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Research Article

Power Research Technique of A Thermomechanical Condition of A Rod of Restricted Length, Variable Section at Influence of Heterogeneous Types of Sources of Heat

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Abstract:

In this work on the basis of energy conservation laws with use square the spline of functions in the local system of coordinates is in number investigated thermo - the intense deformed condition of a horizontal rod of variable section and limited length. At the same time on the areas of transverse sections of two ends of a the rod heat fluxes of identical intensity are brought. Through lateral areas middle (1/3) parts of a the rod there is a heat exchange to a surrounding medium. Other parts of a lateral area are heat-insulated. Distribution laws of temperatures, all being deformations and tension, movements, and also sizes of thermal lengthening and the arising squeezing axial force are defined. Particular regularities are revealed.

1. Introduction

The bearing elements of the modern producer power stations, jet and hydrogen engines, nuclear and thermal power plants, and also technological lines of processing industry are rod stock of variable section. Most of them work at simultaneous influence of diverse types of sources of heat. In this regard reliable work of the above-stated installations and inventories will be depends on thermodurability of the used rod elements of variable section and limited length. Therefore a research thermo - an intense strained state of the rod of variable section and limited length taking into account simultaneous existence of diverse types of sources of heat, local thermal insulations and physicomechanical properties of materials of the rod stock are a relevant task. At the same time the received decisions have to be reliable at the level of fundamental laws of conservation of energy [1,2].

2. Formulatıon of the problem an methods

The horizontal rod of variable section and limited length of L [cm] is considered. The horizontal axis ox coincides with a rod axle. The transverse section of the considered rod is a circle.

The rod section radius on coordinate changes linearly under the   where and a and b constant. At this b–the radius of a transverse section of the left-hand end.

The cross-sectional area of the rod varies according to the formula

$$F(x) = \pi r^2 = \pi(a^2x^2 + 2abx + b^2), \left(0 \le x \le L\right)$$

Physicomechanical and thermal properties of material of a the rod it is characterized by a material elastic modulus, $E[\frac{kG}{cm^2}]$ thermal expansion coefficients, $\alpha[\frac{1}{{}^0K}]$ and heat conductivity $K_{xx}[\frac{Watt}{cm^0K}]$. On the area of transverse sections of two ends of the studied rod it is brought heat fluxes with identical intensity $q[\frac{Watt}{cm^2}]$.

Lateral areas the first $(0 \le x \le l), l = \frac{L}{3}[cm]$, and third $(2l \le x \le 3l = L)$ (1/3) parts of a the rod are heat-insulated.
Through a lateral area of average (1/3) parts $(l \le x \le 2l)$ happen heat exchange to a surrounding medium. At the same time heat exchange coefficient, and environment temperature of $h[\frac{Watt}{cm^{2\circ}K}]$. Here it should be noted that. The settlement scheme of a task is provided in the Figure 1. It is required: to construct approximating square a function spline in a local frame; to formulate a task at the level of the law of conservation of energy in the form of a functional of the total thermal energy for definition the field of temperatures; to define the integrated type of a functional of the total thermal energy; to construct the allowing system of the simple algebraic equations taking into account natural boundary conditions for definition the field of temperatures; In case one end of the studied rod rigid is jammed, and another is free it is necessary to determine the size of its lengthening taking into account simultaneous existence of local thermal insulations, heat exchange, heat fluxes, thermal properties of material of a the rod and its geometry; it is necessary to calculate values of the arising squeezing axial force taking into account geometry and physicomechanical and thermal properties of material of a rod and also existence of diverse types of sources of heat and local thermal insulations in case of jamming of two ends of the considered rod; to define the field of temperature, thermoelastic, temperature and elastic components of deformations and tension; to write a functional potential to energy of elastic deformation taking into account existence the field of temperature for definition the field of movement on the basis of the law of conservation of energy; to define the integrated type of a functional of a potential energy resilient deformations taking into account existence the field of temperature; o construct the allowing system of the simple algebraic equations taking into account natural boundary conditions; to build the field of movement taking into account is jammed of two ends of the studied rod of variable section and limited length.

3. Construction Spline of Functions and of a Functional of Energy

Let's consider one discrete site of a the rod length of l[cm]. The field of temperature longwise of a the rod, we approximate the complete polynom of the second order.

$$T(x) = c_1x^2 + c_2x + c_3 = \phi_i(x)\cdot T_i + \phi_j(x)\cdot T_j + \phi_k(x)\cdot T_k, \quad 0 \le x \le l \tag{1}$$

где $T_i = T(x=0)$; $\phi_j(x) = \frac{4lx - 4x^2}{l^2}$;

$$\phi_k(x) = \frac{2x^2 - lx}{l^2}; \tag{2}$$

These functions we will call square a spline functions in a local frame $(0 \le x \le l)$.With in rod length $(0 \le x \le l)$ the gradient of temperature is defined from (1-2)

$$\frac{\partial T}{\partial x} = \frac{\partial \phi_i}{\partial x}T_i + \frac{\partial \phi_j}{\partial x}T_j + \frac{\partial \phi_k}{\partial x}T_k = \frac{4x-3l}{l^2}T_i + \frac{4l-8x}{l^2}T_j + \frac{4x-l}{l^2}T_k, \quad 0 \le x \le l \tag{3}$$

Using the law of conservation of energy, for the considered task we will write a functional of the total thermal energy [3]

$$J = \int_{F(x=0)} qTds + \int_{V_1} \frac{K_{xx}}{2}\left(\frac{\partial T}{\partial x}\right)^2 dv + \int_{S(l \le x \le 2l)} \frac{h}{2}(T - T_{oc})^2 ds + \int_{V_2} \frac{K_{xx}}{2}\left(\frac{\partial T}{\partial x}\right)^2 dv +$$
$$+ \int_{V_3} \frac{K_{xx}}{2}\left(\frac{\partial T}{\partial x}\right)^2 dv + \int_{F(x=L)} qTds \tag{4}$$

where $F(x=0)$ and $F(x=L)$ - cross-sectional area of the two ends of the investigated rod; Wherein $F(x=0) >> F(x=L)$ - V1, V2 and V3 - volumes of three sections of the rod; For the 1st section $(0 \le x \le l)$ considered rod $T_i = T(x=0) = T_1$; $T_j = T(x = \frac{l}{2}) = T_2$; $T_k = T(x=l) = T_3$; For the 2nd section $(l \le x \le 2l)$ respectively $T_i = T(x=l) = T_3$; $T_j = T(x = \frac{3l}{2}) = T_4$; $T_k = T(x=2l) = T_5$; Finally, for the third section $(2l \le x \le 3l = L)$ will get $T_i = T(x=2l) = T_5$; $T_j = T(x = \frac{5l}{2}) = T_6$; $T_k = T(x=3l=L) = T_7$.
At this designated after an integration (4) has an appearance:

$$J = \pi \cdot b_1^2 \cdot q \cdot T_1 + \frac{\pi \cdot K_{xx}}{2}\Bigg[\left(\frac{a^2 l}{5}+ab_1+\frac{7b_1^2}{3l}\right)\cdot T_1^2 + \left(\frac{32a^2l}{15}+\frac{16ab_1}{3}+\frac{16b_1^2}{3l}\right)\cdot T_2^2 +$$

$$+\left(\frac{26a^2l}{15}+\frac{a(11b_1+3b_2)}{3}+\frac{7(b_1^2+b_2^2)}{3l}\right)\cdot T_3^2 - \left(\frac{4a^2l}{5}+\frac{8ab_1}{3}+\frac{16b_1^2}{3l}\right)\cdot T_1\cdot T_2 + \left(\frac{2a^2l}{5}+\frac{2ab_1}{3}+\frac{2b_1^2}{3l}\right)\cdot T_1 \cdot T_3 -$$

$$-\left(\frac{52a^2l}{15}+8ab_1+\frac{16b_1^2}{3l}\right)\cdot T_2\cdot T_3 + \left(\frac{32a^2l}{15}+\frac{16ab_2}{3}+\frac{16b_2^2}{3l}\right)\cdot T_4^2 + \left(\frac{26a^2l}{15}+\frac{a(11b_2+3b_3)}{3}+\frac{7(b_2^2+b_2^3)}{3l}\right)\cdot T_5^2 -$$

$$-\left(\frac{4a^2l}{5}+\frac{8ab_2}{3}+\frac{16b_2^2}{3l}\right)\cdot T_3\cdot T_4 + \left(\frac{2a^2l}{5}+\frac{2ab_2}{3}+\frac{2b_2^2}{3l}\right)\cdot T_3\cdot T_5 - \left(\frac{52a^2l}{15}+8ab_2+\frac{16b_2^2}{3l}\right)\cdot T_4\cdot T_5 +$$

$$+\left(\frac{32a^2l}{15}+\frac{16ab_3}{3}+\frac{16b_3^2}{3l}\right)\cdot T_6^2 + \left(\frac{23a^2l}{15}+\frac{11ab_3}{3}+\frac{7b_3^2}{3l}\right)\cdot T_7^2 - \left(\frac{4a^2l}{5}+\frac{8ab_3}{3}+\frac{16b_3^2}{3l}\right)\cdot T_5\cdot T_6 + \qquad (5)$$

$$+\left(\frac{2a^2l}{5}+\frac{2ab_3}{3}+\frac{2b_3^2}{3l}\right)\cdot T_5\cdot T_7 - \left(\frac{52a^2l}{15}+8ab_3+\frac{16b_3^2}{3l}\right)\cdot T_6\cdot T_7\Bigg] + h\cdot\pi\cdot\Bigg[\left(\frac{al^2}{60}+\frac{2b_2 l}{15}\right)\cdot T_3^2 +$$

$$+\left(\frac{4al^2}{15}+\frac{8b_2l}{15}\right)\cdot T_4^2 + \left(\frac{7al^2}{60}+\frac{2b_2l}{15}\right)\cdot T_5^2 + \frac{2b_2l}{15}\cdot T_3\cdot T_4 - \left(\frac{al^2}{30}+\frac{b_2l}{15}\right)\cdot T_3\cdot T_5 + \left(\frac{2al^2}{15}+\frac{2b_2l}{15}\right)\cdot T_4\cdot T_5 +$$

$$+\left(\frac{al^2}{2}+b_2l\right)\cdot T_0^2 - \frac{b_2l}{3}\cdot T_3\cdot T_0 - \left(\frac{2al^2}{3}+\frac{4b_2l}{3}\right)\cdot T_4\cdot T_0 - \left(\frac{al^2}{3}+\frac{b_2l}{3}\right)\cdot T_5\cdot T_0 + \pi\left(3al+b\right)^2\cdot q\cdot T_7;$$

Minimizing the last functional on nodal value of temperatures T1,T2,…,T7, we will receive the allowing system of the simple algebraic equations, natural boundary conditions are considered.

$$1)\frac{\partial J}{\partial T_1}=0;\Rightarrow \pi\cdot b_1^2\cdot q+\frac{\pi K_{xx}}{2}\Bigg[2\cdot\left(\frac{a^2l}{5}+ab_1+\frac{7b_1^2}{3l}\right)\cdot T_1-\left(\frac{4a^2l}{5}+\frac{8ab_1}{3}+\frac{16b_1^2}{3l}\right)\cdot T_2+$$

$$+\left(\frac{2a^2l}{5}+\frac{2ab_1}{3}+\frac{2b_1^2}{3l}\right)\cdot T_3\Bigg]=0;$$

$$2)\frac{\partial J}{\partial T_2}=0;\Rightarrow \frac{\pi K_{xx}}{2}\Bigg[2\cdot\left(\frac{32a^2l}{15}+\frac{16ab_1}{3}+\frac{16b_1^2}{3l}\right)\cdot T_2-\left(\frac{4a^2l}{5}+\frac{8ab_1}{3}+\frac{16b_1^2}{3l}\right)\cdot T_1-$$

$$-\left(\frac{52a^2l}{15}+8ab_1+\frac{16b_1^2}{3l}\right)\cdot T_3\Bigg]=0; \qquad (6)$$

$$3)\frac{\partial J}{\partial T_3}=0;\Rightarrow \frac{\pi K_{xx}}{2}\left[\left(\frac{2a^2l}{5}+\frac{2ab_1}{3}+\frac{2b_1^2}{3l}\right)\cdot T_1-\left(\frac{52a^2l}{15}+8ab_1+\frac{16b_1^2}{3l}\right)\cdot T_2+\right.$$
$$+2\cdot\left(\frac{26a^2l}{15}+\frac{a(11b_1+3b_2)}{3}+\frac{7(b_1^2+b_2^2)}{3l}\right)\cdot T_3-\left(\frac{4a^2l}{5}+\frac{8ab_2}{3}+\frac{16b_2^2}{3l}\right)\cdot T_4+$$
$$\left.+\left(\frac{2a^2l}{5}+\frac{2ab_2}{3}+\frac{2b_2^2}{3l}\right)\cdot T_5\right]+h\pi\left[2\cdot\left(\frac{al^2}{60}+\frac{2b_2l}{15}\right)\cdot T_3+\frac{2b_2l}{15}\cdot T_4-\left(\frac{al^2}{30}+\frac{b_2l}{15}\right)\cdot T_5-\right.$$
$$\left.-\frac{b_2l}{3}\cdot T_0\right]_5=0;$$

$$4)\frac{\partial J}{\partial T_4}=0;\Rightarrow h\pi\left[2\cdot\left(\frac{4al^2}{15}+\frac{8b_2l}{15}\right)\cdot T_4+\frac{2b_2l}{15}\cdot T_3+\left(\frac{2al^2}{15}+\frac{2b_2l}{15}\right)\cdot T_5-\right.$$
$$\left.-\left(\frac{2al^2}{3}+\frac{4b_2l}{3}\right)\cdot T_0\right]+\frac{\pi K_{xx}}{2}\left[2\cdot\left(\frac{32a^2l}{15}+\frac{16ab_2}{3}+\frac{16b_2^2}{3l}\right)\cdot T_4-\left(\frac{4a^2l}{5}+\frac{8ab_2}{3}+\frac{16b_2^2}{3l}\right)\cdot\right.$$
$$\left.\cdot T_3-\left(\frac{52a^2l}{15}+8ab_2+\frac{16b_2^2}{3l}\right)\cdot T_5\right]=0;$$

$$5)\frac{\partial J}{\partial T_5}=0;\Rightarrow h\pi\left[2\cdot\left(\frac{7al^2}{60}+\frac{2b_2l}{15}\right)\cdot T_5-\left(\frac{al^2}{30}+\frac{b_2l}{15}\right)\cdot T_3+\left(\frac{2al^2}{15}+\frac{2b_2l}{15}\right)\cdot T_4-\right.$$
$$\left.-\left(\frac{al^2}{3}+\frac{b_2l}{3}\right)\cdot T_0\right]+\frac{\pi K_{xx}}{2}\left[\left(\frac{2a^2l}{5}+\frac{2ab_2}{3}+\frac{2b_2^2}{3l}\right)\cdot T_3-\left(\frac{52a^2l}{15}+8ab_2+\frac{16b_2^2}{3l}\right)\cdot T_4+\right.$$
$$+2\cdot\left(\frac{26a^2l}{15}+\frac{a(11b_2+3b_3)}{3}+\frac{7(b_2^2+b_3^2)}{3l}\right)\cdot T_5-\left(\frac{4a^2l}{5}+\frac{8ab_3}{3}+\frac{16b_3^2}{3l}\right)\cdot T_6+$$
$$\left.+\left(\frac{2a^2l}{5}+\frac{2ab_3}{3}+\frac{2b_3^2}{3l}\right)\cdot T_7\right]=0;$$

$$6)\frac{\partial J}{\partial T_6}=0;\Rightarrow \frac{\pi K_{xx}}{2}\left[2\cdot\left(\frac{32a^2l}{15}+\frac{16ab_3}{3}+\frac{16b_3^2}{3l}\right)\cdot T_6-\left(\frac{4a^2l}{5}+\frac{8ab_3}{3}+\frac{16b_3^2}{3l}\right)\cdot T_5+\right.$$
$$\left.-\left(\frac{52a^2l}{15}+8ab_3+\frac{16b_3^2}{3l}\right)\cdot T_7\right]=0;$$

$$7)\frac{\partial J}{\partial T_7}=0;\Rightarrow \frac{\pi K_{xx}}{2}\left[2\cdot\left(\frac{23a^2l}{15}+\frac{11ab_3}{3}+\frac{7b_3^2}{3l}\right)\cdot T_7+\left(\frac{2a^2l}{5}+\frac{2ab_3}{3}+\frac{2b_3^2}{3l}\right)\cdot T_5-\right.$$
$$\left.-\left(\frac{52a^2l}{15}+8ab_3+\frac{16b_3^2}{3l}\right)\cdot T_6\right]+\pi(3al+b)^2\cdot q=0.$$

Solving this system we define nodal values of temperatures T1,T2,…,T7. On them the distribution law of temperature longwise of each site of the studied rod is under construction. For the first site $(0 \le x \le l)$ of a the rod it has the following appearance:

$$T^{(I)}(x) = \phi_i(x) \cdot T_1 + \phi_j(x) \cdot T_2 + \phi_k(x) \cdot T_3 \qquad (7)$$

Similar to the field of temperatures longwise of the 2nd and 3rd sites of a the rod has an appearance:

$$T^{(II)}(x) = \phi_i(x) \cdot T_3 + \phi_j(x) \cdot T_4 + \phi_k(x) \cdot T_5 \qquad (8)$$

$$T^{(III)}(x) = \phi_i(x) \cdot T_5 + \phi_j(x) \cdot T_6 + \phi_k(x) \cdot T_7 \qquad (9)$$

If one end of a the rod it is rigidly jammed, and another is free, then because of existence of diverse types of sources of heat it is extended. The size of lengthening is defined on the basis of fundamental laws of an applied thermal physics.

$$\Delta l_T = \int_0^L \alpha \cdot T(x) dx. \qquad (10)$$

In case of jamming of two ends of a the rod, the rod can't is extended, and owing to existence of diverse types of sources of heat there is axial R[kG] squeezing efforts. The size of this effort is defined from a consistency relation of deformation [4].

$$R = -\frac{E \cdot F_{av} \cdot \Delta l_T}{L}. \qquad (11)$$

where $F_{av} = \frac{\int_0^L F(x)dx}{L}$ – average area of a transverse section.

In case of jamming of two ends of a the rod there is also a field of distribution of the being deformations and tension. The distribution law of a thermoelastic component of tension is defined according to a Hooke law [4].

$$\sigma(x) = \frac{R}{F(x)}, \quad 0 \le x \le l.$$

(12)

In compliance of a Hooke law it is also possible to construct the distribution law of a thermoelastic component of deformations Ɛ (x) [dimensionless]:

$$\varepsilon(x) = \frac{\sigma(x)}{E}, \quad 0 \le x \le l \qquad (13)$$

In compliance the law of a thermal physics the field of distribution of a temperature component of deformations is under construction:

$\varepsilon_T(x) = -\alpha\, T(x), \; 0 \le x \le l$ (14)

Then according to the generalized Hooke law the distribution law of a temperature component of tension is defined

$$\sigma_T(x) = E \cdot \varepsilon_T(x), \quad 0 \le x \le l \qquad (15)$$

At last in compliance of the theory of a thermoelasticity it is possible to construct distribution laws of elastic components of deformations and tension

$$\varepsilon_x(x) = \varepsilon(x) - \varepsilon_T(x),$$
$$\sigma_x(x) = E \cdot \varepsilon_x(x) = \sigma(x) - \sigma_T(x), \quad 0 \le x \le l$$

(16)

For definition the field of movement we will write a functional potential to energy of elastic deformation in the presence the field of temperatures for one discrete element:

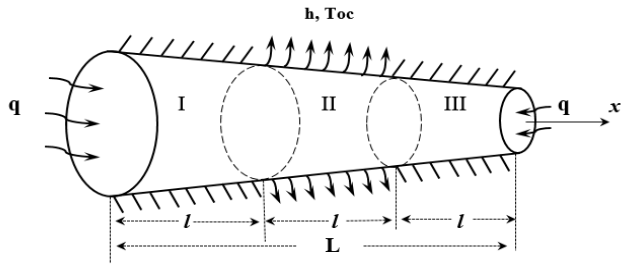

Figure 1. *The settlement scheme of the considered task*

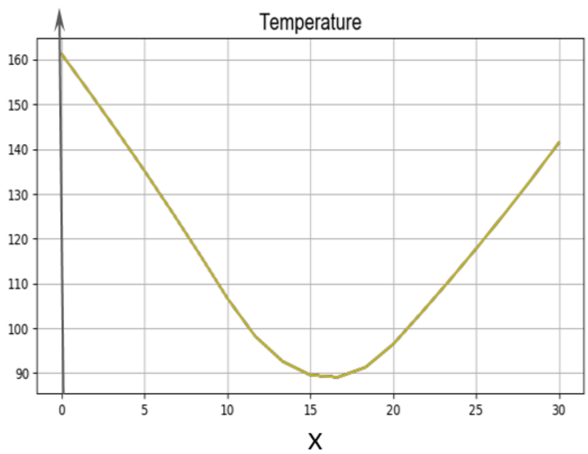

Figure 2. *Dependences of temperature T the length of the rod*

$$\Pi = \int_V \frac{\sigma_x(x)}{2}\varepsilon_x(x)dv - \int_V \alpha E\cdot T(x)\cdot\varepsilon_x(x)dv = \frac{E}{2}\int_v \varepsilon_x^2 dv - \alpha E\int_V T(x)\cdot\varepsilon_x(x)dv =$$

$$= \frac{E\pi}{2}\int_0^l F(x)\left(\frac{\partial U}{\partial x}\right)^2 dx - \alpha E\pi\int_0^l F(x)\cdot T(x)\frac{\partial U}{\partial x}dx = \frac{E\pi}{2}\int_0^l \left(a^2x^2+2abx+b^2\right)\cdot$$

$$\cdot\left[\frac{4x-3l}{l^2}U_i + \frac{4l-8x}{l^2}U_j + \frac{4x-l}{l^2}U_k\right]^2 dx - \alpha E\pi\int_0^l \left(a^2x^2+2abx+b^2\right)\cdot$$

$$\cdot\left[\frac{2x^2-3lx+l^2}{l^2}\cdot U_i\ \frac{4lx-4x^2}{l^2}\cdot U_j + \frac{2x^2-lx}{l^2}\cdot U_k\right]\cdot\left[\frac{4x-3l}{l^2}U_i + \frac{4l-8x}{l^2}U_j + \frac{4x-l}{l^2}U_k\right]^2 dx \quad (17)$$

where $U(x) = \phi_i(x)\cdot U_i + \phi_j(x)\cdot U_j + \phi_k(x)\cdot U_k$, $0 \le x \le l$.

After an integration it has an appearance:

$$\Pi = \frac{E\pi}{2}\Bigg\{\Bigg[\left(\frac{a^2l}{5} + ab_1 + \frac{7b_1^2}{3l}\right)\cdot U_1^2 - \left(\frac{4a^2l}{5} + \frac{8ab_1}{3} + \frac{16b_1^2}{3l}\right)\cdot U_1U_2 + \left(\frac{2a^2l}{5} + \frac{2ab_1}{3} + \frac{2b_1^2}{3l}\right)\cdot U_1U_3 +$$

$$+\left(\frac{32a^2l}{15} + \frac{16ab_1}{3} + \frac{16b_1^2}{3l}\right)\cdot U_2^2 - \left(\frac{52a^2l}{15} + 8ab_1 + \frac{16b_1^2}{3l}\right)\cdot U_2U_3 + \left(\frac{26a^2l}{15} + \frac{a(11b_1+3b_2)}{3} +\right.$$

$$\left.+\frac{7(b_1^2+b_2^2)}{3l}\right)\cdot U_3^2\Bigg] + \Bigg[-\left(\frac{4a^2l}{5} + \frac{8ab_2}{3} + \frac{16b_2^2}{3l}\right)\cdot U_3U_4 + \left(\frac{2a^2l}{5} + \frac{2ab_2}{3} + \frac{2b_2^2}{3l}\right)\cdot U_3U_5 +$$

$$+\left(\frac{32a^2l}{15} + \frac{16ab_2}{3} + \frac{16b_2^2}{3l}\right)\cdot U_4^2 - \left(\frac{52a^2l}{15} + 8ab_2 + \frac{16b_2^2}{3l}\right)\cdot U_4U_5 + \left(\frac{26a^2l}{15} + \frac{a(11b_2+3b_3)}{3} +\right.$$

$$\left.+\frac{7(b_2^2+b_3^2)}{3l}\right)\cdot U_5^2\Bigg] + \Bigg[-\left(\frac{4a^2l}{5} + \frac{8ab_3}{3} + \frac{16b_3^2}{3l}\right)\cdot U_5U_6 + \left(\frac{2a^2l}{5} + \frac{2ab_3}{3} + \frac{2b_3^2}{3l}\right)\cdot U_5U_7 +$$

$$+\left(\frac{32a^2l}{15} + \frac{16ab_3}{3} + \frac{16b_3^2}{3l}\right)\cdot U_6^2 - \left(\frac{52a^2l}{15} + 8ab_3 + \frac{16b_3^2}{3l}\right)\cdot U_6U_7 + \left(\frac{23a^2l}{15} + \frac{11ab_3}{3} + \frac{7b_3^2}{3l}\right)\cdot U_7^2\Bigg]$$

$$-\alpha E\pi\Bigg[-\left(\frac{a^2l^2}{60} + \frac{2ab_1l}{15} + \frac{b_1^2}{2}\right)\cdot T_1U_1 + \left(\frac{a^2l^2}{15} + \frac{4ab_1l}{15} + \frac{2b_1^2}{3}\right)\cdot T_1U_2 - \left(\frac{a^2l^2}{20} + \frac{2ab_1l}{15} + \frac{b_1^2}{6}\right)\cdot T_1U_3 -$$

$$-\left(\frac{a^2l^2}{15} + \frac{2ab_1l}{5} + \frac{2b_1^2}{3}\right)\cdot T_2U_1 - \left(\frac{4a^2l^2}{15} + \frac{8ab_1l}{15}\right)\cdot T_2U_2 + \left(\frac{a^2l^2}{3} + \frac{14ab_1l}{15} + \frac{2b_1^2}{3}\right)\cdot T_2U_3 +$$

$$+\left(\frac{a^2l^2}{12} + \frac{ab_1l}{5} + \frac{b_1^2}{2}\right)\cdot T_3U_1 - \left(\frac{7a^2l^2}{15} + \frac{16ab_1l}{15} + \frac{2b_1^2}{3}\right)\cdot T_3U_2 + \left(\frac{22a^2l^2}{60} + \frac{11al(b_1-b_2)}{15} +\right. \quad (18)$$

$$+\frac{b_1^2-2b_2^2}{2}\Bigg)T_3U_3\Bigg]+\Bigg[\left(\frac{a^2l^2}{15}+\frac{4ab_2l}{15}+\frac{2b_2^2}{3}\right)\cdot T_3U_4-\left(\frac{a^2l^2}{20}+\frac{2ab_2l}{15}+\frac{b_2^2}{6}\right)\cdot T_3U_5-\Bigg(\frac{a^2l^2}{15}+\frac{2ab_2l}{5}+$$

$$+\frac{2b_2^2}{3}\Bigg)\cdot T_4U_3-\left(\frac{4a^2l^2}{15}+\frac{8ab_2l}{15}\right)\cdot T_4U_4+\left(\frac{a^2l^2}{3}+\frac{4ab_2l}{15}+\frac{2b_2^2}{3}\right)\cdot T_4U_5+\left(\frac{a^2l^2}{12}+\frac{ab_2l}{5}+\frac{b_2^2}{6}\right)\cdot T_5U_3-$$

$$-\left(\frac{7a^2l^2}{15}+\frac{16ab_2l}{15}+\frac{2b_2^2}{3}\right)\cdot T_5U_4+\left(\frac{22a^2l^2}{60}+\frac{al(13b_2-2b_3)}{15}+\frac{b_2^2-b_3^2}{2}\right)\cdot T_5U_5\Bigg]+\Bigg[\Bigg(\frac{a^2l^2}{15}+\frac{4ab_3l}{15}+$$

$$+\frac{2b_3^2}{2}\Bigg)\cdot T_5U_6-\left(\frac{a^2l^2}{20}+\frac{2ab_3l}{15}+\frac{b_3^2}{6}\right)\cdot T_5U_7-\left(\frac{a^2l^2}{15}+\frac{2ab_3l}{5}+\frac{2b_3^2}{3}\right)\cdot T_6U_5-\left(\frac{4a^2l^2}{15}+\frac{8ab_3l}{15}\right)\cdot T_6U_6+$$

$$+\left(\frac{a^2l^2}{3}+\frac{14ab_3l}{15}+\frac{2b_3^2}{3}\right)\cdot T_6U_7+\left(\frac{a^2l^2}{12}+\frac{ab_3l}{5}+\frac{b_3^2}{6}\right)\cdot T_7U_5-\left(\frac{7a^2l^2}{15}+\frac{16ab_3l}{15}+\frac{2b_3^2}{3}\right)\cdot T_7U_6+$$

$$+\left(\frac{23a^2l^2}{60}+\frac{13ab_3l}{15}+\frac{b_3^2}{2}\right)\cdot T_7U_7\Bigg]\Bigg\}.$$

For determination of value of movement of sections we minimize a functional of a potential energy on nodal points:

$$1)\ \frac{\partial \Pi}{\partial U_1}=0;\Rightarrow\frac{E\pi}{2}\Bigg[2\cdot\left(\frac{a^2l}{5}+ab_1+\frac{7b_1^2}{3l}\right)\cdot U_1-\left(\frac{4a^2l}{5}+\frac{8ab_1}{3}+\frac{16b_1^2}{3l}\right)\cdot U_2+\left(\frac{2a^2l}{5}+\frac{2ab_1}{3}+\frac{2b_1^2}{3l}\right)\cdot$$

$$\cdot U_3\Bigg]-\alpha E\pi\left[-\left(\frac{a^2l^2}{60}+\frac{2ab_1l}{15}+\frac{b_1^2}{2}\right)\cdot T_1-\left(\frac{a^2l^2}{15}+\frac{2ab_1l}{5}+\frac{2b_1^2}{3}\right)\cdot T_2+\left(\frac{a^2l^2}{12}+\frac{ab_1l}{5}+\frac{b_1^2}{6}\right)\cdot T_3\right]=0;$$

$$2)\ \frac{\partial \Pi}{\partial U_2}=0;\Rightarrow\frac{E\pi}{2}\Bigg[-\left(\frac{4a^2l}{5}+\frac{8ab_1}{3}+\frac{16b_1^2}{3l}\right)\cdot U_1+2\left(\frac{32a^2l}{15}+\frac{16ab_1}{3}+\frac{16b_1^2}{3l}\right)\cdot U_2-\Bigg(\frac{52a^2l}{5}+8ab_1+$$

$$+\frac{16b_1^2}{3l}\Bigg)\cdot U_3\Bigg]-\alpha E\pi\left[\left(\frac{a^2l^2}{15}+\frac{4ab_1l}{15}+\frac{2b_1^2}{3}\right)\cdot T_1-\left(\frac{4a^2l^2}{15}+\frac{8ab_1l}{15}\right)\cdot T_2-\left(\frac{7a^2l^2}{15}+\frac{16ab_1l}{15}+\frac{2b_1^2}{3}\right)\cdot T_3\right]=0;$$

$$3)\ \frac{\partial \Pi}{\partial U_3}=0;\Rightarrow\frac{E\pi}{2}\Bigg[\left(\frac{2a^2l}{5}+\frac{2ab_1}{3}+\frac{2b_1^2}{3l}\right)\cdot U_1-\left(\frac{52a^2l}{15}+8ab_1+\frac{16b_1^2}{3l}\right)\cdot U_2+2\Bigg(\frac{26a^2l}{15}+\frac{a(11b_1+3b_2)}{3}+$$

$$+\frac{7(b_1^2+b_2^2)}{3l}\Bigg)\cdot U_3\Bigg]-\alpha E\pi\Bigg[-\left(\frac{a^2l^2}{20}+\frac{2ab_1l}{15}+\frac{b_1^2}{6}\right)\cdot T_1+\left(\frac{a^2l^2}{3}+\frac{14ab_1l}{15}+\frac{2b_1^2}{3}\right)\cdot T_2+\Bigg(\frac{22a^2l^2}{60}+$$

$$+\frac{al(13b_1-2b_2)}{15}+\frac{b_1^2-b_2^2}{2}\Bigg)\cdot T_3-\left(\frac{a^2l^2}{15}+\frac{2ab_2l}{5}+\frac{2b_2^2}{3}\right)\cdot T_4+\left(\frac{a^2l^2}{12}+\frac{ab_2l}{5}+\frac{b_2^2}{6}\right)\cdot T_5\Bigg]=0;$$

$$4)\frac{\partial \Pi}{\partial U_4}=0;\Rightarrow \frac{E\pi}{2}\left[-\left(\frac{4a^2l}{5}+\frac{8ab_2}{3}+\frac{16b_2^2}{3l}\right)\cdot U_3+2\left(\frac{32a^2l}{15}+\frac{16ab_2}{3}+\frac{16b_2^2}{3l}\right)\cdot U_4-2\left(\frac{52a^2l}{15}+8ab_2+\right.\right.$$

$$\left.\left.+\frac{16b_2^2)}{3l}\right)\cdot U_5\right]-\alpha E\pi\left[\left(\frac{a^2l^2}{15}+\frac{4ab_2l}{15}+\frac{2b_2^2}{3}\right)\cdot T_3-\left(\frac{4a^2l^2}{15}+\frac{8ab_2l}{15}\right)\cdot T_4-\left(\frac{7a^2l^2}{15}+\frac{16ab_2l}{15}+\frac{2b_2^2}{3}\right)\cdot T_5\right]=0;$$

$$5)\frac{\partial \Pi}{\partial U_3}=0;\Rightarrow \frac{E\pi}{2}\left[\left(\frac{2a^2l}{5}+\frac{2ab_2}{3}+\frac{2b_2^2}{3l}\right)\cdot U_3-\left(\frac{52a^2l}{15}+8ab_2+\frac{16b_2^2}{3l}\right)\cdot U_4+2\left(\frac{26a^2l}{15}+\frac{a(11b_2+3b_3)}{3}+\right.\right.$$

$$\left.+\frac{7(b_2^2+b_3^2)}{3l}\right)\cdot U_5-\left(\frac{4a^2l}{5}+\frac{8ab_3}{3}+\frac{16b_3^2}{3l}\right)\cdot U_6+\left(\frac{2a^2l}{5}+\frac{2ab_3}{3}+\frac{2b_3^2}{3l}\right)\cdot U_7\Bigg]-\alpha E\pi\left[-\left(\frac{a^2l^2}{20}+\frac{2ab_2l}{15}+\frac{b_2^2}{6}\right)\cdot T_3+\right.$$

$$+\left(\frac{a^2l^2}{3}+\frac{14ab_2l}{15}+\frac{2b_2^2}{3}\right)\cdot T_4+\left(\frac{22a^2l^2}{60}+\frac{al(13b_2-2b_3)}{15}+\frac{b_2^2-b_3^2}{2}\right)\cdot T_5-\left(\frac{a^2l^2}{15}+\frac{2ab_3l}{5}+\frac{2b_3^2}{3}\right)\cdot T_6+\left(\frac{a^2l^2}{12}+\right.$$

$$\left.\left.+\frac{ab_3l}{5}+\frac{b_3^2}{6}\right)\cdot T_7\right]=0; \tag{19}$$

$$6)\frac{\partial \Pi}{\partial U_6}=0;\Rightarrow \frac{E\pi}{2}\left[-\left(\frac{4a^2l}{5}+\frac{8ab_3}{3}+\frac{16b_3^2}{3l}\right)\cdot U_5+2\left(\frac{32a^2l}{15}+\frac{16ab_3}{3}+\frac{16b_3^2}{3l}\right)\cdot U_6-\left(\frac{52a^2l}{15}+8ab_3+\right.\right.$$

$$\left.\left.+\frac{16b_3^2)}{3l}\right)\cdot U_7\right]-\alpha E\pi\left[\left(\frac{a^2l^2}{15}+\frac{4ab_3l}{15}+\frac{2b_3^2}{3}\right)\cdot T_5-\left(\frac{4a^2l^2}{15}+\frac{8ab_3l}{15}\right)\cdot T_6-\left(\frac{7a^2l^2}{15}+\frac{16ab_3l}{15}+\frac{2b_3^2}{3}\right)\cdot T_7\right]=0;$$

$$7)\frac{\partial \Pi}{\partial U_7}=0;\Rightarrow \frac{E\pi}{2}\left[\left(\frac{2a^2l}{5}+\frac{2ab_3}{3}+\frac{2b_3^2}{3l}\right)\cdot U_5-\left(\frac{52a^2l}{15}+8ab_3+\frac{16b_3^2}{3l}\right)\cdot U_6+2\left(\frac{23a^2l}{15}+\frac{11ab_3}{3}+\frac{7b_3^2}{3l}\right)\cdot U_7\right]-$$

$$-\alpha E\pi\left[-\left(\frac{a^2l^2}{20}+\frac{2ab_3l}{15}+\frac{b_3^2}{6}\right)\cdot T_5+\left(\frac{a^2l^2}{3}+\frac{14ab_3l}{15}+\frac{2b_3^2}{3}\right)\cdot T_6+\left(\frac{23a^2l^2}{60}+\frac{13ab_3l}{15}+\frac{b_3^2}{2}\right)\cdot T_7\right]=0;$$

Solving this system we define values U1, U1,…,U7. Then the field of relocation longwise of each site of the studied rod are defined as follows:

$$\left.\begin{aligned} U^{(I)}(x)&=\phi_i(x)\cdot U_1+\phi_j(x)\cdot U_2+\phi_k(x)\cdot U_3\\ U^{(II)}(x)&=\phi_i(x)\cdot U_3+\phi_j(x)\cdot U_4+\phi_k(x)\cdot U_5\\ U^{(III)}(x)&=\phi_i(x)\cdot U_5+\phi_j(x)\cdot U_6+\phi_k(x)\cdot U_7 \end{aligned}\right\},\qquad 0\le x\le l \tag{20}$$

4. Numerical Results

For approbation of the developed methods and algorithms we will solve the formulated problem at the following input data:

L=30cm; $a=-\frac{1}{10};\ b=12cm$

$\alpha=0,0000125\frac{1}{{}^0K};\quad E=2\cdot10^6\frac{\kappa G}{cm^2};$

$K_{xx}=100\frac{Watt}{cm\cdot{}^0K};\quad h=10\frac{Watt}{cm^2\cdot{}^0K};$

$T_{oc}=40^0K;\ \ q=-500\frac{Watt}{cm^2};$

The received decisions are provided on Figures 2-5.

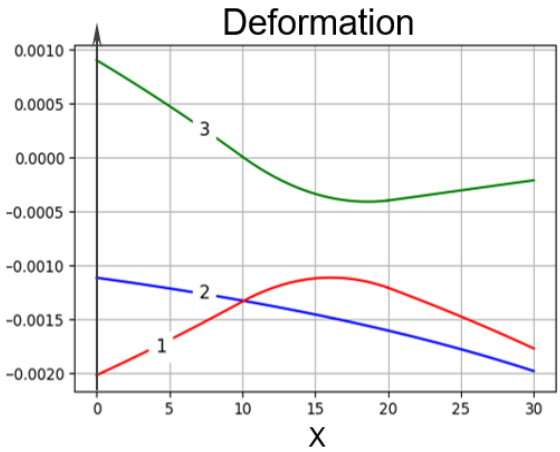

Figure 3. *Dependences of deformation the length of the rod*

$1-\varepsilon(x);\ 3-\varepsilon_x(x);\ 2-\varepsilon_T(x)$

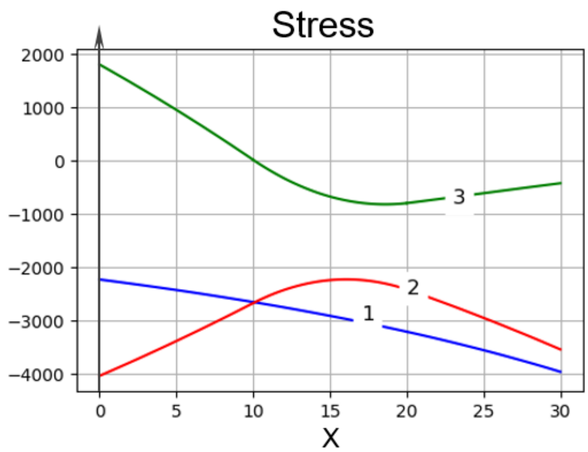

Figure 4. *Dependences tension the length of the rod*

$1-\sigma(x);\ 3-\sigma_x(x);\ \ 2-\sigma_T(x)$

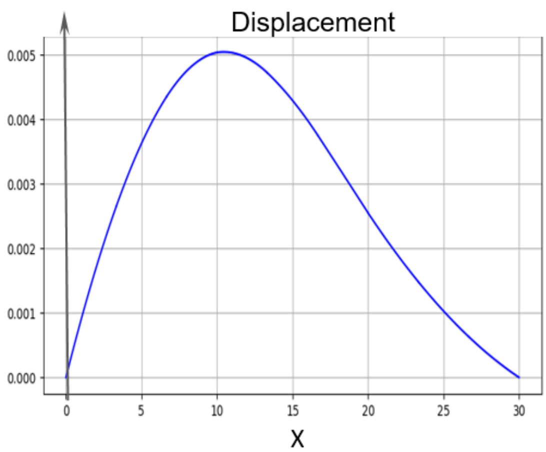

Figure.5*. Dependences movement the length of the rod*

For this example the sectional area of the left-hand end of a the rod is equal F(x=0)=144πcm^2 and right end F(x=L)=81πcm^2 it turns out (x=0)=1,77* (F(x=L)); From the Figure 2 it is visible that value of temperature on the left-hand end of T(x=0)=161.23^{0}K, at that time on the right end of T(x=L)=141.453^{0}K. It is caused by the fact that the sectional area of the right end on 1.777 times is less than left-hand. The value of temperature is equal $T(x=\frac{5l}{3})=89.08^o K,$ in a point $x=\frac{5l}{3}$ it the least in all length of a the rod.

It is caused by the fact that through a lateral area of the site (l≤x≤2l) of a the rod there is a heat exchange to a surrounding medium, and variability of section. From the drawing it is visible that thermoelastic and temperature components of deformation on all length of a the rod have the squeezing character while resilient making deformations $\varepsilon_x(x)$ on the site (0≤x≤l) of a the rod has stretching, and in other sites the squeezing character. It is caused, the fact that the sectional area of the left-hand end of a the rod is maximal, and there brought a heat flux with intensity $q=-500\frac{Watt}{cm^2}$.

The similar phenomena are observed also in the nature. The distribution law of movement is provided in the Figure 5. Where it is visible from the figure that all sections of a rod except jammed move at the left in the rights. At the same time maximal movements corresponds to section with coordinate $x=\frac{4l}{3}$. This process is caused with a big sectional area of the left-hand end of a rod and existence of a heat flux constant intensity $q=-500\frac{Watt}{cm^2}$.

Also this phenomenon is promoted by existence of heat exchange through side the surface of the site (l≤x≤2l). After calculation the field of temperatures is defined the size of lengthening of a the rod for a the rod of jamming of one end:

$$\Delta l_T=\int_0^L \alpha\cdot T(x)dx=0,0432cm.$$

In case of jamming of two ends of a rod there is a squeezing axial force which size equally:

$$R = -\frac{E \cdot F_{av} \cdot \Delta l_T}{L} = -1006573,377[kG].$$

5. Conclusion

The method and computing algorithm is developed of fundamental laws of conservation of energy, and also the program on Python allow to solve a class of tasks of definition established thermo - an intense strained state of a the rod of variable section at influence of diverse types of local sources of heat taking into account existence of thermal insulations, physicomechanical and thermal characteristics of material of a the rod, and also at its difficult geometrical sizes.

The received numerical results differ in a high precision and convergence. Besides the offered approach differs in the universality at the solution of relevant engineering tasks stationary thermoelasticities for the rod stock of variable section.

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