Araştırma Makalesi / Research Article

Some Exact Solutions of Caudrey-Dodd-Gibbon (CDG) Equation and Dodd-Bullough-Mikhailov Equation

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Abstract
Nonlinear partial differential equations have an important place in applied mathematics and physics. Many analytical methods have been found in literature. Using these methods, partial differential equations are transformed into ordinary differential equations. These nonlinear partial differential equations are solved with the help of ordinary differential equations. In this paper, we implemented an improved tanh function Method for some exact solutions of Caudrey-Dodd-Gibbon (CDG) Equation and Dodd-Bullough-Mikhailov Equation.

Keywords: Caudrey-Dodd-Gibbon (CDG) Equation, Dodd-Bullough-Mikhailov Equation, improved tanh function method, exact solutions.

Caudrey-Dodd-Gibbon (CDG) Denklemi ve Dodd-Bullough-Mikhailov Denkleminin Bazı Kesin Çözümleri

Öz
Uygulamalı matematik ve fizikte doğrusal olmayan kısmi diferansiyel denklemler önemli bir yere sahiptir. Literatürde birçok analitik yöntem bulunmuştur. Bu yöntemleri kullanarak, kısmi diferansiyel denklemler, adi diferansiyel denklemlere dönüştürülür. Bu doğrusal olmayan kısmi diferansiyel denklemler, adi diferansiyel denklemlerin yardımıyla çözülmüştür. Bu çalışmada, Caudrey-Dodd-Gibbon (CDG) Denklemi ve Dodd-Bullough-Mikhailov Denkleminin kesin çözümleri için geliştirilmiş tanh fonksiyon metodu sunulmuştur.

Anahtar kelimeler: Caudrey-Dodd-Gibbon (CDG) Denklemi, Dodd-Bullough-Mikhailov Denklemi, geliştirilmiş tanh fonksiyon metodu, tam çözümler.

1. Introduction

Nonlinear partial differential equations (NPDEs) have an important place in applied mathematics and physics [1,2]. Many analytical methods have been found in literature [3-11]. Besides these methods, there are many methods which reach to solution by using an auxiliary equation. Using these methods, partial differential equations are transformed into ordinary differential equations. These nonlinear partial differential equations are solved with the help of ordinary differential equations. These methods are given in [12-25].

In this study, we implemented improved tanh function Method for finding the exact solutions of Caudrey-Dodd-Gibbon (CDG) Equation and Dodd-Bullough-Mikhailov Equation.

2. Analysis of Method

Let's introduce the method briefly. Consider a general partial differential equation of four variables,

$$\varphi(v, v_t, v_x \dots) = 0. \quad (1)$$

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Using the wave variable $(x,t) = v(\emptyset)$, $\emptyset = k(x - wt)$, here k and w are constants. The equation (3) turns into an ordinary differential equation,

$$\varphi'(v', v'', v''', \dots) = 0 \tag{2}$$

With this conversion, we obtain a nonlinear ordinary differential equation for $v(\emptyset)$. We can express the solution of equation (2) as below,

$$v(\emptyset) = \sum_{i=0}^{M} a_i F^i(\emptyset)\,, \tag{3}$$

here n is a positive integer and is found as the result of balancing the highest order linear term and the highest order nonlinear term found in the equation.

If we write these solutions in equation (2), we obtain a system of algebraic equations for $F(\emptyset), F^2(\emptyset), \dots, F^i(\emptyset)$, after, if the coefficients of $F(\emptyset), F^2(\emptyset), \dots, F^i(\emptyset)$ are equal to zero, we can find the constants $k, w, a_0, a_1, \dots, a_n$.

The basic step of the method is to make full use of the Riccati equation satisfying the tanh function and $F(\emptyset)$ solutions. The Riccati equation required in this method is given below

$$F'(\emptyset) = A + BF(\emptyset) + CF^2(\emptyset) \tag{4}$$

here, $F'(\emptyset) = \frac{dF(\emptyset)}{d\emptyset}$ and A, B and C are constants. The authors expressed the solutions of this equation [15].

Example 1.

We consider the Caudrey-Dodd-Gibbon (CDG) Equation,

$$v_t + v_{xxxxxx} + 30vv_{xxx} + 30v_x v_{xx} + 180v^2 v_x = 0. \tag{5}$$

Using the wave variable $v(x,t) = v(z),\ z = k(x - wt)$ Eq. (5) becomes

$$-wv' + k^4 v^{(5)} + 30k^2 vv''' + 30k^2 v'v'' + 180v^2 v' = 0, \tag{6}$$

when balancing $v'v''\ vv'''$with $v^{(5)}$ then $M = 2$ gives. The solution is as follows,

$$v = a_0 + a_1 F + a_2 F^2. \tag{7}$$

If the solution (7) is substituted in equation (6), a system of algebraic equations for k, w, a_0, a_1, a_2 are obtained. The obtained systems of algebraic equations are as follows

$$AB^4k^4a_1 + 22A^2B^2Ck^4a_1 + 16A^3C^2k^4a_1 - Awa_1 + 30AB^2k^2a_0a_1 + 60A^2Ck^2a_0a_1 + 180Aa_0^2a_1 + 30A^2Bk^2a_1^2 + 30A^2B^3k^4a_2 + 120A^3BCk^4a_2 + 180A^2Bk^2a_0a_2 + 60A^3k^2a_1a_2 = 0,$$

$$B^5k^4a_1 + 52AB^3Ck^4a_1 + 136A^2BC^2k^4a_1 - Bwa_1 + 30B^3k^2a_0a_1 + 240ABCk^2a_0a_1 + 180Ba_0^2a_1 + 90AB^2k^2a_1^2 + 120A^2Ck^2a_1^2 + 360Aa_0a_1^2 + 62AB^4k^4a_2 + 584A^2B^2Ck^4a_2 + 272A^3C^2k^4a_2 - 2Awa_2 + 420AB^2k^2a_0a_2 + 480A^2Ck^2a_0a_2 + 360Aa_0^2a_2 + 480A^2Bk^2a_1a_2 + 120A^3k^2a_2^2 = 0. \tag{8}$$

If this system is solved, the coefficients are found as

$$B = 0, a_1 = 0, a_0 = a_0,\ A \neq 0, C \neq 0, a_2 = \frac{3Ca_0}{2A}, k = \frac{i\sqrt{a_2}}{\sqrt{2C}}, k \neq 0, w = 9a_0^2\,, \tag{9}$$

with the help of the Mathematica program. After these operations, The solutions of equation (5) for (9) are as follows:

Solution 1.

$$v_1 = a_0 - \frac{3}{2}a_0\left(\mathrm{Coth}\left[-i\sqrt{-3a_0}\,x + 9ia_0^2\sqrt{-3a_0}\,t\right] \pm \mathrm{Cosech}\left[-i\sqrt{-3a_0}\,x + 9ia_0^2\sqrt{-3a_0}\,t\right]\right)^2$$
$$v_2 = a_0 - \frac{3}{2}a_0\left(\mathrm{Tanh}\left[-i\sqrt{-3a_0}\,x + 9ia_0^2\sqrt{-3a_0}\,t\right] \pm i\mathrm{Sech}\left[-i\sqrt{-3a_0}\,x + 9ia_0^2\sqrt{-3a_0}\,t\right]\right)^2. \quad (10)$$

Solution 2.

$$v_3 = a_0 + \frac{3}{2}a_0\left(\mathrm{Sec}\left[i\sqrt{3a_0}\,x - 9ia_0^2\sqrt{3a_0}\,t\right] \pm \mathrm{Tan}\left[i\sqrt{3a_0}\,x - 9ia_0^2\sqrt{3a_0}\,t\right]\right)^2$$
$$v_4 = a_0 + \frac{3}{2}a_0\left(\mathrm{Cosec}\left[i\sqrt{3a_0}\,x - 9ia_0^2\sqrt{3a_0}\,t\right] \pm \mathrm{Cot}\left[i\sqrt{3a_0}\,x - 9ia_0^2\sqrt{3a_0}\,t\right]\right)^2$$
$$v_5 = a_0 + \frac{3}{2}a_0\left(\mathrm{Cosec}\left[-i\sqrt{3a_0}\,x + 9ia_0^2\sqrt{3a_0}\,t\right] \pm \mathrm{Cot}\left[-i\sqrt{3a_0}\,x + 9ia_0^2\sqrt{3a_0}\,t\right]\right)^2$$
$$v_6 = a_0 + \frac{3}{2}a_0\left(\mathrm{Sec}\left[-i\sqrt{3a_0}\,x + 9ia_0^2\sqrt{3a_0}\,t\right] \pm \mathrm{Cot}\left[-i\sqrt{3a_0}\,x + 9ia_0^2\sqrt{3a_0}\,t\right]\right)^2. \quad (11)$$

Solution 3.

$$v_7 = a_0 - \frac{3}{2}a_0\left(\mathrm{Tanh}\left[\frac{-i\sqrt{-3a_0}}{2}x + \frac{9ia_0^2\sqrt{-3a_0}}{2}t\right]\right)^2$$
$$v_8 = a_0 - \frac{3}{2}a_0\left(\mathrm{Coth}\left[\frac{-i\sqrt{-3a_0}}{2}x + \frac{9ia_0^2\sqrt{-3a_0}}{2}t\right]\right)^2. \quad (12)$$

Solution 4.

$$v_9 = a_0 + \frac{3}{2}a_0\left(\mathrm{Tan}\left[\frac{i\sqrt{3a_0}}{2}x - \frac{9ia_0^2\sqrt{3a_0}}{2}t\right]\right)^2. \quad (13)$$

Solution 5.

$$v_{10} = a_0 + \frac{3}{2}a_0\left(\mathrm{Cot}\left[\frac{-i\sqrt{3a_0}}{2}x + \frac{9ia_0^2\sqrt{3a_0}}{2}t\right]\right)^2. \quad (14)$$

Example 2.

Consider Dodd-Bullough-Mikhailov Equation,

$$u_{tt} - u_{xx} + e^{u} + e^{-2u} = 0. \quad (15)$$

If we make transformation $u = lnv$. Using the wave variable $v(x,t) = v(z)$, $z = k(x - wt)$ then Eq. (15) becomes

$$(k^2w^2 - k^2)vv'' + (-k^2w^2 + k^2)(v')^2 + v^3 + 1 = 0, \quad (16)$$

when balancing $vv''$with v^3 then $M = 2$ gives. The solution is given by

$$u = a_0 + a_1F + a_2F^2. \quad (17)$$

Substituting (17), into Eq. (16), yields a set of algebraic equations for k, w, a_0, a_1, a_2 these systems are finding as

$1 + a_0^3 - ABk^2 a_0 a_1 + ABk^2 w^2 a_0 a_1 + A^2 k^2 a_1^2 - A^2 k^2 w^2 a_1^2 - 2A^2 k^2 a_0 a_2 + 2A^2 k^2 w^2 a_0 a_2 = 0,$
$-B^2 k^2 a_0 a_1 - 2ACk^2 a_0 a_1 + B^2 k^2 w^2 a_0 a_1 + 2ACk^2 w^2 a_0 a_1 + 3a_0^2 a_1 + ABk^2 a_1^2 - ABk^2 w^2 a_1^2 -$
$6ABk^2 a_0 a_2 + 6ABk^2 w^2 a_0 a_2 + 2A^2 k^2 a_1 a_2 - 2A^2 k^2 w^2 a_1 a_2 = 0,$ (18)

if this system is solved, the coefficients are found as

$$B = 0, a_1 = 0, a_0 = \frac{1}{2},\ A \neq 0, C \neq 0, a_2 = \frac{3Ca_0}{A}, k = k, w = \frac{\sqrt{2ACk^2 - 3a_0}}{\sqrt{2}\sqrt{A}\sqrt{C}k}, \quad (19)$$

with the help of the Mathematica program. After these operations, The solutions of equation (15) for (19) are as follows:

Solution 1.

$$u_1 = \mathrm{Ln}\left\{\frac{1}{2} - \frac{3}{2}\left(\mathrm{Coth}\left[\mathrm{kx} + \left(\mathrm{i}\sqrt{-\mathrm{k}^2 - 3}\right)\mathrm{t}\right] \pm \mathrm{Cosech}\left[\mathrm{kx} + \left(\mathrm{i}\sqrt{-\mathrm{k}^2 - 3}\right)\mathrm{t}\right]\right)^2\right\}$$
$$u_2 = \mathrm{Ln}\left\{\frac{1}{2} - \frac{3}{2}\left(\mathrm{Tanh}\left[\mathrm{kx} + \left(\mathrm{i}\sqrt{-\mathrm{k}^2 - 3}\right)\mathrm{t}\right] \pm \mathrm{iSech}\left[\mathrm{kx} + \left(\mathrm{i}\sqrt{-\mathrm{k}^2 - 3}\right)\mathrm{t}\right]\right)^2\right\}. \quad (20)$$

Solution 2.

$$u_3 = \mathrm{Ln}\left\{\frac{1}{2} + \frac{3}{2}\left(\mathrm{Sec}\left[\mathrm{kx} - \left(\sqrt{\mathrm{k}^2 - 3}\right)\mathrm{t}\right] \pm \mathrm{Tan}\left[\mathrm{kx} - \left(\sqrt{\mathrm{k}^2 - 3}\right)\mathrm{t}\right]\right)^2\right\}$$
$$u_4 = \mathrm{Ln}\left\{\frac{1}{2} + \frac{3}{2}\left(\mathrm{Cosec}\left[\mathrm{kx} - \left(\sqrt{\mathrm{k}^2 - 3}\right)\mathrm{t}\right] \pm \mathrm{Cot}\left[\mathrm{kx} - \left(\sqrt{\mathrm{k}^2 - 3}\right)\mathrm{t}\right]\right)^2\right\}$$
$$u_5 = \mathrm{Ln}\left\{\frac{1}{2} + \frac{3}{2}\left(\mathrm{Cosec}\left[\mathrm{kx} + \left(\sqrt{\mathrm{k}^2 - 3}\right)\mathrm{t}\right] \pm \mathrm{Cot}\left[\mathrm{kx} + \left(\sqrt{\mathrm{k}^2 - 3}\right)\mathrm{t}\right]\right)^2\right\}$$
$$u_6 = \mathrm{Ln}\left\{\frac{1}{2} + \frac{3}{2}\left(\mathrm{Sec}\left[\mathrm{kx} + \left(\sqrt{\mathrm{k}^2 - 3}\right)\mathrm{t}\right] \pm \mathrm{Tan}\left[\mathrm{kx} + \left(\sqrt{\mathrm{k}^2 - 3}\right)\mathrm{t}\right]\right)^2\right\}. \quad (21)$$

Solution 3.

$$u_7 = \mathrm{Ln}\left\{\frac{1}{2} - \frac{3}{2}\left(\mathrm{Tanh}\left[\mathrm{kx} + \left(\frac{\mathrm{i}\sqrt{-4\mathrm{k}^2 - 3}}{2}\right)\mathrm{t}\right]\right)^2\right\}$$
$$u_8 = \mathrm{Ln}\left\{\frac{1}{2} - \frac{3}{2}\left(\mathrm{Coth}\left[\mathrm{kx} + \left(\frac{\mathrm{i}\sqrt{-4\mathrm{k}^2 - 3}}{2}\right)\mathrm{t}\right]\right)^2\right\}. \quad (22)$$

Solution 4.

$$u_9 = \mathrm{Ln}\left\{\frac{1}{2} + \frac{3}{2}\left(\mathrm{Tan}\left[\mathrm{kx} - \left(\frac{\sqrt{4\mathrm{k}^2 - 3}}{2}\right)\mathrm{t}\right]\right)^2\right\}. \quad (23)$$

Solution5.

$$u_{10} = \mathrm{Ln}\left\{\frac{1}{2} + \frac{3}{2}\left(\mathrm{Cot}\left[\mathrm{kx} + \left(\frac{\sqrt{4\mathrm{k}^2 - 3}}{2}\right)\mathrm{t}\right]\right)^2\right\}. \quad (24)$$

3. Conclusion

We used the improved tanh function method to find the exact solutions of Caudrey-Dodd-Gibbon (CDG) Equation and Dodd-Bullough-Mikhailov Equation. This method has been successfully applied to solve some nonlinear wave equations and can be used to many other nonlinear equations or coupled ones.

4. Explanations and Graphical Presentments of the Found Solutions

The graphs of some of the solutions of Equation (15) are as follows

a) b)

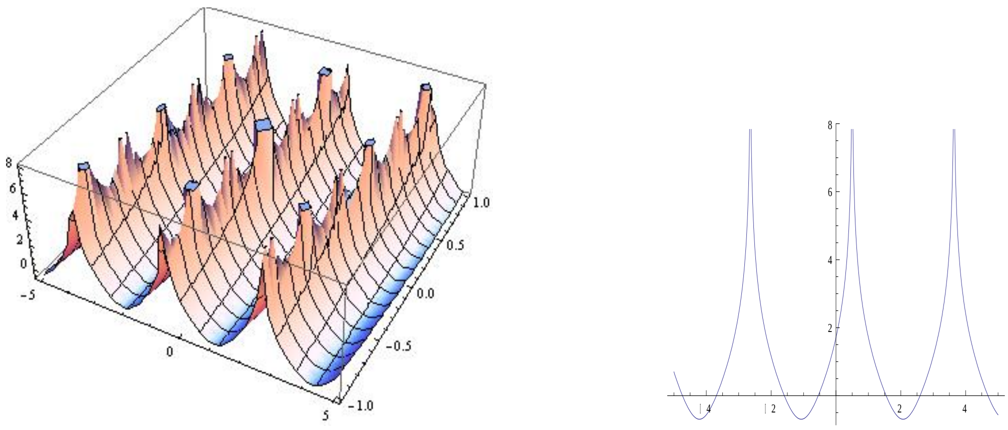

Figure 1. a) The 3D surfaces of Eq.(21-b)for the value k=2 within the interval $-5 \leq x \leq 5, -1 \leq t \leq 1$
b) The 2D surfaces of Eq.(21-b)for the values k=2,t=1 within the interval $-5 \leq x \leq 5$

a) b)

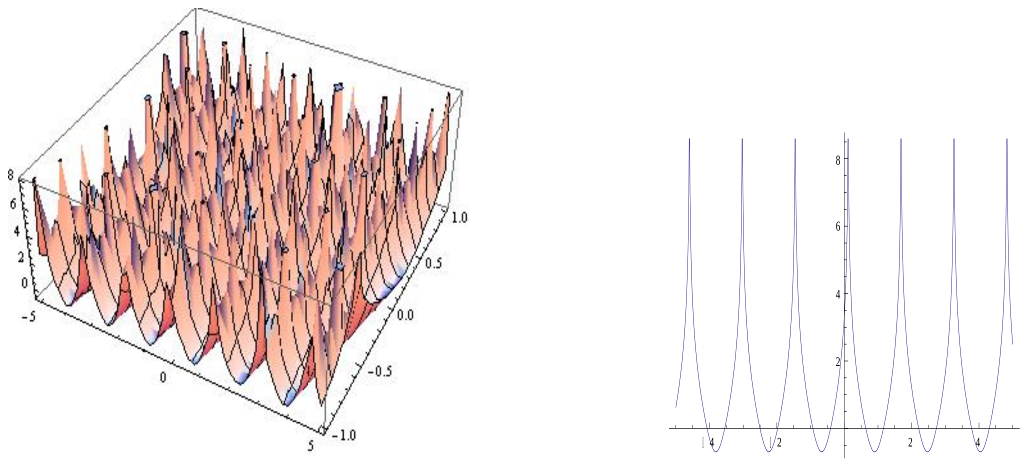

Figure 2. a) The 3D surfaces of Eq.(23)for the value k=2 within the interval $-5 \leq x \leq 5, -1 \leq t \leq 1$
b) The 2D surfaces of Eq.(23)for the values k=2,t=1 within the interval $-5 \leq x \leq 5$

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