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On Lightly Nano ω-Closed Sets

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Abstaract − In this paper, we introduce and investigate the concepts of lightly nano ω-closed sets and lightly nano ω-open sets in a nano topological spaces, which are weaker form of lightly nano-closed sets and lightly nano-open sets and relationships among related ng-closed sets are investigated.

Keywords − *Lightly nano-closed sets, ng-closed sets and lightly nano ω-closed set.*

1 Introduction

Thivagar *et al.* [4] introduced the concept of nano topological spaces with respect to a subset X of a universe U. We study the relationships between some near nano open sets in nano topological spaces.

In this paper, we introduce and investigate the concepts of lightly nano ω-closed sets and lightly nano ω-open sets in a nano topological spaces, which are weaker form of lightly nano-closed sets and lightly nano-open sets and relationships among related ng-closed sets are investigated.

2 Preliminaries

Definition 2.1. *[7] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.*

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1. *The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \subseteq X\}$, where $\tau_R(x)$ denotes the equivalence class determined by x.*

2. *The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \cap X \neq \phi\}$.*

3. *The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.*

Definition 2.2. *[4] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms:*

1. *U and $\phi \in \tau_R(X)$,*

2. *The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,*

3. *The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.*

Thus $\tau_R(X)$ is a topology on U called the nano topology with respect to X and $(U, \tau_R(X))$ is called the nano topological space. The elements of $\tau_R(X)$ are called nano-open sets (briefly n-open sets). The complement of a n-open set is called n-closed.

In the rest of the paper, we denote a nano topological space by $(U, \mathcal{N})$, where $\mathcal{N} = \tau_R(X)$. The nano-interior and nano-closure of a subset A of U are denoted by n-int(A) and n-cl(A), respectively.

Definition 2.3. *A subset H of a space $(U, \mathcal{N})$ is called*

1. *nano α-open set (briefly nα-open) [4] if $H \subseteq$ n-int(n-cl(n-int(H))).*

2. *nano semi-open set [4] if $H \subseteq$ n-cl(n-int(H)).*

3. *nano pre-open set [4] if $H \subseteq$ n-int(n-cl(H)).*

4. *nano semi-preopen set [9] if $H \subseteq$ n-cl(n-int(n-cl(H))).*

5. *nano regular-open set (briefly nr-open) [4] if $H =$ n-int(n-cl(H)).*

6. *nano nowhere dense (briefly n-nowhere dense) [5] if n-int(n-cl(H)) $= \phi$.*

The complements of the above mentioned sets are called their respective closed sets.

Definition 2.4. *A subset H of a space $(U, \mathcal{N})$ is called*

1. *nano g-closed (briefly ng-closed) [2] if $n\text{-}cl(H) \subseteq G$, whenever $H \subseteq G$ and G is n-open.*

2. *nano sg-closed set (briefly nsg-closed) [3] if $n\text{-}scl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano semi open.*

3. *nano αg-closed (briefly nαg-closed) [11] if $n\text{-}\alpha cl(H) \subseteq G$ whenever $H \subseteq G$ and G is n-open.*

4. *nano gα-closed (briefly ngα-closed) [11] if $n\text{-}\alpha cl(H) \subseteq G$ whenever $H \subseteq G$ and G is nα-open.*

5. *nano gsp-closed (briefly ngsp-closed) [9] if $n\text{-}spcl(H) \subseteq G$ whenever $H \subseteq G$ and G is n-open.*

3 On lightly nano closed sets

Definition 3.1. *A subset R of a space $(U, \mathcal{N})$, is called*

1. *lightly nano closed (briefly $\mathcal{L}$-n-closed) set if $n\text{-}cl(R) \subseteq S$ whenever $R \subseteq S$ and S is nano semi-open.*

   The complement of a $\mathcal{L}$-n-closed set is said to be $\mathcal{L}$-n-open.

2. *lightly nano ω-closed (briefly $\mathcal{L}$-nω-closed) set if $n\text{-}cl(n\text{-}int(R)) \subseteq S$ whenever $R \subseteq S$ and S is nano semi-open.*

   The complement of a $\mathcal{L}$-nω-closed set is said to be $\mathcal{L}$-nω-open.

Recall, we denote the class of $\mathcal{L}$-n-closed sets in $(U, \mathcal{N})$ by $\mathcal{K}$ i.e., $\mathcal{K} = \{R \subseteq U : R$ is $\mathcal{L}$-n-closed in $(U, \mathcal{N})\}$.

Theorem 3.2. *In a space $(U, \mathcal{N})$, the following relations are true for a subset R of U.*

1. *R is n-closed $\Rightarrow$ R is $\mathcal{L}$-n-closed.*

2. *R is $\mathcal{L}$-n-closed $\Rightarrow$ R is ng-closed.*

3. *R is $\mathcal{L}$-n-closed $\Rightarrow$ R is nsg-closed.*

4. *R is $\mathcal{L}$-n-closed $\Rightarrow$ R is ngα-closed.*

Proof. 1. Let R be every n-closed set and S be every nano semi-open set such that $R \subseteq S$. Then $n\text{-}cl(R) \subseteq S$. Since $n\text{-}cl(R) = R$ and hence R is $\mathcal{L}$-n-closed.

2. Let $R \in \mathcal{K}$ and S be every n-open set such that $R \subseteq S$. Since every n-open set is nano semi-open and R is $\mathcal{L}$-n-closed set, we have $n\text{-}cl(R) \subseteq S$ and hence R is ng-closed.

3. Let $R \in \mathcal{K}$ and S be every nano semi-open set containing R. Then $n\text{-}scl(R) \subseteq n\text{-}cl(R) \subseteq S$, since R is $\mathcal{L}$-n-closed. Therefore R is nsg-closed.

4. Let $R \in \mathcal{K}$ and S be every $n\alpha$-open set containing R. Since every $n\alpha$-open set is nano semi-open and since $n\text{-}\alpha cl(R) \subseteq n\text{-}cl(R)$, we have by hypothesis, $n\text{-}\alpha cl(R) \subseteq n\text{-}cl(R) \subseteq S$ and so R is $ng\alpha$-closed.

Proposition 3.3. *In a space $(U, \mathcal{N})$, the following relations are true for a subset R of U.*

1. R is $\mathcal{L}$-n-closed $\Rightarrow$ R is $\mathcal{L}$-$n\omega$-closed.
2. R is n-closed $\Rightarrow$ R is $\mathcal{L}$-$n\omega$-closed.
3. R is ng-closed $\Rightarrow$ R is $\mathcal{L}$-$n\omega$-closed.

Proof. 1. Let $R \subseteq S$ where S is nano semi-open and R is $\mathcal{L}$-n-closed. $n\text{-}cl(n\text{-}\mathrm{int(R)}) \subseteq n\text{-}cl(R) \subseteq S$. This proves R is $\mathcal{L}$-$n\omega$-closed.

2. Let R is n-closed. Also S is nano semi-open. Therefore $R \subseteq n\text{-}cl(n\text{-}int(R)) \subseteq S$ which shows that R is $\mathcal{L}$-$n\omega$-closed.

3. Let R is ng-closed. Since S is nano semi-open. Therefore $n\text{-}cl(n\text{-}int(R)) \subseteq R \subseteq S$. Thus R is $\mathcal{L}$-$n\omega$-closed.

Proposition 3.4. *In a space $(U, \mathcal{N})$, the following relations are true for a subset R of U.*

1. *R is nr-closed $\Rightarrow$ R is $\mathcal{L}$-$n\omega$-closed.*
2. *R is $\mathcal{L}$-$n\omega$-closed $\Rightarrow$ R is $ngsp$-closed.*

Proof. 1. Let every nr-closed set can be as R and S be a nano semi-open set containing R. Since R is nr-closed we have $R = n\text{-}cl(n\text{-}int(R)) \subseteq S$ and hence R is $\mathcal{L}$-$n\omega$-closed.

2. Let every $\mathcal{L}$-$n\omega$-closed set can be as R and S be a n-open set containing R. Then S is a nano semi-open set containing R, so $n\text{-}cl(n\text{-}int(R)) \subseteq S$. Since S is n-open we get $n\text{-}int(n\text{-}cl(n\text{-}int(R))) \subseteq S$ which implies $n\text{-}spcl(R) \subseteq S$, hence R is $ngsp$-closed.

Remark 3.5. *These relations are shown in the diagram.*

$$\begin{tikzcd}
 & & & ng\alpha\textbf{-}\textit{\textbf{closed}} & \\
 & n\textbf{-}\textit{\textbf{closed}} \arrow[rr] \arrow[dr] & & \mathcal{L}\textbf{-}n\textbf{-}\textit{\textbf{closed}} \arrow[u] \arrow[r] \arrow[dl] \arrow[d] & nsg\textbf{-}\textit{\textbf{closed}} \\
nr\textbf{-}\textit{\textbf{closed}} \arrow[rr] & & \mathcal{L}\textbf{-}n\omega\textbf{-}\textit{\textbf{closed}} \arrow[d] & ng\textbf{-}\textit{\textbf{closed}} \arrow[l] & \\
 & & ngsp\textbf{-}\textit{\textbf{closed}} & &
\end{tikzcd}$$

The converses of each statement in Theorem 3.2, Propositions 3.3 and 3.4 are not true as shown in the following Example.

Example 3.6. *Let* $U = \{1_a, 1_b, 1_c\}$ *with* $U/R = \{\{1_a\}, \{1_b, 1_c\}\}$ *and* $X = \{1_c\}$. *Then* $\mathcal{N} = \{\phi, U, \{1_b, 1_c\}\}$. *Let* $A = \{1_a, 1_b\}$ *be* $\mathcal{L}$*-*n*-closed but not* n*-closed.*

Example 3.7. *Let* $U = \{1_a, 1_b, 1_c, 1_d\}$ *with* $U/R = \{\{1_a\}, \{1_b, 1_c, 1_d\}\}$ *and* $X = \{1_a\}$. *Then* $\mathcal{N} = \{\phi, U, \{1_a\}\}$. *Then*

1. ng-closed $\nrightarrow$ $\mathcal{L}$-n-closed.

   Let us consider $B = \{1_b\}$ is ng-closed. Then

   $$R = n\text{-}cl(B) = n\text{-}cl(\{1_b\}) = \{1_b, 1_c, 1_d\}$$

   Therefore $R \nsubseteq S$, since S is nano semi open. Hence B is not $\mathcal{L}$-n-closed.

2. ng-closed $\nrightarrow$ $\mathcal{L}$-$n\omega$-closed.

   Let us consider $C = \{1_a, 1_b, 1_c\}$ is ng-closed. Then

   $$R = n\text{-}cl(n\text{-}int(C)) = n\text{-}cl(n\text{-}int(\{1_a, 1_b, 1_c\})) = n\text{-}cl(\{1_a\}) = U$$

   Therefore $R \nsubseteq S$, since S is nano semi open. Hence C is not $\mathcal{L}$-$n\omega$-closed.

3. nsg-closed $\nrightarrow$ $\mathcal{L}$-n-closed.

   Let us consider $D = \{1_c\}$ is nsg-closed. Then

   $$R = n\text{-}cl(D) = n\text{-}cl(\{1_c\}) = \{1_b, 1_c, 1_d\}$$

   Therefore $R \nsubseteq S$, since S is nano semi open. Hence D not $\mathcal{L}$-n-closed.

4. $ng\alpha$-closed $\nrightarrow$ $\mathcal{L}$-n-closed.

   Let us consider $E = \{1_a, 1_c\}$ is $ng\alpha$-closed. Then

   $$R = n\text{-}cl(E) = n\text{-}cl(\{1_a, 1_c\}) = U$$

   Therefore $R \nsubseteq S$, since S is nano semi open. Hence E is not $\mathcal{L}$-n-closed.

5. $\mathcal{L}$-$n\omega$-closed $\nrightarrow$ $\mathcal{L}$-n-closed.

   Let us consider $F = \{1_c, 1_d\}$ is $\mathcal{L}$-$n\omega$-closed. Then

   $$R = n\text{-}cl(F) = n\text{-}cl(\{1_c, 1_d\}) = U$$

   Therefore $R \nsubseteq S$, since S is nano semi open. Hence F is not $\mathcal{L}$-n-closed.

6. $\mathcal{L}$-$n\omega$-closed $\nrightarrow$ n-closed.

   Let us consider $J = \{1_d\}$ is $\mathcal{L}$-$n\omega$-closed. Then $J \nsubseteq \mathcal{N}'$. Hence J is not n-closed.

7. $\mathcal{L}$-$n\omega$-closed $\nrightarrow$ nr-closed.

   Let us consider $K = \{1_b, 1_c, 1_d\}$ is $\mathcal{L}$-$n\omega$-closed. Then

$$R = n\text{-}cl(n\text{-}int(\{1_b, 1_c, 1_d\})) = \phi$$

   Therefore $K \neq R$. Hence K is not nr-closed.

8. $ngsp$-closed $\nrightarrow$ $\mathcal{L}$-$n\omega$-closed.

   Let us consider $I = \{1_a, 1_d\}$ is $ngsp$-closed. Then

$$R = n\text{-}cl(n\text{-}int(\{1_a\})) = U$$

   Therefore $R \nsubseteq S$, since S is nano semi open. Hence I is not $\mathcal{L}$-$n\omega$-closed.

Theorem 3.8. *In a space $(U, \mathcal{N})$ is both n-closed and $n\alpha g$-closed, then it is $\mathcal{L}$-$n\omega$-closed.*

Proof. Let $n\alpha g$-closed set can be as R and S be a n-open set containing R. Then $S \supseteq n\text{-}\alpha cl(R) = R \cup n\text{-}cl(n\text{-}int(n\text{-}cl(R)))$. Since R is n-closed, we have $S \supseteq n\text{-}cl(n\text{-}int(R))$ and hence R is $\mathcal{L}$-$n\omega$-closed.

Theorem 3.9. *In a space $(U, \mathcal{N})$ is both n-open and $\mathcal{L}$-$n\omega$-closed, then it is n-closed.*

Proof. Since R is both n-open and $\mathcal{L}$-$n\omega$-closed, $R \supseteq n\text{-}cl(n\text{-}int(R)) = n\text{-}cl(R)$ and hence R is n-closed.

Corollary 3.10. *In a space $(U, \mathcal{N})$ is both n-open and $\mathcal{L}$-$n\omega$-closed, then it is both nr-open and nr-closed.*

Theorem 3.11. *A set R is $\mathcal{L}$-$n\omega$-closed $\iff$ $n\text{-}cl(n\text{-}int(R)) - R$ contains no non-empty nano semi-closed set.*

Proof. Necessity. Let M be a nano semi-closed set such that $M \subseteq n\text{-}cl(n\text{-}int(R)) - R$. Since M^c is nano semi-open and $R \subseteq M^c$, from the definition of $\mathcal{L}$-$n\omega$-closed set it follows that $n\text{-}cl(n\text{-}int(R)) \subseteq M^c$. Hence $M \subseteq (n\text{-}cl(n\text{-}int(R)))^c$. This implies that $M \subseteq (n\text{-}cl(n\text{-}int(R))) \cap (n\text{-}cl(n\text{-}int(R)))^c = \phi$.

Sufficiency. Let $R \subseteq T$, where T is nano semi-open set subset in U. If $n\text{-}cl(n\text{-}int(R))$ is not contained in T, then $n\text{-}cl(n\text{-}int(R)) \cap T^c$ is a non-empty nano semi-closed subset of $n\text{-}cl(n\text{-}int(R)) - R$, we obtain a contradiction.

Theorem 3.12. *Let $(U, \mathcal{N})$ be a space and $K \subseteq R \subseteq U$. If K is $\mathcal{L}$-$n\omega$-closed set relative to R and R is and $\mathcal{L}$-$n\omega$-closed subset of U then K is $\mathcal{L}$-$n\omega$-closed set relative to U.*

Proof. Let $K \subseteq S$ and S be a nano semi-open. Then $K \subseteq R \cap S$. Since K is $\mathcal{L}$-$n\omega$-closed relative to R, we have $n\text{-}cl_R(n\text{-}int_R(K)) \subseteq R \cap S$. That is $R \cap n\text{-}cl(n\text{-}int(K)) \subseteq R \cap S$. We have $R \cap n\text{-}cl(n\text{-}int(K)) \subseteq S$ and then $[R \cap n\text{-}cl(n\text{-}int(K))] \cup (n\text{-}cl(n\text{-}int(K)))^c \subseteq S \cup (n\text{-}cl(n\text{-}int(K)))^c$. Since R is $\mathcal{L}$-$n\omega$-closed, we have $n\text{-}cl(n\text{-}int(R)) \subseteq S \cup (n\text{-}cl(n\text{-}int(K)))^c$. Since $n\text{-}cl(n\text{-}int(K))$ is not contained in $(n\text{-}cl(n\text{-}int(K)))^c$ we get $R \supseteq n\text{-}cl(n\text{-}int(K))$. Thus K is $\mathcal{L}$-$n\omega$-closed set relative to U.

Corollary 3.13. *If R is both n-open and $\mathcal{L}$-$n\omega$-closed and P is n-closed in a space $(U, \mathcal{N})$, then $R \cap P$ is $\mathcal{L}$-$n\omega$-closed.*

Proof. Since P is n-closed, we have $R \cap P$ is n-closed in R. Therefore $n\text{-}cl_R(R \cap P) = R \cap P$ in R. Let $R \cap P \subseteq S$, where S is nano semi-open in R. Then $n\text{-}cl_R(n\text{-}int_R(R \cap P)) \subseteq S$ and hence $R \cap P$ is $\mathcal{L}$-$n\omega$-closed in R. By Theorem 3.18, $R \cap P$ is $\mathcal{L}$-$n\omega$-closed.

Theorem 3.14. *If R is $\mathcal{L}$-$n\omega$-closed and $R \subseteq K \subseteq n\text{-}cl(n\text{-}int(R))$, then K is $\mathcal{L}$-$n\omega$-closed.*

Proof. Since $R \subseteq K$ we have $n\text{-}cl(n\text{-}int(K)) - K \subseteq n\text{-}cl(n\text{-}int(R)) - R$. By Theorem 3.11, $n\text{-}cl(n\text{-}int(R)) - R$ contains no non-empty nano semi-closed set and so $n\text{-}cl(n\text{-}int(K)) - K$ contains no non-empty nano semi-closed, so K is $\mathcal{L}$-$n\omega$-closed.

Theorem 3.15. *If a subset R of a space $(U, \mathcal{N})$ is every n-nowhere dense, then it is $\mathcal{L}$-$n\omega$-closed.*

Proof. Since $n\text{-}int(R) \subseteq n\text{-}int(n\text{-}cl(R))$ and R is n-nowhere dense, $n\text{-}int(R) = \phi$. Therefore $n\text{-}cl(n\text{-}int(R)) = \phi$ and hence R is $\mathcal{L}$-$n\omega$-closed.

Remark 3.16. *The converse of Theorem 3.15 are not true as shown in the following Example.*

Example 3.17. *In Example 3.6, then $J = \{1_a, 1_b\}$ is $\mathcal{L}$-$n\omega$-closed but not n-nowhere dense.*

Proposition 3.18. *In a space $(U, \mathcal{N})$, R is n-open $\Rightarrow$ R is $\mathcal{L}$-$n\omega$-open.*

Proof. Let every n-open set can be as R in a space U. Then R^c is n-closed in U. By Proposition 3.11(2) follows that R^c is $\mathcal{L}$-$n\omega$-closed in U. Hence R is $\mathcal{L}$-$n\omega$-open.

Remark 3.19. *The converse of Proposition 3.18 are not true as shown in the following Example.*

Example 3.20. *In Example 3.7, then $M = \{1_a, 1_b, 1_c\}$ is $\mathcal{L}$-$n\omega$-open but not n-open.*

Proposition 3.21. *A subset R of a space $(U, \mathcal{N})$, in the following results are true*

1. If R is $\mathcal{L}$-n-open then R is $\mathcal{L}$-$n\omega$-open.
2. If R is ng-open then R is $\mathcal{L}$-$n\omega$-open.
3. If R is $\mathcal{L}$-$n\omega$-open then R is $ngsp$-open.

Remark 3.22. *The converse of Proposition 3.21 are not true as shown in the following Example.*

Example 3.23. *In Example 3.6, then*

1. $\{1_a, 1_b\}$ is $\mathcal{L}$-$n\omega$-open but not $\mathcal{L}$-n-open.
2. $\{1_a, 1_c\}$ is $\mathcal{L}$-$n\omega$-open but not ng-open.

Example 3.24. *In Example 3.7, then* $\{1_c\}$ *is ngsp-open but not* $\mathcal{L}$*-n*ω*-open.*

Theorem 3.25. *A subset R be a space U is* $\mathcal{L}$*-n*ω*-open if* $Q \subseteq n\text{-}int(n\text{-}cl(R))$ *whenever* $Q \subseteq R$ *and Q is nano semi-closed.*

Proof. Let every $\mathcal{L}$-$n\omega$-open can be as R. Then R^c is $\mathcal{L}$-$n\omega$-closed. Let Q be a nano semi-closed set contained in R. Then Q^c is a nano semi-open set in U containing R^c. Since R^c is $\mathcal{L}$-$n\omega$-closed, we have $n\text{-}cl(n\text{-}int(R^c)) \subseteq Q^c$. Therefore $Q \subseteq n\text{-}int(n\text{-}cl(R))$.

Conversely, we suppose that $Q \subseteq n\text{-}int(n\text{-}cl(R))$ whenever $Q \subseteq R$ and Q is nano semi-closed. Then Q^c is a nano semi-open set containing R^c and $Q^c \supseteq (n\text{-}int(n\text{-}cl(R)))^c$. It follows that $Q^c \supseteq n\text{-}cl(n\text{-}int(R^c))$. Hence R^c is $\mathcal{L}$-$n\omega$-closed and so R is $\mathcal{L}$-$n\omega$-open.

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