\nabla=(D_1,D_2,...,D_n),D_i=\frac{\partial}{\partial x_i}$$

the function k and a are a real valued continuous function of bounded variation in $\mathbb{R}$ and the function $f(t,x,u,v)$ is also a real valued continuous function defined on $J\times\overline{\Omega}\times B\times B$ and there exist a constant $L>0$ such that

$$\|f(t,x,u_1,v_1)-f(t,x,u_2,v_2)\|\le L[\|u_1-u_2\|+\|v_1-v_2\|]$$

for $x\in\Omega$and $u_i,v_i\in B,i=1,2$. $u:J\times\Omega\to\mathbb{R}$ is unknown function and u_0 is its initial value. Again, we consider that $h,g:[0,\infty)\times\Omega\times B\times B\to\mathbb{R}$ is continuous and there exist constants $M>0$ and $N>0$ such that

$$\|h(t,x,u,\xi)-h(t,x,v,\eta)\|\le M[\|u-v\|+\|\xi-\eta\|]$$

$$\|g(t,x,u,\xi)-g(t,x,v,\eta)\|\le N[\|u-v\|+\|\xi-\eta\|]$$

for $x\in\Omega$ and $u,v,\xi,\eta\in B$.

Let $\frac{n}{2l-1}<p<\infty$ and $X=L^p(\Omega)$ with the usual norm

$$\|u\|_p=\left[\int_\Omega |u|^p\,dx\right]^{\frac{1}{p}}$$

then integrodifferential equation (10) can be reformed as an abstract integrodifferential (1) in Banach space X, where $A(t,u)v=A(t,x,u;D)v$ with domain

$$D(A(t,u))=\{v\in W_p^2(\Omega),v(t,x)=0,(t,x)\in(0,T]\times\partial\Omega\}$$

and

$$f\left(t,u,\int_0^t a(t-s)\,g(s,u(s))\,ds\right)=f(t,x,u(t,x)K(u)(t,x))$$

$$\int_0^t k(t-s)\,h(s,u(s))\,ds=G(u)(t,x).$$

We take note of that the assumption $(H_1)-(H_{10})$ are satisfied hence we may exert the finding of earlier part to assure the existence of unique classical solution of (10).

References

[1] S. Abbas and D. Bahuguna, Existence of solutions to a quasilinear functional differential equations, Electronic Journal of Differential Equations, Vol. 2009(2009), no. 164, pp. 1–8.
[2] H. Aman, Quasilinear evolution equations and parabolic systems, Transactions of the American mathematical society, 293 (1986), no.1, 191-227.
[3] E. H. Anderson, M. J. Anderson and W.T. England, Nonhomogeneous quasilinear evolution equations, Journal of integral equations, 3(1981), no.2, 175-184.
[4] D. Bahuguna, Quasilinear integrodifferential equations in Banach spaces, *Nonlinear Analysis* 24 (1995), 175-183.
[5] D. Bahuguna, Regularity solutions to quasilinear integrodifferential equations in Banach spaces, *Appl. Anal.* 62 (1996), 1-9.
[6] K. Balachandran and D.G. Park, Existence of solutions of quasilinear integrodifferential evolution equations in Banach spaces, Bull. Korean Math. Soc., 46(2009), no.4, 691-700.
[7] R. S. Dubey, Existence of a Regular solution to quasilinear implicit integrodifferential equations in Banach space, Nonlinear Dynamics and Systems Theory,11(2) (2011) 137–146.
[8] R. Haloi, D. Bahuguna and D. N. Pandey, Electronic Journal of Differential Equations, Vol. 2012 (2012), ~~No~~ no. 13, pp. 1–10.
[9] S. Kato, Nonhomogeneous quasilinear evolution equations in Banach spaces, Nonlinear Analysis,9(1985), 1061-1071.
[10] T. Kato, Quasilinear equations of evolution with application to partial differential equations, Lecture Notes in Math. 448(1975), 25-70.
[11] A. Pazy, Semigroup of Linear Operators and Applications to Partial Differential Equations, Springer Verlag, New York, 1983.
[12] F. P. Samuel and K. Balachandran, Existence of solutions of quasilinear integrodifferential evolution equations with impulsive conditions, Thai Journal of Mathematics, 9(2011), no.1, 139-152.
[13] F. P. Samuel, T.R. Lisso and K.Z. Kaunda, Existence solutions for quasilinear evolution integrodifferential equations with infinite delay, International Journal of Engineering and Technical Research (IJETR), 2(4), April 2014.
[14] N. Sanekata, Abstract quasilinear equations of evolution in nonreflexive Banach spaces, Hiroshima Mathematical Journal, 19(1989), 109-139.
[15] K. Yosida, Functional Analysis, springer (1968).