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Original Article

## A New Operation on Soft Sets: Extended Difference of Soft Sets

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**Abstract** — Soft Set Theory, which has been considered as an adequate mathematical device, was proposed by Molodtsov to deal with ambiguities and uncertainties. Several operations on soft sets were defined in many soft set papers. This study is based on the paper "On operations of Soft Sets" by Sezgin and Atagün [Comput. Math. Appl. 61 (2011) 1457-1467]. In this paper, we define a new operation on soft sets, called extended difference and investigate its relationship between extended difference and restricted difference and some other operations of soft sets.

**Keywords** — *Soft sets, Restricted union, Extended union, Restricted intersection, Extended intersection, Restricted difference, Extended difference.*

## 1 Introduction

In different areas, Mathematicians and Scientists have been facing several ambiguities and uncertainties in the problems of computer science, statistics, different branches of engineering, environmental sciences, economics, medical sciences, sociology and many other different fields of sciences. In the past, many of the theories were presented to overcome these uncertainties. But Molodtsov [10] has found that these theories have their own built-in deadlocks. The main problem shared by those theories is their conflict with the parametrization tools. So, to overcome these deadlocks properly, in 1999, Molodtsov [10] suggested a fully new approach that is soft set theory which acts as a breakthrough for those deadlocks. In this theory, a soft set could be a parameterized group of subsets of the universal set and also the drawback of setting the membership operation does not appear. It gives us a lot of choices in

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the problem to solve them easily. Now the development in the field of soft set theory is increasing day by day.

In 2002, Maji et al. [9] applied the soft sets to decision creating problems using rough mathematics and then in 2003, Maji and Biswas [8] introduced many operations of soft sets. And then, many authors [1, 2, 4, 5, 6, 7, 11, 13] also studied the different soft operations.

In 2011, Sezgin and Atagün [12] discussed the fundamental theorems about operations about soft sets i.e; union and intersection of soft sets and other operations. In that paper, they defined union and intersection operation of soft sets both with restricted and extended condition but defined the difference operation only with restricted condition. They did not define difference operation with extended condition. Here in this paper, we have defined a new operation on soft sets called *extended difference* and also proved some of its properties. Moreover, we have also proved the interesting result which shows the relationship between extended difference with the restricted difference. The main objective of this paper is to make soft set theory more effective and solid by enhancing the conceptual feature of operations on soft sets.

## 2 Preliminary

In view this chapter, we review a few fundamental assumptions in soft set theory. From now on,  $U$  is a basic universe set,  $E$  is set of all feasible parameters in the below discussion with reference to  $U$ . We denote the power set on  $U$  (i.e; the set of consisting all the subsets of  $U$ ) by  $P(U)$ ,  $A$  is a subset of  $E$ . Generally parameters are the numeric values, attributes of elements in  $U$ . Molodtsov [10], illustrated the soft set in such a way as following:

**Definition 2.1.** ([10]) Let  $U$  be the fundamental universe,  $E$  is a set about parameters and  $D$  be a subset of  $E$ . The pair  $(L, D)$  is known to be a soft set over  $U$ , when  $L$  is mapping of  $D$  within set of every subsets of set  $U$ .

For  $e \in D$ ,  $L(e)$  is may be regarded as a set of  $e$ -elements of soft set  $(L, D)$ .

**Definition 2.2.** ([8]) Let  $(L, D)$  and  $(K, J)$  be two soft sets over the same universe  $U$ , then  $(K, J)$  is soft subset on  $(L, D)$  if it satisfies:

(i)  $J \subseteq D$

(ii)  $K(e) \subseteq L(e), \forall e \in J$

and it is denoted by  $(K, J) \tilde{\subseteq} (L, D)$  and also if  $(L, D)$  is soft subset on  $(K, J)$  then  $(K, J)$  is said to be a soft superset of  $(L, D)$  and it is denoted by  $(K, J) \tilde{\supseteq} (L, D)$ .

**Definition 2.3.** ([8]) Let  $(L, D)$  and  $(K, J)$  be two soft sets over the identical universe  $U$ .  $(L, D)$  and  $(K, J)$  are called soft equal sets if  $(L, D)$  is soft subset of  $(K, J)$  and  $(K, J)$  is soft subset of  $(L, D)$ .

**Definition 2.4.** ([3]) The relative complement of a soft set  $(K, B)$  is shown by  $(K, B)^r$  and is illustrated as  $(K, B)^r = (K^r, B)$ , where  $K^r : B \rightarrow P(U)$  is a mapping assigned as  $K^r(e) = U \setminus K(e)$ ,  $\forall e \in B$ .

**Definition 2.5.** ([8]) A soft set  $(K, B)$  over  $U$  is known as a null soft set shown as  $\Phi_B$  if for all  $e \in B$ ,  $K(e) = \emptyset$ .

**Definition 2.6.** ([8]) Let  $(L, D)$  be a soft set over a universe  $U$ . Then,  $(L, D)$  is known as an absolute soft set if for all  $e \in D$ ,  $L(e) = U$  and it is denoted by  $\mathcal{U}_D$ .

**Definition 2.7.** ([8]) Let  $(L, D)$  and  $(K, Z)$  be soft sets over an identical universe  $U$ , then “ $(L, D)AND(K, Z)$ ” denoted by  $(L, D) \wedge (K, Z)$  and is expressed as  $(L, D) \wedge (K, Z) = (H, D \times Z)$ , where  $H(\alpha, \beta) = L(\alpha) \cap K(\beta)$ ,  $\forall(\alpha, \beta) \in D \times Z$ .

**Definition 2.8.** ([8]) Let  $(L, D)$  and  $(K, Z)$  be two soft sets over a common universe  $U$ , then “ $(L, D)OR(K, Z)$ ” shown as  $(L, D) \vee (K, Z)$  and is expressed as  $(L, D) \vee (K, Z) = (H, D \times Z)$ , where  $H(\alpha, \beta) = L(\alpha) \cup K(\beta)$ ,  $\forall(\alpha, \beta) \in D \times Z$ .

**Definition 2.9.** ([3]) Let  $(L, D)$  and  $(K, Z)$  be two soft sets over an identical universe  $U$ , where  $D \cap Z \neq \phi$ . The restricted union of  $(L, D)$  and  $(K, Z)$  is shown by  $(L, D) \cup_R (K, Z)$  and expressed as  $(L, D) \cup_R (K, Z) = (H, C)$ , when  $C = D \cap Z$  and for all  $e \in C$ ,  $H(e) = L(e) \cup K(e)$ .

**Definition 2.10.** ([8]) Let  $(L, D)$  and  $(K, Z)$  be two soft sets over an identical universe  $U$ . The extended union of  $(L, D)$  and  $(K, Z)$  is expressed as the soft set  $(I, O)$  fulfilling the situations: (i) $O = D \cup Z$ ; (ii)for all  $e \in O$ ,

$$I(e) = \begin{cases} L(e) & \text{if } e \in D \setminus Z, \\ K(e) & \text{if } e \in Z \setminus D, \\ L(e) \cup K(e) & \text{if } e \in D \cap Z. \end{cases}$$

This relation is shown by  $(L, D) \tilde{\cup} (K, Z) = (I, O)$ .

**Definition 2.11.** ([3]) Let  $(L, D)$  and  $(K, Z)$  be two soft sets over an identical universe  $U$ , where  $D \cap Z \neq \phi$ . The restricted intersection of  $(L, D)$  and  $(K, Z)$  shown by  $(L, D) \cap_R (K, Z)$  and is expressed as  $(L, D) \cap_R (K, Z) = (H, C)$ , where  $C = D \cap Z$  and for all  $e \in C$ ,  $H(e) = L(e) \cap K(e)$ .

**Definition 2.12.** ([3]) Let  $(L, D)$  and  $(K, Z)$  be two soft sets over an identical universe  $U$ . The extended intersection of  $(L, D)$  and  $(K, Z)$  is expressed as the soft set  $(I, O)$  fulfilling situations: (i) $C = D \cup Z$ ; (ii)for all  $e \in O$ ,

$$I(e) = \begin{cases} L(e) & \text{if } e \in D \setminus Z, \\ K(e) & \text{if } e \in Z \setminus D, \\ L(e) \cap K(e) & \text{if } e \in D \cap Z. \end{cases}$$

This relation is shown by  $(L, D) \tilde{\cap} (K, Z) = (I, O)$ .

**Definition 2.13.** ([12]) Let  $(L, D)$  and  $(K, Z)$  be two soft sets over an identical universe  $U$ , where  $D \cap Z \neq \phi$ . The restricted difference of  $(L, D)$  and  $(K, Z)$  is shown by  $(L, D) \sim_R (K, Z)$ , and expressed as  $(L, D) \sim_R (K, Z) = (H, C)$ , where  $C = D \cap Z$  and for all  $e \in C$ ,  $H(e) = L(e) \setminus K(e)$ .

**Definition 2.14.** ([12]) Let  $(L, D)$  and  $(K, Z)$  be two soft sets over an identical universe  $U$ , where  $D \cap Z \neq \phi$ . The restricted symmetric difference of  $(L, D)$  and  $(K, Z)$  is shown by  $(L, D) \tilde{\Delta} (K, Z)$ , and expressed as  $(L, D) \tilde{\Delta} (K, Z) = ((L, D) \cup_R (K, Z)) \sim_R ((L, D) \cap_R (K, Z)) = (T, C)$ , where  $C = D \cap Z$ .

### 3 Properties of operations of soft sets and their correlations with one another

As the fundamental properties and theorems related to operations of soft sets such as restricted union, extended union, restricted intersection, extended intersection, restricted difference we refer to the paper Sezgin and Atagün [12], Maji et al. [8], Ali et al. [3] and Pei and Miao [11]. Now we are ready to give the definition of extended difference of soft sets and its basic properties.

**Definition 3.1.** Let  $(X, D)$  and  $(P, E)$  be the two soft sets over an identical universe  $U$ . The extended difference of  $(X, D)$  and  $(P, E)$  can be expressed as the soft set  $(L, C)$  fulfilling the situations as under: (i)  $C = D \cup E$ ; (ii) for all  $e \in C$ ,

$$L(e) = \begin{cases} X(e) & \text{if } e \in D \setminus E, \\ P(e) & \text{if } e \in E \setminus D, \\ X(e) \setminus P(e) & \text{if } e \in D \cap E. \end{cases}$$

Thus, the relation is shown by  $(X, D) \sim_E (P, E) = (L, C)$ .

**Example 3.2.** Let  $E$  be the universe set of parameters,  $D, B$  be the subsets of  $E$  such that  $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$ ,  $D = \{e_1, e_2, e_3, e_4\}$  and  $B = \{e_3, e_4, e_5\}$ . Assume that  $(X, D)$  and  $(K, B)$  are two soft sets over common universe  $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$  as following:  $(X, D) = \{(e_1, \{h_2, h_4\}), (e_2, \{h_1, h_3\}), (e_3, \{h_3, h_5\}), (e_4, \{h_1, h_6\})\}$ ,  $(P, B) = \{(e_3, \{h_4, h_5\}), (e_4, \{h_1, h_2\}), (e_5, \{h_2, h_5\})\}$ , where  $C = D \cup B = \{e_1, e_2, e_3, e_4, e_5\}$ .

Now let  $(X, D) \sim_E (P, B) = (L, D \cup B)$ , where

$$L(e) = \begin{cases} X(e) & \text{if } e \in D \setminus B, \\ P(e) & \text{if } e \in B \setminus D, \\ X(e) \setminus P(e) & \text{if } e \in D \cap B. \end{cases}$$

and for all  $e \in D \cup B = \{e_1, e_2, e_3, e_4, e_5\}$ . Since  $D \setminus B = \{e_1, e_2\}$ ,  $L(e_1) = X(e_1) = \{h_2, h_4\}$ ,  $L(e_2) = X(e_2) = \{h_1, h_3\}$ . Since  $B \setminus D = \{e_5\}$ ,  $L(e_5) = P(e_5) = \{h_2, h_5\}$  and since  $D \cap B = \{e_3, e_4\}$ ,  $L(e_3) = X(e_3) \setminus P(e_3) = \{h_3, h_5\} \setminus \{h_4, h_5\} = \{h_3\}$ ,  $L(e_4) = X(e_4) \setminus P(e_4) = \{h_1, h_6\} \setminus \{h_1, h_2\} = \{h_6\}$ . Hence,  $(X, D) \sim_E (P, B) = (L, D \cup B) = \{(e_1, \{h_2, h_4\}), (e_2, \{h_1, h_3\}), (e_3, \{h_3\}), (e_4, \{h_6\}), (e_5, \{h_2, h_5\})\}$ .

**Theorem 3.3.** Let  $(P, D), (P, V), (R, J), (S, I), (X, Z)$  be two soft sets over a common universe  $U$ . Then, we have the following:

- a)  $(P, D) \sim_E \Phi_D = (P, D)$ .
- b)  $(P, D) \sim_E (P, D) = \Phi_D$ .
- c)  $\mathcal{U}_D \sim_E (P, D) = (P, D)^r$ .
- d) Left distribution of restricted intersection over extended difference:  
 $(P, V) \cap_R ((R, J)) \sim_E (S, I) = ((P, V) \cap_R (R, J)) \sim_E ((P, V) \cap_R (S, I))$ .
- e) Right distribution of restricted intersection over extended difference:  
 $((X, Z) \sim_E (R, J)) \cap_R (S, I) = ((X, Z) \cap_R (S, I)) \sim_E ((R, J) \cap_R (S, I))$ .

f) Right distribution of restricted difference over extended difference:

$$((P, Z) \sim_E (R, J)) \sim_R (W, I) = ((P, Z) \sim_R (W, I)) \sim_E ((R, J) \sim_R (W, I)).$$

*Proof.* **a)** Let  $\Phi_D = (M, D)$  and  $(P, D) \sim_E \Phi_D = (P, D) \sim_E (M, D) = (H, D)$ , where

$$H(e) = \begin{cases} P(e) & \text{if } e \in D \setminus D, \\ M(e) & \text{if } e \in D \setminus D, \\ P(e) \setminus M(e) & \text{if } e \in D \cap D = D. \end{cases}$$

and for all  $e \in D \cup D$ . Since  $M(e) = \phi$  for all  $e \in D \cup D$ , it follows that  $H(e) = P(e) \setminus \phi = P(e)$ . This means that  $P$  and  $H$  are the same mappings. This completes the proof.

**b)** Let  $(P, D) \sim_E (P, D) = (H, D)$ , where

$$H(e) = \begin{cases} P(e) & \text{if } e \in D \setminus D, \\ P(e) & \text{if } e \in D \setminus D, \\ P(e) \setminus P(e) & \text{if } e \in D \cap D = D. \end{cases}$$

and for all  $e \in D \cup D$ . Hence,  $H(e) = P(e) \setminus P(e) = \emptyset$ . This completes the proof.

**c)** Let  $\mathcal{U}_D = (G, D)$  and  $\mathcal{U}_D \sim_E (P, D) = (G, D) \sim_E (P, D) = (W, D)$ , where

$$W(e) = \begin{cases} G(e) & \text{if } e \in D \setminus D, \\ P(e) & \text{if } e \in D \setminus D, \\ G(e) \setminus P(e) & \text{if } e \in D \cap D = D. \end{cases}$$

for all  $e \in D \cup D$ . Since  $G(e) = U$  for all  $e \in D \cup D$ , it follows that  $W(e) = U \setminus P(e) = P^r(e)$ , which completes the proof.

**d)** For the left hand side of the property, let  $(R, J) \sim_E (S, I) = (T, J \cup I)$ , where

$$T(e) = \begin{cases} R(e) & \text{if } e \in J \setminus I, \\ S(e) & \text{if } e \in I \setminus J, \\ R(e) \setminus S(e) & \text{if } e \in J \cap I. \end{cases}$$

for all  $e \in J \cup I$ .

First let  $(P, V) \cap_R (T, J \cup I) = (X, V \cap (J \cup I))$ , where  $X(e) = P(e) \cap T(e)$  for all  $e \in V \cap (J \cup I)$ .

Due the main features of set theory and according to the expressions of  $X$  also with  $T$  and suppose  $T$  is a piecewise function, we write the following equalities for the mapping  $X$ :

$$X(e) = \begin{cases} P(e) \cap R(e) & \text{if } e \in V \cap (J \setminus I) = (V \cap J) \setminus (V \cap I), \\ P(e) \cap S(e) & \text{if } e \in V \cap (I \setminus J) = (V \cap I) \setminus (V \cap J), \\ P(e) \cap (R(e) \setminus S(e)) & \text{if } e \in V \cap (J \cap I) \end{cases}$$

for all  $e \in V \cap (J \cup I)$ .

For the right hand side of the property, let  $(P, V) \cap_R (R, J) = (D, V \cap J)$ , where  $D(e) = P(e) \cap R(e)$  for all  $e \in V \cap J \neq \phi$ . Suppose that  $(P, V) \cap_R (S, I) = (O, V \cap I)$ , where  $O(e) = P(e) \cap S(e)$  for all  $e \in V \cap I \neq \phi$ . Assume that  $(D, V \cap J) \sim_E (O, V \cap I) = (Z, (V \cap J) \cup (V \cap I))$ , where

$$Z(e) = \begin{cases} D(e) & \text{if } e \in (V \cap J) \setminus (V \cap I), \\ O(e) & \text{if } e \in (V \cap I) \setminus (V \cap J), \\ D(e) \setminus O(e) & \text{if } e \in (V \cap J) \cap (V \cap I) = V \cap (J \cap I) \end{cases}$$

for all  $e \in (V \cap J) \cup (V \cap I)$ . Due to considering the expressions of  $D$  and  $O$ , we write the mapping  $Z$  as below:

$$Z(e) = \begin{cases} P(e) \cap R(e) & \text{if } e \in (V \cap J) \setminus (V \cap I), \\ P(e) \cap S(e) & \text{if } e \in (V \cap I) \setminus (V \cap J), \\ (P(e) \cap R(e)) \setminus (P(e) \cap S(e)) & \text{if } e \in V \cap (J \cap I). \end{cases}$$

It shows that  $X$  and  $Z$  are the identical mapping when we are assuming the attributes of operations about set theory. Hence the proof is completed.

e) For the left hand side of the property, let  $(X, Z) \sim_E (R, J) = (T, Z \cup J)$ , where

$$T(e) = \begin{cases} X(e) & \text{if } e \in Z \setminus J, \\ R(e) & \text{if } e \in J \setminus Z, \\ X(e) \setminus R(e) & \text{if } e \in Z \cap J \end{cases}$$

for all  $e \in Z \cup J$ .

First let  $(T, Z \cup J) \cap_R (S, I) = (Q, (Z \cup J) \cap I)$ , where  $Q(e) = T(e) \cap S(e)$  for all  $e \in (Z \cup J) \cap I$ . Due the main features of set theory and according to the expressions of  $Q$  also with  $T$  and suppose that  $T$  is a piecewise function, also we write the following equalities for the mapping  $Q$ :

$$Q(e) = \begin{cases} X(e) \cap S(e) & \text{if } e \in (Z \setminus J) \cap I = (Z \cap I) \setminus (J \cap I), \\ R(e) \cap S(e) & \text{if } e \in (J \setminus Z) \cap I = (J \cap I) \setminus (Z \cap I), \\ (X(e) \setminus R(e)) \cap S(e) & \text{if } e \in (Z \cap J) \cap I \end{cases}$$

for all  $e \in (Z \cup J) \cap I$ .

For the right hand side of the property, let  $(X, Z) \cap_R (S, I) = (M, Z \cap I)$ , where  $M(e) = X(e) \cap S(e)$  for all  $e \in Z \cap I$ . Assume  $(R, J) \cap_R (S, I) = (O, J \cap I)$ , where  $O(e) = R(e) \cap S(e)$  for all  $e \in J \cap I$ . Let  $(M, Z \cap I) \sim_E (O, J \cup I) = (W, (Z \cap I) \cup (J \cap I))$ , where

$$W(e) = \begin{cases} M(e) & \text{if } e \in (Z \cap I) \setminus (J \cap I), \\ O(e) & \text{if } e \in (J \cap I) \setminus (Z \cap I), \\ M(e) \setminus O(e) & \text{if } e \in (Z \cap I) \cap (J \cap I) \end{cases}$$

for all  $e \in (Z \cap I) \cup (J \cap I)$ . By assuming the main expressions of  $M$  and  $O$ , we can rewrite the mapping  $W$  as below:

$$W(e) = \begin{cases} X(e) \cap S(e) & \text{if } e \in (Z \cap I) \setminus (J \cap I), \\ R(e) \cap S(e) & \text{if } e \in (J \cap I) \setminus (Z \cap I), \\ (X(e) \cap S(e)) \setminus (R(e) \cap S(e)) & \text{if } e \in (Z \cap I) \cap (J \cap I) \end{cases}$$

for all  $e \in (Z \cap J) \cup (Z \cap I)$ . This leads that  $Q$  and  $W$  are the identical mapping. Hence this completes the proof.

f) For the left hand side of the property, let  $(P, Z) \sim_E (R, J) = (T, Z \cup J)$ , where

$$T(e) = \begin{cases} P(e) & \text{if } e \in Z \setminus J, \\ R(e) & \text{if } e \in J \setminus Z, \\ P(e) \setminus R(e) & \text{if } e \in Z \cap J \end{cases}$$

for all  $e \in Z \cup J$ .

First let  $(T, Z \cup J) \sim_R (W, I) = (Q, (Z \cup J) \cap I)$ , where  $Q(e) = T(e) \setminus W(e)$  for all  $e \in (Z \cup J) \cap I$ . Due the main features of set theory and according to the expressions of  $Q$  also with  $T$  and suppose that  $T$  is a piecewise function, we can write the following equalities for the mapping  $Q$ :

$$Q(e) = \begin{cases} P(e) \setminus W(e) & \text{if } e \in (Z \setminus J) \cap I = (Z \cap I) \setminus (J \cap I), \\ R(e) \setminus W(e) & \text{if } e \in (J \setminus Z) \cap I = (J \cap I) \setminus (Z \cap I), \\ (P(e) \setminus R(e)) \setminus W(e) & \text{if } e \in (Z \cap J) \cap I \end{cases}$$

for all  $e \in (Z \cup J) \cap I$ .

For the right hand side of the property, let  $(P, Z) \sim_R (W, I) = (M, Z \cap I)$ , where  $M(e) = P(e) \setminus W(e)$  for all  $e \in Z \cap I$ . Assume  $(R, J) \sim_R (W, I) = (O, J \cap I)$ , where  $O(e) = R(e) \setminus W(e)$  for all  $e \in J \cap I$ . Let  $(M, Z \cap I) \sim_E (O, J \cup I) = (X, (Z \cap I) \cup (J \cap I))$ , where

$$X(e) = \begin{cases} M(e) & \text{if } e \in (Z \cap I) \setminus (J \cap I), \\ O(e) & \text{if } e \in (J \cap I) \setminus (Z \cap I), \\ M(e) \setminus O(e) & \text{if } e \in (Z \cap I) \cap (J \cap I) \end{cases}$$

for all  $e \in (Z \cap I) \cup (J \cap I)$ . By assuming the main expressions of  $M$  and  $O$ , we can rewrite the mapping  $X$  as below:

$$X(e) = \begin{cases} P(e) \setminus W(e) & \text{if } e \in (Z \cap I) \setminus (J \cap I), \\ R(e) \setminus W(e) & \text{if } e \in (J \cap I) \setminus (Z \cap I), \\ (P(e) \setminus W(e)) \setminus (R(e) \setminus W(e)) & \text{if } e \in (Z \cap I) \cap (J \cap I) \end{cases}$$

for all  $e \in (Z \cap J) \cup (Z \cap I)$ . This leads that  $Q$  and  $X$  are the identical mapping. Hence this completes the proof.

Now, we give a corresponding example of part (g) of above Theorem.

**Example 3.4.** Suppose that  $E$  is the universe set of parameters and  $Z, J$  and  $I$  are the subsets of  $E$  such that  $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$ ,  $Z = \{e_1, e_2, e_3, e_5\}$ ,  $J = \{e_4, e_5, e_6\}$  and  $I = \{e_2, e_5, e_6, e_7\}$ .

Suppose that  $(P, Z)$ ,  $(R, J)$  and  $(W, I)$  be three soft sets over a common universe  $U = \{h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8, h_9\}$  such that

$$(P, Z) = \{(e_1, \{h_1, h_2, h_9\}), (e_2, \{h_4, h_5, h_6\}), (e_3, \phi), (e_5, \{h_7, h_8, h_9\})\},$$

$(R, J) = \{(e_4, \{h_3, h_4, h_7\}), (e_5, \{h_7, h_8, h_9\}), (e_6, \{h_7, h_8\})\}$ ,  
 $(W, I) = \{(e_2, \{h_4, h_5\}), (e_5, \{h_3, h_8\}), (e_6, \{h_1, h_3, h_5, h_6\}), (e_7, \{h_4, h_6, h_8\})\}$ .  
 For the left hand side of the equality, let  $(P, Z) \sim_E (R, J) = (T, Z \cup J)$ , where

$$T(e) = \begin{cases} P(e) & \text{if } e \in Z \setminus J, \\ R(e) & \text{if } e \in J \setminus Z, \\ P(e) \setminus R(e) & \text{if } e \in Z \cap J \end{cases}$$

for all  $e \in Z \cup J = \{e_1, e_2, e_3, e_4, e_5, e_6\}$ .

Since  $Z \setminus J = \{e_1, e_2, e_3\}$ ,  $T(e_1) = P(e_1) = \{h_1, h_2, h_9\}$ ,  $T(e_2) = P(e_2) = \{h_4, h_5, h_6\}$ ,  $T(e_3) = P(e_3) = \phi$ , since  $J \setminus Z = \{e_4, e_6\}$ ,  $T(e_4) = R(e_4) = \{h_3, h_4, h_7\}$ ,  $T(e_6) = R(e_6) = \{h_7, h_8\}$  and since  $Z \cap J = \{e_5\}$ ,  $T(e_5) = P(e_5) \setminus R(e_5) = \{h_7, h_8, h_9\} \setminus \{h_7, h_8, h_9\} = \phi$ . So,

$$(T, Z \cup J) = \{(e_1, \{h_1, h_2, h_9\}), (e_2, \{h_4, h_5, h_6\}), (e_3, \phi), (e_4, \{h_3, h_4, h_7\}), (e_5, \phi), (e_6, \{h_7, h_8\})\}.$$

Now let  $(T, Z \cup J) \sim_R (W, I) = (Q, (Z \cup J) \cap I)$ , where  $Q(e) = T(e) \setminus W(e)$  for all  $e \in (Z \cup J) \cap I$ . By the main features of set theory and the definitions of  $Q$  along with  $T$  and considering that  $T$  is a piecewise function, we can write the below equalities for  $Q$ :

$$Q(e) = \begin{cases} P(e) \setminus W(e) & \text{if } e \in (Z \setminus J) \cap I = (Z \cap I) \setminus (J \cap I), \\ R(e) \setminus W(e) & \text{if } e \in (J \setminus Z) \cap I = (J \cap I) \setminus (Z \cap I), \\ (P(e) \setminus R(e)) \setminus W(e) & \text{if } e \in (Z \cap J) \cap I \end{cases}$$

for all  $e \in (Z \cup J) \cap I = \{e_2, e_5, e_6\}$ .

Since  $(Z \setminus J) \cap I = (Z \cap I) \setminus (J \cap I) = \{e_2\}$ ,  $Q(e_2) = P(e_2) \setminus W(e_2) = \{h_4, h_5, h_6\} \setminus \{h_4, h_5\} = \{h_6\}$ , since  $(J \setminus Z) \cap I = (J \cap I) \setminus (Z \cap I) = \{e_6\}$ ,  $Q(e_6) = R(e_6) \setminus W(e_6) = \{h_7, h_8\} \setminus \{h_1, h_3, h_5, h_6\} = \{h_7, h_8\}$  and since  $(Z \cap J) \cap I = Z \cap I \cap J = \{e_5\}$ ,  $Q(e_5) = (P(e_5) \setminus R(e_5)) \setminus W(e_5) = \phi \setminus \{h_3, h_8\} = \phi$ . So,

$$(Q, (Z \cup J) \cap I) = ((P, Z) \sim_E (R, J)) \sim_R (W, I) = \{(e_2, \{h_6\}), (e_5, \phi), (e_6, \{h_7, h_8\})\}.$$

For the right hand side of the equality let  $(P, Z) \sim_R (W, I) = (M, Z \cap I)$ , where  $M(e) = P(e) \setminus W(e)$  for all  $e \in Z \cap I = \{e_2, e_5\}$ , then  $M(e_2) = P(e_2) \setminus W(e_2) = \{h_4, h_5, h_6\} \setminus \{h_4, h_5\} = \{h_6\}$ ,  $M(e_5) = P(e_5) \setminus W(e_5) = \{h_7, h_8, h_9\} \setminus \{h_3, h_8\} = \{h_7, h_9\}$ .

Now let  $(R, J) \sim_R (W, I) = (O, J \cap I)$ , where  $O(e) = R(e) \setminus W(e)$  for all  $e \in J \cap I = \{e_5, e_6\}$ . Then,  $O(e_5) = R(e_5) \setminus W(e_5) = \{h_7, h_8, h_9\} \setminus \{h_3, h_8\} = \{h_7, h_9\}$ ,  $O(e_6) = R(e_6) \setminus W(e_6) = \{h_7, h_8\} \setminus \{h_1, h_3, h_5, h_6\} = \{h_7, h_8\}$ .

Now let  $(M, Z \cap I) \sim_E (O, J \cap I) = (X, (Z \cap I) \cup (J \cap I))$ , where



$$X(e) = \begin{cases} M(e) & \text{if } e \in (Z \cap I) \setminus (J \cap I), \\ O(e) & \text{if } e \in (J \cap I) \setminus (Z \cap I), \\ M(e) \setminus O(e) & \text{if } e \in (Z \cap I) \cap (J \cap I) \end{cases}$$

for all  $e \in (Z \cap I) \cup (J \cap I) = \{e_2, e_5, e_6\}$ . Since  $(Z \cap I) \setminus (J \cap I) = \{e_2\}$ ,  $X(e_2) = M(e_2) = P(e_2) \setminus W(e_2) = \{h_6\}$ , since  $(J \cap I) \setminus (Z \cap I) = \{e_6\}$ ,  $X(e_6) = O(e_6) = R(e_6) \setminus W(e_6) = \{h_7, h_8\}$  and since  $(Z \cap I) \cap (J \cap I) = \{e_5\}$ ,  $X(e_5) = M(e_5) \setminus O(e_5) = (P(e_5) \setminus (W(e_5))) \setminus (R(e_5) \setminus W(e_5)) = \phi$ . So,  $(X, (Z \cap I) \cup (J \cap I)) = ((P, Z) \sim_R (W, I)) \sim_E ((R, J) \sim_R (W, I)) = \{(e_2, \{h_6\}), (e_5, \phi), (e_6, \{h_7, h_8\})\}$ . Since  $Q$  and  $X$  are the same mappings,  $((P, Z) \sim_E (R, J)) \sim_R (W, I) = ((P, Z) \sim_R (W, I)) \sim_E ((R, J) \sim_R (W, I))$  is satisfied.

## 4 Conclusion and Future Work

Here in this work, we have illustrated a brief analytical review of operations of soft sets. We have defined the extended difference of soft sets and also proved some of its properties. Moreover, we have shown the relationship between extended difference and the restricted difference and some other operations of soft sets. The main objective of this paper is to make soft set theory more effective and solid by enhancing the conceptual feature of operations on soft sets. One can may define the extended symmetric difference and can also construct a property which shows a relationship or connects of extended symmetric difference with restricted symmetric difference.

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