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ON EQUITABLE CHROMATIC NUMBER OF TADPOLE GRAPH $T_{m,n}$

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ABSTRACT. Graph coloring is a special case of graph labeling. Proper vertex k-coloring of a graph $G$is to color all the vertices of a graph with different colors in such a way that no two adjacent vertices are assigned with the same color. In a vertex coloring of G, the set of vertices with the same color is called color class. An equitable k-coloring of a graph G is a proper k-coloring in which any two color classes differ in size by atmost one. In this paper we give results regarding the equitable coloring of central, middle, total and line graphs of Tadpole graph which is obtained by connecting a cycle graph and a path graph with a bridge.

1. Introduction

A graph G is a set of vertex V connected by edges E. All graphs considered in this paper are finite and simple (i.e) undirected, loop-less and without multiple edges.

Everytime when we have to divide a system with binary conflict relations into equal or almost equal conflict - free subsystems we can model this situation by means of equitable graph coloring [6]. The notion of equitable colorability was introduced by Meyer [9]. However an earlier work of Hajnal and Szemeredi [7] showed that a graph G with degree $\Delta(G)$ is equitably k-colorable if $k > \Delta(G) + 1$.

If the set of vertices of a graph G can be partitioned into k classes $V_1, V_2, ..., V_k$ such that each V_i is an independent set and the condition $||V_i| - |V_j|| \leq 1$ holds for every pair (i, j) then G is said to be equitably k-colorable. The smallest integer k for which G is equitably k-colorable [5] is known as the equitable chromatic number of G and is denoted by $\chi_=(G)$.

A proper vertex coloring of a graph G is m-bounded if each color appears on atmost m vertices [4, 8]. Every equitable k-coloring of a graph G is $\lceil n(G/k)\rceil$-

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bounded, where $n(G)$ denotes the number of vertices of G and a k-coloring is a coloring with k color classes. A graph G may have an equitable k-coloring but not an equitable $(k+1)$-coloring. There are two parameters of a graph G related to equitable coloring, equitable chromatic number and equitable chromatic threshold. The equitable chromatic number of G denoted by $\chi_{=}(G)$ is the minimum $k$such that G is equitably k-colorable. The equitable chromatic threshold of G denoted by $\chi^*_{=}(G)$ is the minimum $k^{'}$ such that $k \geq k^{'}$ where G is equitably k-colorable [10].

In this paper we investigate the equitable chromatic number of central, middle, total and line graphs of Tadpole graph $T_{m,n}$.

2. Preliminaries

Let G be a simple and undirected graph and let its vertex set and edge set be denoted by $V(G)$ and $E(G)$ respectively. The *centralgraph* of G[1, 2], denoted by $C(G)$ is obtained by subdividing each edge of G exactly once and joining all the non-adjacent vertices of G in $C(G)$.

Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. The *middlegraph* [2, 3] of G is denoted by $M(G)$ is defined as follows: The vertex set of $M(G)$ is $V(G) \cup E(G)$ in which two vertices x, y are adjacent in $M(G)$ if the following condition holds

(1) $x, y \in E(G)$ and x, y are adjacent in G.
(2) $x \in V(G)$, $y \in E(G)$ and they are incident in G.

Let G be a graph with vertex set $V(G)$ and edge set $E(G)$.The *totalgraph* [2, 3] of G is denoted by $T(G)$ is defined as follows: The vertex set of $T(G)$ is $V(G) \cup E(G)$ in which two vertices x, y are adjacent in $T(G)$ if the following condition holds

(1) x, y are in $V(G)$ and x is adjacent to y in G.
(2) x, y are in $E(G)$ and x, y are adjacent in G.
(3) x is in $V(G)$, y is in $E(G)$ and x, y are adjadent in G.

The *linegraph*[2, 3] of a graph G denoted by $L(G)$ is a graph whose vertices are the edges of G and if $u, v \in E(G)$ then $uv \in E(L(G))$ if u and v share a vertex in G.

The (m, n)-tadpole graph denoted by $T_{m,n}$ also called dragon graph is the graph obtained by joining a cycle graph C_m to a path graph P_n with a bridge.

In the *tadpolegraph* $T_{m,n}$, let m denote the number of vertices of cycle graph C_m and n denote the number of vertices of path graph P_n. Let the vertices of cycle graph and path graph be denoted by $\{v_i : 1 \leq i \leq m\}$ and $\{p_j : 1 \leq j \leq n\}$ respectively and let the edges of cycle graph and path graph be denoted by $\{e_i : 1 \leq i \leq m\}$ and $\left\{e^{'}_j : 1 \leq j \leq n\right\}$ respectively. Throughout this paper, the path graph is joined from the vertex v_1 of the cycle graph.

3. Equitable Coloring of central graph of Tadpole Graph $T_{m,n}$

Theorem 3.1. *For a Tadpole Graph $T_{m,n}$, where m and n are any two positive integers such that $m > 4$ and $n \geq 1$, the equitable chromatic number of central graph is*

$$\chi_{=}[C(T_{m,n})] = \begin{cases} \frac{m+n}{2} & \text{if } m \text{ is even, } n \text{ is even} \\ \frac{m+n+1}{2} & \text{if } m \text{ is even, } n \text{ is odd} \\ \frac{m+n+1}{2} & \text{if } m \text{ is odd, } n \text{ is even} \\ \frac{m+n+2}{2} & \text{if } m \text{ is odd, } n \text{ is odd} \end{cases}$$

Proof. Let $V = \{v_1, v_2, ...v_m, p_1, p_2, ...p_n\}$ and $E = \left\{e_1, e_2, ...e_m, e_1^{'}, e_2^{'}, ...e_n^{'}\right\}$ denote the vertex set and edge set of the Tadpole graph $T_{m,n}$ respectively .

By the definition of central graph on $T_{m,n}$, the edges of the cycle graph v_iv_{i+1} $(1 \leq i \leq m)$ are subdivided exactly once by e_i $(1 \leq i \leq m)$, the edges of the path graph p_jp_{j+1} $(1 \leq j \leq n)$ are subdivided by $e_j^{'}$ $(2 \leq j \leq n)$ and the edge v_1p_1 by $e_1^{'}$.

$V(C(T_{m,n})) = \{v_i : 1 \leq i \leq m\} \cup \{p_j : 1 \leq j \leq n\} \cup \{e_i : 1 \leq i \leq m\} \cup \left\{e_j^{'} : 1 \leq j \leq n\right\}$

The two positive integers m and n may be either even or odd. The equitable coloring on Tadpole graph $T_{m,n}$ (*m and n being even or odd*) are assigned in the following manner.

Case 1: m and n are even

Assign the colors $\{c_1, c_1, c_2, c_2, c_3, c_3, ...c_{\frac{m}{2}}, c_{\frac{m}{2}}\}$ to the consecutive vertices of the cycle graph $\{v_1, v_2, v_3, v_4, v_5, v_6, ..., v_{m-1}, v_m\}$ and the colors $\{c_{\frac{m}{2}+1}$, $c_{\frac{m}{2}+1}$, $c_{\frac{m}{2}+2}$, $c_{\frac{m}{2}+2}$,...$\}$ to the vertices of the path graph $\{p_1, p_2, p_3, p_4, ...\}$. Now assign the colors $\{c_2$, c_3, c_4, ..., $c_{\frac{m+n}{2}}\}$ to the edges of the cycle graph $\{e_1$, e_2, e_3, ..., $e_{\frac{m+n}{2}-1}\}$ respectively. The remaining consecutive edges of the cycle graph are assigned the colors $\{c_1, c_2, c_3, ...\}$. The edges of the path graph $\left\{e_1^{'}, e_2^{'}, e_3^{'}, ..., e_n^{'}\right\}$ are assigned with the following colors:

$e_1^{'}$ as $c_{\frac{m+n}{2}}$

$e_2^{'}$ as c_1

$\left\{e_n^{'}, e_{n-1}^{'}, e_{n-2}^{'}, ...\right\}$ as $\left\{c_{\frac{m+n}{2}-1}, c_{\frac{m+n}{2}-2}, c_{\frac{m+n}{2}-3}, ...\right\}$ respectively.

Now we partition the vertex set $V(C(T_{m,n}))$ as follows:

$$\begin{aligned} V_1 &= \left\{v_1, v_2, e_{m-3}, e_2^{'}\right\} \\ V_2 &= \{v_3, v_4, e_1, e_{m-2}\} \\ V_3 &= \{v_5, v_6, e_2, e_{m-1}\} \end{aligned}$$

$$\begin{aligned} V_4 &= \{v_7, v_8, e_3, e_m\} \\ &\vdots \\ V_{\frac{m+n}{2}} &= \left\{p_{n-1}, p_n, e_1^{'}, e_{n-1}\right\} \end{aligned}$$

Clearly, $V_1, V_2, ...V_{\frac{m+n}{2}}$ are independent sets of $C(T_{m,n})$. Also $|V_1| = |V_2| = ... = |V_{\frac{m+n}{2}}| = 4$. $||V_i| - |V_j|| = 0 \;\; \forall \;\; i \neq j$. Hence

$$\chi_{=}(C(T_{m,n})) \leq \frac{m+n}{2}.$$

For each i, V_i is non-adjacent with V_{i-1} and V_{i+1} and an easy check shows that $\chi_{=}(C(T_{m,n})) \geq \frac{m+n}{2}$. Therefore, $\chi_{=}(C(T_{m,n})) = \frac{m+n}{2}$.

Case 2: m is even and n is odd

Assign the colors $\left\{c_1, c_1, c_2, c_2, c_3, c_3, ...c_{\frac{m}{2}}, c_{\frac{m}{2}}\right\}$ to the consecutive vertices of the cycle graph $\{v_1, v_2, v_3, ...v_{m-1}, v_m\}$ similar to case 1. The colors $\{c_{\frac{m}{2}+1}, c_{\frac{m}{2}+1}, c_{\frac{m}{2}+2}, c_{\frac{m}{2}+2}, ..., c_{\frac{m+n-1}{2}}, c_{\frac{m+n+1}{2}}\}$ are assigned to the vertices of the path graph $\{p_1, p_2, p_3, ..., p_n\}$ respectively. As introduced in case 1, the colors $\{c_2, c_3, c_4, ..., c_{\frac{m+n+1}{2}}\}$ are assigned to the edges of the cycle graph $\left\{e_1, e_2, e_3, ..., e_{\frac{m+n-1}{2}}\right\}$ respectively.

The remaining consecutive edges of the cycle graph are assigned the colors $\{c_1, c_2, c_3, ...\}$. The edges of the path graph $\left\{e_1^{'}, e_2^{'}, e_3^{'}, ..., e_n^{'}\right\}$ are assigned with the following colors:
$e_1^{'}$ as $c_{\frac{m+n+1}{2}}$
$e_2^{'}$ as c_1
$\left\{e_n^{'}, e_{n-1}^{'}, e_{n-2}^{'}, ...\right\}$ as $\left\{c_{\frac{m+n-3}{2}}, c_{\frac{m+n-5}{2}}, c_{\frac{m+n-7}{2}}, ...\right\}$ respectively.
Now we partition the vertex set $V(C(T_{m,n}))$ as follows:

$$\begin{aligned} V_1 &= \left\{v_1, v_2, e_{m-1}, e_2^{'}\right\} \\ V_2 &= \{v_3, v_4, e_m\} \\ V_3 &= \left\{v_5, v_6, e_3^{'}\right\} \\ V_4 &= \left\{v_7, v_8, e_3, e_4^{'}\right\} \\ &\vdots \\ V_{\frac{m+n+1}{2}} &= \left\{p_n, e_1^{'}, e_6\right\} \end{aligned}$$

Clearly, $V_1, V_2, ...V_{\frac{m+n+1}{2}}$ are independent sets of $C(T_{m,n})$. Also $|V_1|, |V_2|, ..., |V_{\frac{m+n+1}{2}}|$ is either 3 or 4. So $||V_i| - |V_j|| \leq 1 \; \forall \; i \neq j$. Hence

$$\chi_{=}(C(T_{m,n})) \leq \frac{m+n+1}{2}.$$

For each i, V_i is non-adjacent with V_{i-1} and V_{i+1} and an easy check shows that $\chi_{=}(C(T_{m,n})) \geq \frac{m+n+1}{2}$. Therefore, $\chi_{=}(C(T_{m,n})) = \frac{m+n+1}{2}$.

Case 3: m is odd and n is even

Assign the colors $\left\{c_1, c_1, c_2, c_2, ...c_{\frac{m-1}{2}}, c_{\frac{m-1}{2}}, c_{\frac{m+1}{2}}\right\}$ to the consecutive vertices of the cycle graph $\{v_1, v_2, v_3, v_4 ... v_{m-2}, v_{m-1}, v_m\}$ respectively and the vertices of the path graph $\{p_1, p_2, ..., p_{n-1}, p_n\}$ are assigned with the colors $\{c_{\frac{m+1}{2}+1}$, $c_{\frac{m+1}{2}+1}$, ..., $c_{\frac{m+n+1}{2}+1}$, $c_{\frac{m+n+1}{2}}\}$ respectively. Now the edges of the cycle graph $\{e_1$, e_2, e_3, ..., $e_{\frac{m+n-1}{2}}\}$ are assigned with the colors $\{c_2$, c_3, c_4, ..., $c_{\frac{m+n+1}{2}}\}$. The remaining consecutive edges are assigned the colors $\{c_1, c_2, c_3, ...\}$. The edges of the path graph are $\{e_1^{'}$, $e_2^{'}$, ..., $e_n^{'}\}$ assigned with the following colors:

$e_1^{'}$ as $c_{\frac{m+n+1}{2}}$

$e_2^{'}$ as c_1

$\left\{e_n^{'}, e_{n-1}^{'}, e_{n-2}^{'}, ...\right\}$ with $\left\{c_{\frac{m+n+1}{2}-1}, c_{\frac{m+n+1}{2}-2}, c_{\frac{m+n+1}{2}-3}, ...\right\}$ respectively.

Now the vertex set $V(C(T_{m,n}))$ are partitioned as follows:

$$\begin{aligned}
V_1 &= \left\{v_1, v_2, e_6, e_2^{'}\right\} \\
V_2 &= \{v_3, v_4, e_1, e_7\} \\
V_3 &= \{v_5, v_6, e_2\} \\
V_4 &= \left\{v_7, e_3, e_3^{'}\right\} \\
&\vdots \\
V_{\frac{m+n+1}{2}} &= \left\{p_{n-1}, p_n, e_1^{'}, e_{m-2}\right\}
\end{aligned}$$

Clearly, $V_1, V_2, ...V_{\frac{m+n+1}{2}}$ are independent sets of $C(T_{m,n})$. Also $|V_1|, |V_2|, ..., |V_{\frac{m+n+1}{2}}|$ is either 3 or 4. So $||V_i| - |V_j|| \leq 1 \; \forall \; i \neq j$. Hence

$$\chi_{=}(C(T_{m,n})) \leq \frac{m+n+1}{2}.$$

For each i, V_i is non-adjacent with V_{i-1} and V_{i+1} and an easy check shows that $\chi_{=}(C(T_{m,n})) \geq \frac{m+n+1}{2}$. Therefore, $\chi_{=}(C(T_{m,n})) = \frac{m+n+1}{2}$.

Case 4: m is odd and n is odd

Assign the colors $\left\{c_1, c_1, c_2, c_2, c_3, c_3, ...c_{\frac{m-1}{2}}, c_{\frac{m-1}{2}}, c_{\frac{m+1}{2}}\right\}$ to the consecutive vertices of the cycle graph $\{v_1, v_2, v_3, ..., v-m-2, v_{m-1}, v_m\}$ respectively and the vertices of the path graph $\{p_1, p_2, ..., p_{n-2}, p_{n-1}, p_n\}$ are assigned with the colors $\{c_{\frac{m+n-2}{2}}, c_{\frac{m+n-2}{2}}, \ldots, c_{\frac{m+n}{2}}, c_{\frac{m+n}{2}}, c_{\frac{m+n+2}{2}}\}$ respectively. Now the edges of the cycle graph $\left\{e_1, e_2, e_3, ..., e_{\frac{m+n}{2}}\right\}$ are assigned with the colors $\left\{c_2, c_3, c_4, ...c_{\frac{m+n+2}{2}}\right\}$. The remaining edges are assigned the colors $\{c_1, c_2, c_3, ...\}$. The edges of the path graph are assigned with the following colors:

$e_1^{'}$ as $c_{\frac{m+n+2}{2}}$

$e_2^{'}$ as c_1

$\left\{e_n^{'}, e_{n-1}^{'}, e_{n-2}^{'}, ...\right\}$ with $\left\{c_{\frac{m+n+2}{2}}, c_{\frac{m+n}{2}}, c_{\frac{m+n}{2}}, ...\right\}$ respectively.

Now we partition the vertex set $V(C(T_{m,n}))$ as follows:

$$\begin{aligned} V_1 &= \left\{v_1, v_2, e_{m-1}, e_2^{'}\right\} \\ V_2 &= \{v_3, v_4, e_m\} \\ &\vdots \\ V_{\frac{m+n+2}{2}} &= \left\{p_n, e_1^{'}, e_{m-2}\right\} \end{aligned}$$

Clearly, $V_1, V_2, ...V_{\frac{m+n+2}{2}}$ are independent sets of $C(T_{m,n})$. Also $|V_1|, |V_2|, ..., |V_{\frac{m+n+2}{2}}|$ is either 3 or 4. So $||V_i| - |V_j|| \leq 1 \ \forall \ i \neq j$. Hence

$$\chi_=(C(T_{m,n})) \leq \frac{m+n+2}{2}.$$

For each i, V_i is non-adjacent with V_{i-1} and V_{i+1} and an easy check shows that $\chi_=(C(T_{m,n})) \geq \frac{m+n+2}{2}$. Therefore, $\chi_=(C(T_{m,n})) = \frac{m+n+2}{2}$. □

4. Equitable Coloring of middle graph of Tadpole Graph $T_{m,n}$

Theorem 4.1. *The equitable chromatic number of the middle graph of tadpole graph $T_{m,n}$, where m and n are any two positive integers such that $m \geq 4$ and $n \geq 1$ is $\chi_=[M(T_{m,n})] = 4$*

Proof. Let $V = \{v_i : (1 \leq i \leq m)\}$ and $\{p_j : (1 \leq j \leq n\}$ be the vertices of the cycle graph and path graph respectively of the Tadpole graph $(T_{m,n})$. Let $\{e_i : (1 \leq i \leq m)\}$ and $\left\{e_j^{'} : (1 \leq j \leq n\right\}$ denote the edges of the cycle graph and path graph respectively. By the definition of Middle graph on $T_{m,n}$ each edge v_iv_{i+1} $(1 \leq i \leq m)$ are subdivided by the vertex e_i $(1 \leq i \leq m-1)$ and the edge v_mv_1 by e_m. Each edge p_jp_{j+1} is subdivided by $e_{j+1}^{'}$ $(1 \leq j \leq n)$ and the edge v_1p_1 by $e_1^{'}$. Clearly,

$$\begin{aligned} V(M(T_{m,n})) &= \{v_i : 1 \leq i \leq m\} \cup \{p_j : 1 \leq j \leq n\} \\ &\cup \{e_i : 1 \leq i \leq m\} \cup \left\{e_j^{'} : 1 \leq j \leq n\right\} \end{aligned}$$

Thus 4 colors $\{c_1, c_2, c_3, c_4\}$ are assigned to $V(M(T_{m,n}))$. The assigning of colors is done in the following three cases.

Case 1: m is even and n is odd or even

Assign the colors $\{c_1, c_2, c_1, c_2, ..., c_1, c_2\}$ to the consecutive vertices of $\{v_1, v_2, v_3, v_4, ..., v_{m-1}, v_m\}$, the colors $\{c_3, c_4, c_3, c_4, ..., c_3, c_4\}$ to the consecutive vertices $\{e_1, e_2, e_3, e_4, ..., e_{m-1}, e_m\}$ and the colors $\{c_1, c_2, c_3, c_4, c_1, c_2, ...\}$ to the consecutive vertices $\left\{v_1, e_1^{'}, p_1, e_2^{'}, p_2, e_3^{'}, p_3, ...\right\}$. The colors c_1 is used $\frac{m}{2} + \lfloor \frac{n}{2} \rfloor$ times, c_2 is used $\frac{m}{2} + \lceil \frac{n}{2} \rceil$ times, c_3 is used $\frac{m}{2} + \lceil \frac{n}{2} \rceil$ times and c_4 is used $\frac{m}{2} + \lfloor \frac{n}{2} \rfloor$ times such that each color class differ in value by atmost one which satisfies equitable coloring.

Case 2: m is odd and n is even

Assign the colors $\{c_1, c_2, c_1, c_2, ..., c_1, c_2\}$ to the consecutive vertices of $\{v_1, v_2, v_3, ...v_{m-2}, v_{m-1}\}$, the colors $\{c_3, c_4, c_3, c_4, ..., c_3, c_4\}$ to the consecutive vertices $\{e_1, e_2, e_3, ..., e_{m-2}, e_{m-1}\}$ and the colors $\{c_4, c_3, c_2, c_1, c_4, c_3, ...\}$ to the consecutive vertices $\{e_1^{'}, p_1, e_2^{'}, p_2, e_3^{'}, p_3, ...\}$. The colors c_3 and c_2 are assigned to the vertices v_m and e_m respectively. The color c_1 is used $\lfloor \frac{m}{2} \rfloor + \frac{n}{2}$ times, c_2 is used $\lceil \frac{m}{2} \rceil + \frac{n}{2}$ times, c_3 is used $\lceil \frac{m}{2} \rceil + \frac{n}{2}$ times and c_4 is used $\lfloor \frac{m}{2} \rfloor + \frac{n}{2}$ times such that each color classes differ in value by atmost one which satisfies equitable coloring.

Case 3: m is odd and n is odd

Assign the colors $\{c_1, c_2, c_1, c_2, ..., c_1, c_2\}$ to the consecutive vertices of $\{v_1, v_2, v_3, v_4, ...v_{m-4}, v_{m-3}\}$, and the colors $\{c_1, c_4, c_3\}$ to the remaining vertices $\{v_{m-2}, v_{m-1}, v_m\}$ respectively, the colors $\{c_3, c_4, c_3, c_4, ..., c_3, c_4\}$ to the consecutive edges $\{e_1, e_2, e_3, e_4, ..., e_{m-4}, e_{m-3}\}$ and the colors $\{c_2, c_1, c_4\}$ to the remaining edges $\{e_{m-2}, e_{m-1}, e_m\}$ respectively, the colors $\{c_1, c_2, c_3, c_4, c_1, c_2, ...\}$ to the consecutive vertices $\{v_1, e_1^{'}, p_1, e_2^{'}, p_2, e_3^{'}, ...\}$. The colors c_2 and c_3 are assigned to the vertices v_m and e_m respectively. The color c_1 is used $\lceil \frac{m}{2} \rceil + \lfloor \frac{n}{2} \rfloor$ times, c_2 is used $\lfloor \frac{m}{2} \rfloor + \lceil \frac{n}{2} \rceil$ times, c_3 is used $\lfloor \frac{m}{2} \rfloor + \lceil \frac{n}{2} \rceil$ times and c_4 is used $\lceil \frac{m}{2} \rceil + \lfloor \frac{n}{2} \rfloor$ times such that each color classes differ in value by atmost one which satisfies equitable coloring. Thus 4 colors are assigned to $V[M(T_{m,n})]$ in all the three follows that $\chi_=(M(T_{m,n})) \leq 4$ since $\left\{e_1, v_1, e_m, e_1^{'}\right\}$ is complete, we have $\chi_=(M(T_{m,n})) \geq 4$. Therefore, $\chi_=(M(T_{m,n})) = 4$. □

5. Equitable Coloring of total graph of Tadpole Graph $T_{m,n}$

Theorem 5.1. *The equitable chromatic number of the total graph of tadpole graph $T_{m,n}$, where m and n are any two positive integers such that $m \geq 4$ and $n \geq 1$ is $\chi_{=}[T(T_{m,n})] = 4$*

Proof. For a tadpole graph $T_{m,n}$, let $V(T_{m,n}) = \{v_i : (1 \leq i \leq m)\} \cup \{p_j : (1 \leq j \leq n\}$ and $E(T_{m,n}) = \{e_i : (1 \leq i \leq m)\} \cup \left\{e_j^{'} : (1 \leq j \leq n\right\}$. By the definition of total graph on $T_{m,n}$, we have

$$\begin{aligned} V[T(T_{m,n})] &= V(T_{m,n}) \cup E(T_{m,n}) \\ &= \{v_i : (1 \leq i \leq m)\} \cup \{p_j : (1 \leq j \leq n\} \\ &\quad \cup \{e_i : (1 \leq i \leq m)\} \cup \left\{e_j^{'} : (1 \leq j \leq n\right\}. \end{aligned}$$

The 4 colors $\{c_1, c_2, c_3, c_4\}$ are assigned to $V(T(T_{m,n}))$. The proof of this theorem follows as in theorem 4.1. Therefore, $\chi_{=}[T(T_{m,n})] = 4$ □

6. Equitable Coloring of line graph of Tadpole Graph $T_{m,n}$

Theorem 6.1. *The equitable chromatic number of the line graph of tadpole graph $T_{m,n}$, where m and n are any two positive integers such that $m \geq 3$ and $n \geq 1$ is $\chi_{=}[L(T_{m,n})] = 3$*

Proof. Let the vertices of the tadpole graph $T_{m,n}$ be $V(T_{m,n}) = \{v_i : (1 \leq i \leq m)\} \cup \{p_j : (1 \leq j \leq n\}$ and the edges be $E(T_{m,n}) = \{e_i : (1 \leq i \leq m)\} \cup \left\{e_j^{'} : (1 \leq j \leq n\right\}$. By the definition of line graph

$$V[L(T_{m,n})] = E(T_{m,n}) = \{e_i : (1 \leq i \leq m)\} \cup \left\{e_j^{'} : (1 \leq j \leq n\right\},$$

where e_i and $e_j^{'}$ represents the edges of the cycle graph and path graph respectively. The colors $\{c_1, c_2, c_3\}$ are assigned to $V[L(T_{m,n})]$. Based on the number of vertices $\{e_i : (1 \leq i \leq m)\}$ of the cycle graph in the line graph of tadpole graph we have the following conditions:

1. $m \ mod \ 3 = 0$

Here we assign the colors $\{c_1, c_2, c_3, c_1, c_2, c_3, ..., c_1, c_2, c_3\}$ to the consecutive vertices of $\{e_1, e_2, e_3, ...e_{m-1}, e_m\}$, the cycle graph and the colors $\{c_2, c_3, c_1, c_2, c_3, c_1, ...\}$ to the consecutive vertices of $\left\{e_1^{'}, e_2^{'}, e_3^{'}, e_4^{'}, ...\right\}$ of the path graph.

2. $m \ mod \ 3 = 1$

In this case we assign the colors $\{c_1, c_2, c_3, c_1, c_2, c_3, ..., c_2, c_3, c_2\}$ to the consecutive vertices of the cycle graph $\{e_1, e_2, e_3, ...e_{m-1}, e_m\}$, and the colors $\{c_3,\ c_1,\ c_2,\ c_3,\ c_1,\ c_2, ...\}$ to the consecutive vertices of $\left\{e_1^{'}, e_2^{'}, e_3^{'}, e_4^{'}, ...\right\}$.

3. $m \ mod \ 3 = 0$

Here we assign the colors $\{c_1, c_2, c_3, c_1, c_2, c_3, ..., c_1, c_2\}$ to the consecutive vertices of the cycle graph $\{e_1, e_2, e_3, ...e_{m-1}, e_m\}$, and the colors $\{c_3, c_1, c_2, c_3, c_1, ...\}$ to the consecutive vertices of the path graph $\left\{e_1^{'}, e_2^{'}, e_3^{'}, e_4^{'}, ...\right\}$.

An easy check shows that the coloring is equitable in all the three cases. Since 3 colors are assigned, it follows that $\chi_{=}[L(T_{m,n})] \leq 3$. Since $\left\{e_1, e_m, e_1^{'}\right\}$ is complete, we have $\chi_{=}[L(T_{m,n})] \geq 3$. Hence $\chi_{=}[L(T_{m,n})] = 3$. □

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