New Trends in Mathematical Sciences
http://dx.doi.org/10.20852/ntmsci.2017.201

On new traveling wave solutions of the Hirota-Satsuma coupled KdV equation

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Received: 22 February 2017, Accepted: 24 April 2017

Published online: 26 August 2017.

Abstract: The improved $(G^{'}/G)$-expansion method is applied to reach the different type soliton solitions of the Hirota-Satsuma Coupled KdV (HSCKdV) equation. It is obtained hyperbolic, triangular, periodic wave and kink soliton solitions of this equation. The method is an effective one to reach the different types of solutions of nonlinear partial differential equations and systems. Finally, the numerical simulations add to these obtained solutions.

Keywords: Improved $(G^{'}/G)$-expansion method, Hirota-Satsuma coupled KdV equation, soliton.

1 Introduction

The mathematical modeling of scientific systems generally is expressed by nonlinear evolution equations. Therefore, it is crucial to reach general solutions of these corresponding nonlinear equations. Thus, the general solutions of these equations provide much information about the character and the structure of the equations for researchers. Many effective methods have been improved to provide much information for physicians and engineers. We recall that most of these methods use the wave variable transformation to reduce the nonlinear PDE to ODE in order to acquire the solution. Several are tanh [1], $G^{'}/G$-expansion [2], Jacobi elliptic function [3], mapping [4], tanh-sech [5],exp-function [6] and first integral methods [7]. All of these methods are effective methods for acquiring traveling wave solutions NPDE. For more details see [8,9,10].

The improved $(G^{'}/G)$-expansion method has been presented to the literature by Liu et al.[7]. This method has been successfully implemented to NPDE and some fractional differential equations which are new type of equations. The aim of this paper is to find travelling wave solutions of the HSCKdV equation by using improved $(G^{'}/G)$-expansion method.

This paper is organized as follows: the HSCKdV equation is given in Section 2 and improved $(G^{'}/G)$-expansion method is described in Section 3. This method has been applied to the HSCKdV equation [6] in Section 4. Then we give some conclusions in the last Section.

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2 Mathematical analysis

In this work, we consider the generalized HSCKdV equation [6]

$$\begin{aligned} u_t &= \frac{1}{2}u_{xxx} - 3uu_x + 3(vw)_x \\ v_t &= -v_{xxx} + 3uv_x \\ w_t &= -w_{xxx} + 3uw_x \end{aligned} \tag{1}$$

where the subscripts t and x denote differention. With $w = v^*$and $w = v$, Eq.(1) reduces, respectively, to a new complex coupled KdV equation [6] and Hirota-Satsuma equation [12].

Eq. (1) has been studied to obtain analitical and numerical solutions by many authors. Such as, numerically, variational iteration method [13],two-dimensional differential transform method [14,15], Adomian's method [16], homotopy analysis method [17]. Analitically, improve Riccati equation method [18], fractional sub-equation method [19], homogeneous balance method [20], etc.

3 Improved (G'/G)-expansion method

In this method, we assume that

$$u(\xi) = \sum_{i=0}^{m} a_i F_i(\xi), \tag{2}$$

is solution function such that

$$F(\xi) = \frac{G'(\xi)}{G(\xi)}, \tag{3}$$

where $G(\xi)$ is given by

$$G(\xi)G''(\xi) = AG^2(\xi) + BG(\xi)G'(\xi) + C(G'(\xi))^2. \tag{4}$$

$G(\xi)$ is the solution of the second-order nonlinear diferential equation such that A, B, C are real parameters. If we try to find the solution of the (3), then we find the following four cases [7].

Case 1: If $B \neq 0$, $\Delta = B^2 + 4A - 4AC \geq 0$, then

$$F(\xi) = \frac{B}{2(1-c)} + \frac{B\sqrt{\Delta}}{2(1-c)} \frac{c_1 e^{\frac{\sqrt{\Delta}}{2}\xi} + c_2 e^{\frac{\sqrt{\Delta}}{2}\xi}}{c_1 e^{\frac{\sqrt{\Delta}}{2}\xi} + c_2 e^{\frac{\sqrt{\Delta}}{2}\xi}}, \tag{3.4}$$

such that c_1, c_2 are real parameters.

Case 2: If $B \neq 0$ and $\Delta = B^2 + 4A - 4AC < 0$, then

$$F(\xi) = \frac{B}{2(1-c)} + \frac{B\sqrt{-\Delta}}{2(1-c)} \frac{ic_1 \cos\frac{\sqrt{-\Delta}}{2}\xi - ic_2 \sin\frac{\sqrt{-\Delta}}{2}\xi}{ic_1 \sin\frac{\sqrt{-\Delta}}{2}\xi + ic_2 \cos\frac{\sqrt{-\Delta}}{2}\xi}. \tag{3.5}$$

Case 3: If $B = 0$ and $\Delta = A(c-1) \geq 0$, then

$$F(\xi) = \frac{\sqrt{\Delta}}{(1-c)} \frac{c_1 \cos(\sqrt{\Delta})\xi + c_2 \sin(\sqrt{\Delta})\xi}{c_1 \sin(\sqrt{\Delta})\xi - c_2 \cos(\sqrt{\Delta})\xi}. \tag{3.6}$$

Case 4: If $B=0$ and $\Delta=A(c-1)<0$, then

$$F(\xi)=\frac{\sqrt{\Delta}}{(1-c)}\frac{ic_1\cosh(\sqrt{-\Delta})\xi-c_2\sinh(\sqrt{-\Delta})\xi}{ic_1\sinh(\sqrt{-\Delta})\xi-c_2\cosh(\sqrt{-\Delta})\xi}. \tag{3.7}$$

Let apply the presented improved $(G'/G)-$expansion method to Hirota-Satsuma coupled KdV equation. If we use

$$u(x,t)=u(\xi),\quad v(x,t)=v(\xi),\quad w(x,t)=w(\xi),\quad \xi=x-kt,$$

transformations in

$$\begin{aligned} u_t &= \frac{1}{2}u_{xxx}-3uu_x+3(vw)_x \\ v_t &= -v_{xxx}+3uv_x \\ w_t &= -w_{xxx}+3uw_x \end{aligned} \tag{5}$$

such that k is wave velocity, then we get

$$\begin{cases} -ku'-\frac{1}{2}u'''+3u\,u'-3(vw)'=0, \\ -kv'+v'''-3uv'=0, \\ -kw'+w'''-3uw'=0. \end{cases} \tag{6}$$

We can take

$$m_1=2,\quad m_2=m_3=1,$$

by order balancing because of the higher order linear and nonlinear terms in (3) [21]. Thus for (9)

$$\begin{cases} u(\xi)=a_0+a_1F(\xi)+a_2F^2(\xi), \\ v(\xi)=b_0+b_1F(\xi), \\ w(\xi)=c_0+c_1F(\xi), \end{cases} \tag{7}$$

such a solution can be searched, where $a_0,a_1,a_2,b_0,b_1,c_0,c_1$ are constants that will be determined and $F(\xi)$ is the solution function of the (3). If we take necessary derivatives in (10) and put in (9), then the following algebraic equation system is obtained.

$$\begin{aligned}
&\left(\frac{G'}{G}\right)^0 : A^2a_1+3Aa_0a_1-3A^2a_2B-\frac{1}{2}Aa_1B^2-A^2a_1C-3Ab_1c_0-3Ab_0c_1-Aa_1k=0 \\
&\left(\frac{G'}{G}\right)^1 : A^2a_1^2+8A^2a_2+6Aa_0a_2+4Aa_1B+3a_0a_1B-7Aa_2B^2-\frac{1}{2}a_1B^3-8A^2a_2C \\
&\qquad -4Aa_1BC-3Bb_1c_0-3Bb_0c_1-6Ab_1c_1-2Aa_2k-a_1Bk=0 \\
&\left(\frac{G'}{G}\right)^2 : -4Aa_1-3a_0a_1+9Aa_1a_2+3a_1^2B+26Aa_2B+6a_0a_2B+\frac{7}{2}a_1B^2-4a_2B^3+8Aa_1C+3a_0a_1C-26Aa_2BC \\
&\qquad -\frac{7}{2}D_1B^2C-4Aa_1C^2+3b_1c_0-3b_1Cc_0+3b_0c_1-6Bb_1c_1-3b_0Cc_1+a_1k-2a_2Bk-a_1Ck=0 \\
&\left(\frac{G'}{G}\right)^3 : -3a_1^2-20Aa_2-6a_0a_2+6Aa_2^2-6a_1B+9a_1a_2B+19a_2B^2+3a_1^2C+40Aa_2C \\
&\qquad +6a_0a_2C+12a_1BC-19a_2B^2C-20Aa_2C^2-6a_1BC^2+6b_1c_1-6b_1Cc_1+2a_2k-2a_2Ck=0
\end{aligned}$$

$$\left(\frac{G'}{G}\right)^4 : 3a_1 - 9a_1a_2 - 27A^2B + 6a_2^2B - 9a_1C + 9a_1a_2C + 54a_2BC + 9a_1C^2 - 27a_2BC^2 - 3a_1C^3 = 0$$

$$\left(\frac{G'}{G}\right)^5 : 12a_2 - 6a_2^2 - 36a_2C + 6a_2^2C + 36a_2C^2 - 12a_2C^3 = 0$$

$$\left(\frac{G'}{G}\right)^0 : -2A^2b_1 - 3Aa_{0b_1} + AB^2b_1 + 2A^2b_1C - Ab_1k = 0$$

$$\left(\frac{G'}{G}\right)^1 : -3Aa_1b_1 - 8ABb_1 - 3a_0Bb_1 + B^3b_1 + 8ABb_1C - Bb_1k = 0$$

$$\left(\frac{G'}{G}\right)^2 : 8Ab_1 + 3a_0b_1 - 3Aa_2b_1 - 3a_1Bb_1 - 7B^2b_1 - 16Ab_1C - 3a_0b_1C + 7B^2b_1C + 8Ab_1C^2$$
$$+ b_1k - 16Ab_1C - 3a_0b_1C + 7B^2b_1C + 8Ab_1C^2 + b_1k - b_1Ck = 0$$

$$\left(\frac{G'}{G}\right)^3 : 3a_1b_1 + 12Bb_1 - 3a_2Bb_1 - 3a_1b_1C - 24Bb_1C + 12Bb_1C^2 = 0$$

$$\left(\frac{G'}{G}\right)^4 : -6b_1 + 3a_2b_1 + 18b_1C - 3a_2b_1C - 18b_1C^2 + 6b_1C^3 = 0$$

$$\left(\frac{G'}{G}\right)^0 : -2A^2c_1 - 3Aa_0c_1 + AB^2c_1 + 2A^2Cc_1 - Ac_1k = 0$$

$$\left(\frac{G'}{G}\right)^1 : -3Aa_1c_1 - 8ABc_1 - 3a_0Bc_1 + B^3c_1 + 8ABCc_1 - Bc_1k = 0$$

$$\left(\frac{G'}{G}\right)^2 : 8Ac_1 + 3a_0c_1 - 3Aa_2c_1 - 3a_1Bc_1 - 7B^2c_1 - 16ACc_1 - 3a_0Cc_1$$
$$+ 7B^2Cc_1 + 8AC^2c_1 + c_1k - Cc_1k = 0$$

$$\left(\frac{G'}{G}\right)^3 : 3a_1c_1 + 12Bc_1 - 3a_2Bc_1 - 3a_1Cc_1 - 24BCc_1 + 12BC^2c_1 = 0$$

$$\left(\frac{G'}{G}\right)^4 : -6c_1 + 3a_2c_1 + 18Cc_1 - 3a_2Cc_1 - 18C^2c_1 + 6C^3c_1 = 0.$$

If we solve this algebraic equation system by Mathematica, then we acquire the desired constants as

$$\begin{array}{l} B = 0 \\ a_0 = \frac{1}{3}\left(-2A + B^2 + 2AC - k\right), \quad a_1 = 0, \quad a_2 = 2\left(1 - 2C + C^2\right) \\ b_0 = 0, \quad b_1 \neq 0 \\ c_0 = 0, \; c_1 = \frac{4(1-C)^2(A(1-C)+k)}{3b_1} \end{array} \tag{8}$$

and

$$\begin{array}{l} B \neq 0 \\ a_0 = \frac{1}{3}\left(-2A + 2AC - k\right), \quad a_1 = 2\left(-B + BC\right), \quad a_2 = 2\left(1 - 2C + C^2\right) \\ b_0 = 0, \; b_1 \neq 0 \\ c_0 = \frac{(1-C)B\left(-B^2 - 4A(1-C) + 4k\right)}{3b_1}, \quad c_1 = \frac{-c_0 + Cc_0}{B}. \end{array} \tag{9}$$

If these constants are put in (3.10), then the complete solutions of Hirota-Satsuma coupled KdV equation are given by

Case 1:

$$u(x,t)=\frac{1}{3}(-2A(1-C)-k)-\frac{B^2}{2}\left[1-\Delta\left(\frac{c_1e^{\frac{\sqrt{\Delta}}{2}\xi}+c_2e^{-\frac{\sqrt{\Delta}}{2}\xi}}{c_1e^{\frac{\sqrt{\Delta}}{2}\xi}-c_2e^{-\frac{\sqrt{\Delta}}{2}\xi}}\right)^2\right] \tag{10}$$

$$v(x,t)=\frac{b_1B}{2(1-C)}\left[1+\sqrt{\Delta}\left(\frac{c_1e^{\frac{\sqrt{\Delta}}{2}\xi}+c_2e^{-\frac{\sqrt{\Delta}}{2}\xi}}{c_1e^{\frac{\sqrt{\Delta}}{2}\xi}-c_2e^{-\frac{\sqrt{\Delta}}{2}\xi}}\right)\right], \quad b_1\neq 0 \tag{11}$$

and

$$w(x,t)=\frac{B(1-C)\left(-B^2-4A(1-C)+4k\right)}{6b_1}\left[1-\sqrt{\Delta}\left(\frac{c_1e^{\frac{\sqrt{\Delta}}{2}\xi}+c_2e^{-\frac{\sqrt{\Delta}}{2}\xi}}{c_1e^{\frac{\sqrt{\Delta}}{2}\xi}-c_2e^{-\frac{\sqrt{\Delta}}{2}\xi}}\right)\right]. \tag{12}$$

If we take $-c_1=c_2$ in (13)–(15), then we will obtain the following soliton solutions as

$$u(x,t)=\frac{1}{3}(-2A(1-C)-k)-\frac{B^2}{2}\left[1-\Delta\tanh^2\frac{\sqrt{\Delta}}{2}\xi\right] \tag{13}$$

$$v(x,t)=\frac{b_1B}{2(1-C)}\left[1+\sqrt{\Delta}\tanh\frac{\sqrt{\Delta}}{2}\xi\right] \tag{14}$$

and

$$w(x,t)=\frac{B(1-C)\left(-B^2-4A(1-C)+4k\right)}{6b_1}\left[1-\sqrt{\Delta}\tanh\frac{\sqrt{\Delta}}{2}\xi\right]. \tag{15}$$

If we use

$$\tanh^2 y=1-\operatorname{sec}h^2 y$$

in Eq. (16), then we get another single wave solution as

$$u(x,t)=\frac{1}{3}(-2A(1-C)-k)+\frac{B^2}{2}(\Delta-1)-\frac{B^2}{2}\Delta\operatorname{sec}h^2\frac{\sqrt{\Delta}}{2}\xi. \tag{16}$$

If we take $c_1=c_2$ in (13)–(15), then we acquire

$$u(x,t)=\frac{1}{3}(-2A(1-C)-k)-\frac{B^2}{2}\left[1-\Delta\coth^2\frac{\sqrt{\Delta}}{2}\xi\right] \tag{17}$$

$$v(x,t)=\frac{b_1B}{2(1-C)}\left[1+\sqrt{\Delta}\coth\frac{\sqrt{\Delta}}{2}\xi\right] \tag{18}$$

and

$$w(x,t)=\frac{B(1-C)\left(-B^2-4A(1-C)+4k\right)}{6b_1}\left[1-\sqrt{\Delta}\coth\frac{\sqrt{\Delta}}{2}\xi\right]. \tag{19}$$

Case 2:

$$u(x,t)=\left(\frac{1}{3}-2A(1-C)-k\right)-\frac{B^2}{2}\left[1+\Delta\left[\frac{ic_1\cos\frac{\sqrt{-\Delta}}{2}\xi-c_2\sin\frac{\sqrt{-\Delta}}{2}\xi}{ic_1\sin\frac{\sqrt{-\Delta}}{2}\xi+c_2\cos\frac{\sqrt{-\Delta}}{2}\xi}\right]^2\right] \tag{20}$$

$$v(x,t)=\frac{b_1B}{2(1-C)}\left[1+\sqrt{-\Delta}\frac{ic_1\cos\frac{\sqrt{-\Delta}}{2}\xi-c_2\sin\frac{\sqrt{-\Delta}}{2}\xi}{ic_1\sin\frac{\sqrt{-\Delta}}{2}\xi+c_2\cos\frac{\sqrt{-\Delta}}{2}\xi}\right],\quad b_1\neq 0 \tag{21}$$

and

$$w(x,t)=\frac{B(1-C)\left(-B^2-4A(1-C)+4k\right)}{6b_1}\left[1-\sqrt{-\Delta}\frac{ic_1\cos\frac{\sqrt{-\Delta}}{2}\xi-c_2\sin\frac{\sqrt{-\Delta}}{2}\xi}{ic_1\sin\frac{\sqrt{-\Delta}}{2}\xi+c_2\cos\frac{\sqrt{-\Delta}}{2}\xi}\right]. \tag{22}$$

Case 3:

$$u(x,t)=\frac{1}{3}\left(-2A(1-C)+B^2-k\right)+2\Delta\left[\frac{c_1\cos\left(\sqrt{\Delta}\,\xi\right)+c_2\sin\left(\sqrt{\Delta}\,\xi\right)}{c_1\sin\left(\sqrt{\Delta}\,\xi\right)-c_2\cos\left(\sqrt{\Delta}\,\xi\right)}\right]^2, \tag{23}$$

$$v(x,t)=\frac{b_1\sqrt{\Delta}}{(1-C)}\left[\frac{c_1\cos\left(\sqrt{\Delta}\,\xi\right)+c_2\sin\left(\sqrt{\Delta}\,\xi\right)}{c_1\sin\left(\sqrt{\Delta}\,\xi\right)-c_2\cos\left(\sqrt{\Delta}\,\xi\right)}\right],\quad b_1\neq 0 \tag{24}$$

and

$$w(x,t)=\frac{4(1-C)(A(1-C)+k)}{3b_1}\left[\sqrt{\Delta}\frac{c_1\cos\left(\sqrt{\Delta}\,\xi\right)+c_2\sin\left(\sqrt{\Delta}\,\xi\right)}{c_1\sin\left(\sqrt{\Delta}\,\xi\right)-c_2\cos\left(\sqrt{\Delta}\,\xi\right)}\right]. \tag{25}$$

Case 4:

$$u(x,t)=\frac{1}{3}\left(-2A(1-C)+B^2-k\right)-2\Delta\left[\frac{ic_1\cosh\left(\sqrt{-\Delta}\,\xi\right)-c_2\sinh\left(\sqrt{-\Delta}\,\xi\right)}{ic_1\sinh\left(\sqrt{-\Delta}\,\xi\right)-c_2\cosh\left(\sqrt{-\Delta}\,\xi\right)}\right]^2, \tag{26}$$

$$v(x,t)=\frac{b_1\sqrt{\Delta}}{(1-C)}\left[\frac{ic_1\cosh\left(\sqrt{-\Delta}\,\xi\right)-c_2\sinh\left(\sqrt{-\Delta}\,\xi\right)}{ic_1\sinh\left(\sqrt{-\Delta}\,\xi\right)-c_2\cosh\left(\sqrt{-\Delta}\,\xi\right)}\right],\quad b_1\neq 0 \tag{27}$$

and

$$w(x,t)=\frac{4(1-C)(A(1-C)+k)}{3b_1}\left[\sqrt{-\Delta}\frac{ic_1\cosh\left(\sqrt{-\Delta}\,\xi\right)-c_2\sinh\left(\sqrt{-\Delta}\,\xi\right)}{ic_1\sinh\left(\sqrt{-\Delta}\,\xi\right)-c_2\cosh\left(\sqrt{-\Delta}\,\xi\right)}\right]. \tag{28}$$

If we take $c_1=0,\ \ c_2\neq 0$ and $c_1\neq 0,\ \ c_2=0$ in (23)-(25) respectively, then we acquire

$$u_1(x,t)=\frac{1}{3}-2A(1-C)-k-\frac{B^2}{2}\left[1-\Delta\tan\frac{\sqrt{-\Delta}}{2}(x-kt)\right]^2 \tag{29}$$

$$u_2(x,t)=\frac{1}{3}-2A(1-C)-k-\frac{B^2}{2}\left[1+\Delta\cot\frac{\sqrt{-\Delta}}{2}(x-kt)\right]^2 \tag{30}$$

$$v_1(x,t)=\frac{b_1B}{2(1-C)}\left[1-\sqrt{-\Delta}\tan\frac{\sqrt{-\Delta}}{2}(x-kt)\right],b_1\neq 0. \tag{31}$$

$$v_2(x,t)=\frac{b_1B}{2(1-C)}\left[1+\sqrt{-\Delta}\cot\frac{\sqrt{-\Delta}}{2}(x-kt)\right],\ b_1\neq 0 \tag{32}$$

$$w_1(x,t)=\frac{B(1-C)\left(-B^2-4A(1-C)+4k\right)}{6b_1}\left[1+\sqrt{-\Delta}\tan\frac{\sqrt{-\Delta}}{2}(x-kt)\right] \tag{33}$$

and

$$w_2(x,t)=\frac{B(1-C)\left(-B^2-4A(1-C)+4k\right)}{6b_1}\left[1-\sqrt{-\Delta}\cot\frac{\sqrt{-\Delta}}{2}(x-kt)\right]. \tag{34}$$

If we take $c_1 = 0, \ \ c_2 \neq 0$ and $c_1 \neq 0, \ \ c_2 = 0$ in (26)-(28) respectively, then we get

$$u_3(x,t) = \frac{1}{3}\left(-2A(1-C)+B^2-k\right) - 2\Delta \tan^2\left[\sqrt{\Delta}\ (x-kt)\right] \tag{35}$$

$$u_4(x,t) = \frac{1}{3}\left(-2A(1-C)+B^2-k\right) + 2\Delta \cot^2\left[\sqrt{\Delta}\ (x-kt)\right] \tag{36}$$

$$v_3(x,t) = -\frac{b_1\sqrt{\Delta}}{(1-C)}\tan\left[\sqrt{\Delta}\ (x-kt)\right],\ b_1 \neq 0 \tag{37}$$

$$v_4(x,t) = \frac{b_1\sqrt{\Delta}}{(1-C)}\cot\left[\sqrt{\Delta}\ (x-kt)\right],\ b_1 \neq 0 \tag{38}$$

$$w_3(x,t) = -\frac{4(1-C)(A(1-C)+k)}{3b_1}\sqrt{\Delta}\tan\left[\sqrt{\Delta}\ (x-kt)\right] \tag{39}$$

and

$$w_4(x,t) = \frac{4(1-C)(A(1-C)+k)}{3b_1}\cot\left[\sqrt{\Delta}\ (x-kt)\right]. \tag{40}$$

If we take $c_1 = 0, \ \ c_2 \neq 0$ and $c_1 \neq 0, \ \ c_2 = 0$ in (3.28)-(3.30) respectively, then we obtain

$$u_5(x,t) = \frac{1}{3}\left(-2A(1-C)+B^2-k\right) - 2\Delta \tanh^2\left[\sqrt{-\Delta}\ (x-kt)\right], \tag{41}$$

$$u_6(x,t) = \frac{1}{3}\left(-2A(1-C)+B^2-k\right) - 2\Delta \coth^2\left[\sqrt{-\Delta}\ (x-kt)\right] \tag{42}$$

$$v_5(x,t) = \frac{b_1\sqrt{\Delta}}{(1-C)}\tanh\left[\sqrt{-\Delta}\ (x-kt)\right],\ b_1 \neq 0, \tag{43}$$

$$v_6(x,t) = \frac{b_1\sqrt{\Delta}}{(1-C)}\coth\left[\sqrt{-\Delta}\ (x-kt)\right],\ b_1 \neq 0, \tag{44}$$

$$w_5(x,t) = \frac{4(1-C)(A(1-C)+k)}{3b_1}\sqrt{-\Delta},\tanh\left[\sqrt{-\Delta}\ (x-kt)\right] \tag{45}$$

and

$$w_6(x,t) = \frac{4(1-C)(A(1-C)+k)}{3b_1}\sqrt{-\Delta}\coth\left[\sqrt{-\Delta}\ (x-kt)\right]. \tag{46}$$

Analytic solutions of Hirota–Satsuma coupled KdV equation (HSCKdV) given by (8) were investigated by improved $\left(\frac{G^I}{G}\right)$ −expansion method presented by Liu at al. [7], in this work. We obtained complex, single wave, soliton, trigonometric and singular solutions of (8) by this method. In addition, we showed that these solutions ensure the system (8) by Mathematica. HSCKdV equation has been solved by many authors by using various methods. HSCKdV equation was solved by Feng and Li [21] by using Fan sub equation method with balancing terms $m = 2$ and $m = 1$. Feng ve Zhang [22] acquired jacobi elliptic solutions of HSCKdV equation by auxiliary function method. Yan [23] found wave solutions of HSCKdV equation by using

$$u = \alpha v^2 + \beta v + \gamma, w = Av + B,$$

transformations by extended Jacobi elliptic function method such that α, β, γ, A and B are constants that will be determined. We obtained some of the solitons that have been found by the methods that we mentioned and also we found different solutions in this work.

4 Numerical simulations

We will discuss the physical properties of the results of the results of the HSCKdV equation. Some obtained solıtions are shown in Figures 1-6. These figures have the following physical comments.

The shapes of Eqs. (13)-(15) and (16)-(18) are represented in Figs.1-6. Eq. (13) is singular solitary wave solution. In Fig.1 both 3D and 2D graphs describe the singular solitary wave solution, the wave speed is $k = 0.05$ within $-4 \leq x, t \leq 4$. Eq. (14) is singular king solution and is given Fig.2 In Fig.2,both 3D and 2D graphs describe the behaviour of $v(x,t)$, the wave speed is $k = 0.1$ within $-10 \leq x, t, \leq 10$. Eq. (15) is singular kink solution and is given Fig.3. Eq. (16) is non-topological soliton solutions and are given Figs. 5 and 6, respectively.

Results of this work are new soliton solutions of HSCKdV equation. The results are in terms of hyperbolic, triangular and exponential functions, and hence they all produce topological solutions (16)-(18), bell-shaped solitary wave solutions (19)-(21), periodic wave solutions (32)-(33) and similar waves. These solutions are useful in study shallow water waves. The integrability of NLPDEs can be studied by using the improved $(G'/G)-$expansion method. The obtained soliton solutions of this study are going to be useful for interested in the HSCKdV equation.

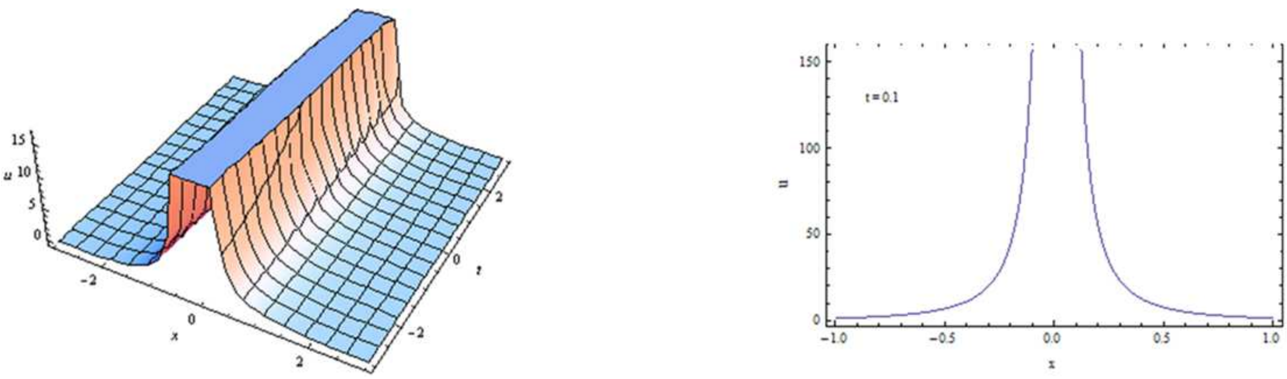

Fig. 1: Three dimensional and two dimensional graphics of the soliton solution of $u(x,t)$ that given by (13) such that $A = 2$, $B = C = 1$, $k = 0.05$, $c_1 = c_2 = 0.1$.

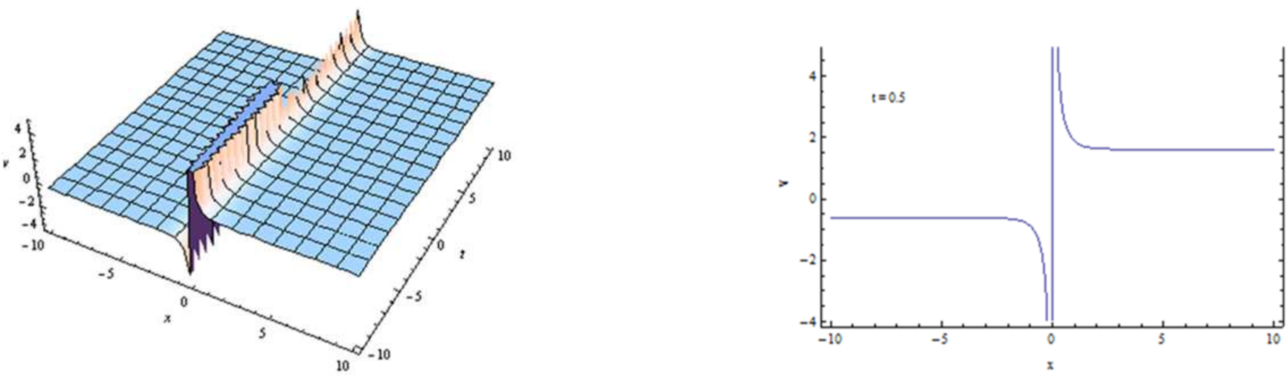

Fig. 2: Three dimensional and two dimensional graphics of soliton solution of $v(x,t)$ that given by (14) such that $A = 2$, $B = 1$, $C = 0.5$, $k = 0.01$, $c_1 = c_2 = 0.1$, $b_1 = 0.5$.

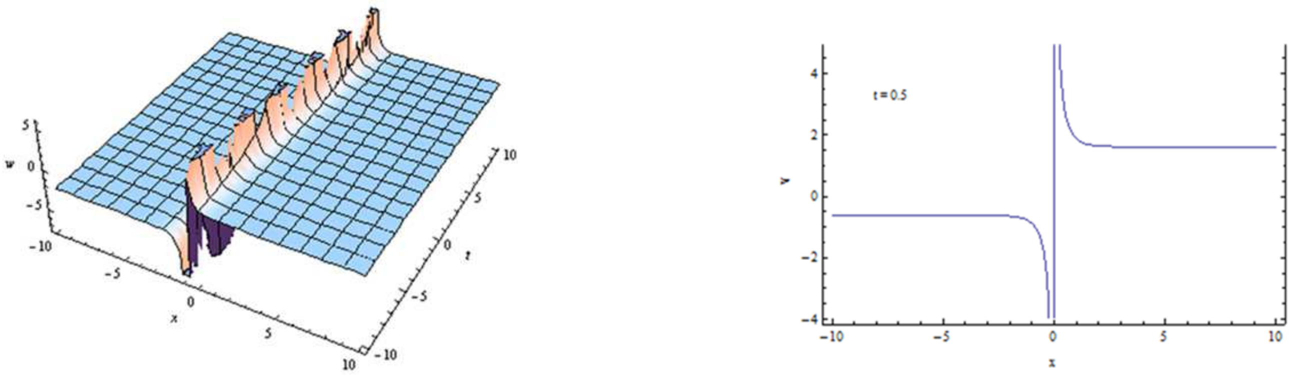

Fig. 3: Three dimensional and two dimensional graphics of soliton solution of $w(x,t)$ that given by (15) such that $A=2$, $B=1$, $C=0.5$, $k=0.1$, $c_1=c_2=0.1$, $b_1=0.5$.

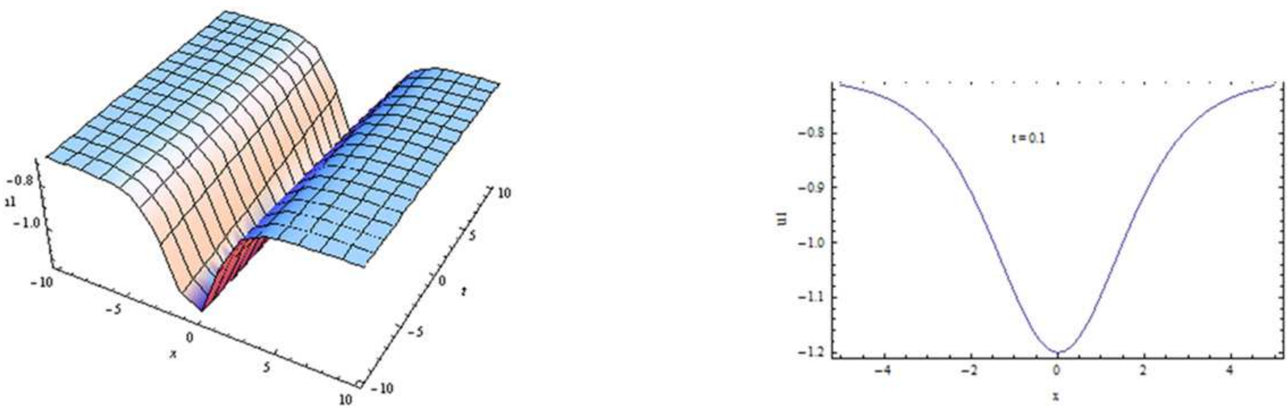

Fig. 4: Three dimensional and two dimensional graphics of the soliton solution of $u(x,t)$ that given by (16) such that $A=2$, $B=C=1$, $k=0.05$, $c_1=c_2=0.1$.

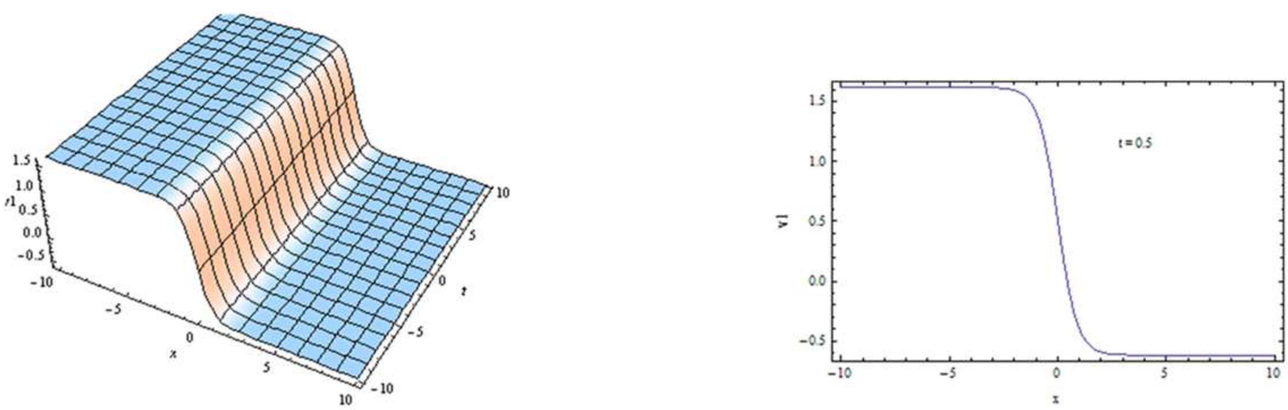

Fig. 5: Three dimensional and two dimensional graphics of the soliton solution of $v(x,t)$ that given by (17) such that $A=2$, $B=1$, $C=0.5$, $k=0.01$, $c_1=c_2=0.1$, $b_1=0.5$.

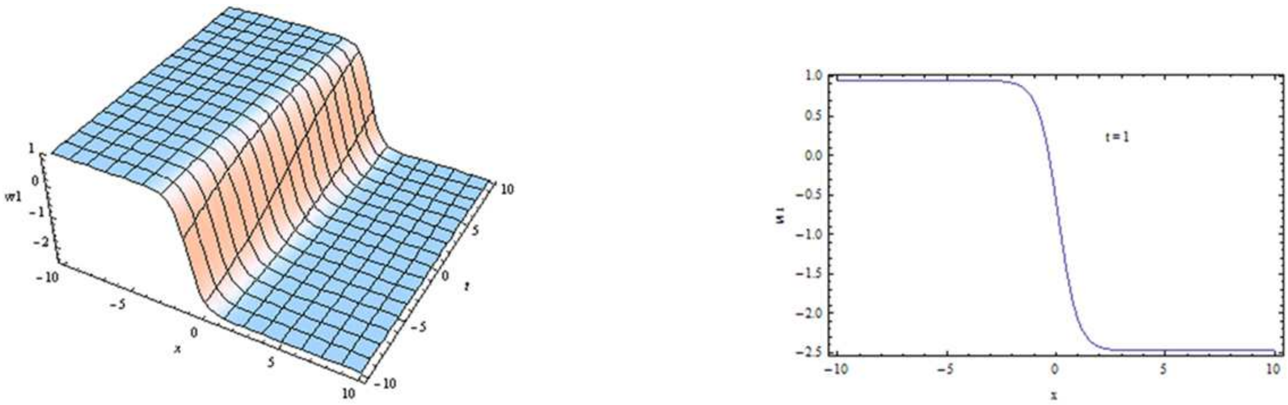

Fig. 6: Three dimensional and two dimensional graphics of the soliton solution of $w(x,t)$ that given by (18) such that $A=2$, $B=1$, $C=0.5$, $k=0.1$, $c_1=c_2=0.1$.

5 Conclusion

we used the improved $\left(\frac{G^I}{G}\right)$ −expansion method for acquiring several new exact solutions for the HSCKdV equation. We have acquired different types solutions which are denoted in terms of trigonometric, triangular, algebric, periodic wave and singular soliton solutions. Some of our reached solutions are new as our research from literature. Consequently, the improved $\left(\frac{G^I}{G}\right)$ −expansion method is crucial one to construct different types of the soliton solutions of the NPDEs and systems. In addition, this system with time-dependent coefficient and the stochactic perturbation terms will be reported in next works. Additionally, the numerical results that are obtained in this study are in conjuction with the analytical development here (see Figures 1-6).

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors have contributed to all parts of the article. All authors read and approved the final manuscript.

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