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Research Article

PI Controller Design for Second Order Plus Time Delay Plants

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ABSTRACT

Analytical design scheme of a Proportional Integral controllers for the stability and performance of time delay systems in the second order is presented in this paper. The method proposed in the study achieves general computation equations for mentioned systems. Inspired from the Bode's ideal transfer function characteristics, gain crossover frequency and phase margin specifications are considered for the system. Then, these specifications are used to obtain the parameters of the controller. Analytically derived formulas by the proposed method are tested on some existing second order plus time delay plants in the literature and the results are graphically given in the illustrative examples section. It is observed that the tuning method satisfies desired gain crossover frequency and phase margin specifications.

Keywords: *Second Order Plus Time Delay Plant, PI Controller, Bode's Ideal Transfer Function*.

İkinci Derece Zaman Gecikmeli Sistemler için PI Denetleyici Tasarımı

ÖZET

Bu çalışmada ikinci dereceden zaman gecikmeli sistemlerin kararlılığı ve performansı için oransal integral denetleyicilerin analitik tasarımı verilmiştir. Çalışmada önerilen yöntem söz konusu sistemler için genelleştirilmiş hesaplama eşitliklerini elde etmektedir. Bode'nin ideal transfer fonksiyonunun karakteristiğinden esinlenerek, kazanç kesim frekansı ve faz payı şartları göz önüne alınmıştır. Daha sonra, bu şartlar denetleyicinin parametrelerini elde etmek için kullanılmıştır. Önerilen yöntemle analitik olarak türetilen formüller literatürde var olan bazı ikinci derece zaman gecikmeli sistemler üzerinde test edilmiş ve sonuçlar grafiksel olarak açıklayıcı örnekler bölümünde sergilenmiştir. Parametre ayarlama yönteminin istenen kazanç kesim frekansı ve faz payı şartlarını sağladığı gözlemlenmiştir.

Anahtar Kelimeler: *İkinci Derece Zaman Gecikmeli Sistem, Oransal Integral Denetleyici, Bode'nin İdeal Transfer Fonksiyonu.*

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I. INTRODUCTION

Due to its efficient performance, Proportional Integral (PI) controllers are widely used in real world processes. A PI controller is a variation of the well-known proportional integral derivative (PID) controller which do not have the derivative operator. The literature has numerous studies implemented on PI controllers. For instance, as a similar study, tuning of PI controllers based on gain and phase margin specifications is presented in [1]. A comparative analysis of fuzzy logic and PI speed control is studied in [2]. A paper related to self-tuning fuzzy PI control is given in [3]. Miao et al. optimized PI parameters in [4]. A comparative study of cascaded PI-PD controllers applied on a coupled tank system can be found in [5] and Onat et al. presented a PI tuning method for first order plus time delay (FOPTD) plants in [6]. This list can be extended.

Second order plus time delay (SOPTD) transfer functions are used in modeling a considerable number of industrial processes in the literature. SOPTD models are widely used in chemistry [7], electronics [8], control processes [9] etc. Thus, control of these plants is a challenging area of research. Better design methods are studied inspired from this idea [10-13].

This paper intends to present a tuning method of PI controllers for the stability of plants described by SOPTD transfer functions. Parameters are tuned to satisfy desired phase crossover frequency and phase margin specifications. General components of a Bode diagram are reminded. The method gives generalized parameters of the PI controller for SOPTD plants. Efficiency of the proposed equations are tested in the illustrative examples section with existing plants in the literature and the results are graphically given. First example considers a SOPTD plant provided from a study related to closed-loop identification of SOPTD model of multivariable systems by optimization method [9]. The second example considers another SOPTD model provided from the same study.

Remaining parts of this paper is organized as follows. Section 2 gives remindful information about PI controllers and SOPTD plans. Section 3 presents the computation process of the PI controller. Illustrative examples clarify the process in section 4 and concluding remarks are given in section 5.

II. PI CONTROLLERS AND SOPTD PLANTS

This section gives brief information about transfer functions of a PI controller and a SOPTD plant. General properties of a Bode diagram is also reminded. Following equation denotes the general representation of a SOPTD plant.

$$P(s)=\frac{K}{(T_1s+1)(T_2s+1)}\mathrm{e}^{-Ls} \tag{1}$$

where, K is the gain, T is the time constant and L is the delay. Similarly, transfer function of a PI controller is given as follows.

$$C(s)=k_p+\frac{k_i}{s}=\frac{k_ps+k_i}{s} \tag{2}$$

Figure 1 shows the closed loop scheme of the system implemented in this paper.

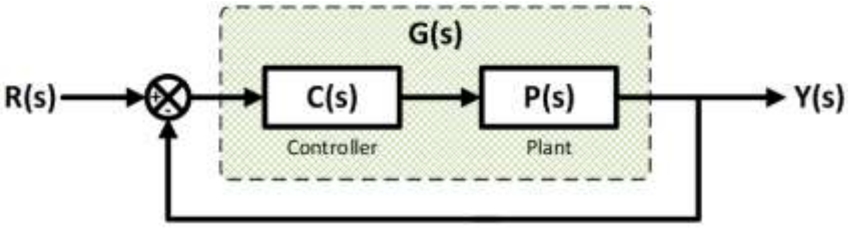

Figure 1. *Block diagram of the closed loop system.*

Thus, the system is,

$$G(s) = C(s)P(s) \tag{3}$$

where, $R(s)$ is the input signal and $Y(s)$ is the output signal. $P(s)$ is the transfer function of the SOPTD plant in Eq. 1 and $C(s)$ is the PI controller in Eq. 2. Figure 2 shows an example of Bode diagram of an open loop system.

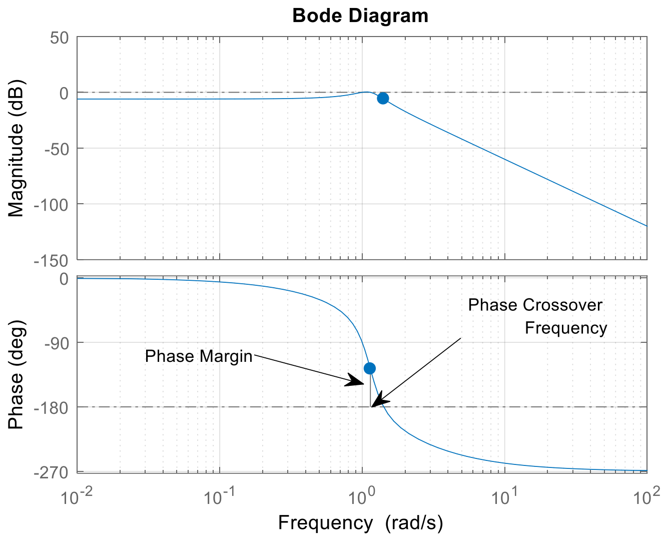

Figure 2. *Example of a Bode diagram.*

It would be useful to describe the components of a Bode diagram. The frequency value that the gain curve crosses the $0dB$ line is the gain crossover frequency and denoted as ω_c in this paper. Difference of the phase value with the -180 degrees line at ω_c is the phase margin and denoted as ϕ_m .

Now, the desired gain and phase specifications can be given.

III. Design Specifications of PI Controllers for SOPTD Plants

In order to analyze the system in the frequency domain, Laplace operator s should be replaced with $j\omega$ in Eq. 3 as,

$$G(j\omega) = C(j\omega)P(j\omega)\,. \tag{4}$$

With the help of Eq. 4, response of the system in the frequency domain can be written as,

$$\begin{aligned} P(j\omega) &= \frac{K}{\left(T_1(j\omega)+1\right)\left(T_2(j\omega)+1\right)}\mathrm{e}^{-L(j\omega)} = \frac{K}{(1+jT_1\omega)(1+jT_2\omega)}\mathrm{e}^{-jL\omega} \\ &= \left(\frac{K-KT_1T_2\omega^2}{\left(1+T_1^2\omega^2\right)\left(1+T_2^2\omega^2\right)} + j\left(-\frac{K\left(T_1+T_2\right)\omega}{\left(1+T_1^2\omega^2\right)\left(1+T_2^2\omega^2\right)}\right)\right)\mathrm{e}^{-jL\omega}\,. \\ &= \left|P(j\omega)\right|e^{j\angle P(j\omega)} = \sqrt{\frac{K^2}{\left(1+T_1^2\omega^2\right)\left(1+T_2^2\omega^2\right)}}\,e^{-j(\arctan\left(\frac{K\left(T_1+T_2\right)\omega}{K-KT_1T_2\omega^2}\right)+L\omega)} \end{aligned} \tag{5}$$

Similarly, frequency response of the PI controller is,

$$C(j\omega) = k_p + \frac{k_i}{j\omega} = k_p - \frac{jk_i}{\omega}\,. \tag{6}$$

Magnitude and phase of the SOPTD plant are obtained in the following way.

$$\left|P(j\omega)\right| = \sqrt{\frac{K^2}{\left(1+T_1^2\omega^2\right)\left(1+T_2^2\omega^2\right)}}\,, \tag{7}$$

$$\angle P(j\omega) = -\arctan\left(\frac{K\left(T_1+T_2\right)\omega}{K-KT_1T_2\omega^2}\right) - L\omega\,. \tag{8}$$

Likewise, magnitude and phase of the PI controller are,

$$\left|C(j\omega)\right| = \sqrt{k_p^{\,2} + \left(-\frac{k_i}{\omega}\right)^2} = \sqrt{\frac{k_i^{\,2} + k_p^{\,2}\omega^2}{\omega^2}}\,, \tag{9}$$

$$\angle C(j\omega) = \arctan\left(\frac{-\frac{k_i}{\omega}}{k_p}\right) = -\arctan\left(\frac{k_i}{k_p\omega}\right). \tag{10}$$

Therefore, absolute magnitude and phase of the system can be written as follows.

$$\left|G(j\omega)\right| = \left|C(j\omega)P(j\omega)\right| = \left|C(j\omega)\right|\left|P(j\omega)\right| \tag{11}$$

$$\angle G(j\omega) = \angle C(j\omega)P(j\omega) = \angle C(j\omega) + \angle P(j\omega) \tag{12}$$

Assume the gain crossover frequency as ω_c and the phase margin as ϕ_m. Then, following gain and phase specifications are desired to be satisfied.

$$\left|G(j\omega_c)\right| = 1 \tag{13}$$

$$\angle G(j\omega_c) = \phi_m - \pi \tag{14}$$

Considering Eq. 11 and Eq. 13, gain specification of the system is,

$$\left|G(j\omega_c)\right| = \left|C(j\omega_c)\right|\left|P(j\omega_c)\right| = \sqrt{\frac{k_i^2 + k_p^2\omega_c^2}{\omega_c^2}}\sqrt{\frac{K^2}{\left(1+T_1^2\omega_c^2\right)\left(1+T_2^2\omega_c^2\right)}} = 1\,. \tag{15}$$

Similarly, considering Eq. 12 and Eq. 14 phase margin specification of the system is,

$$\angle G(j\omega_c) = \angle C(j\omega_c) + \angle P(j\omega_c) = -\arctan\left(\frac{k_i}{k_p\omega_c}\right) - \arctan\left(\frac{K\left(T_1+T_2\right)\omega_c}{K - KT_1T_2\omega_c^2}\right) - L\omega_c = \phi_m - \pi\,. \tag{16}$$

Together solution of Eq. 15 and Eq. 16 leads to the following parameters of the PI controller.

$$k_p = \pm\frac{\sqrt{1+T_1^2\omega_c^2}\sqrt{1+T_2^2\omega_c^2}}{K\sqrt{1+\tan\left(\phi_m + L\omega_c + \arctan\left(\frac{K\left(T_1+T_2\right)\omega_c}{K-KT_1T_2\omega_c^2}\right)\right)^2}}\,, \tag{17}$$

$$k_i = \mp\frac{\omega_c\sqrt{1+T_1^2\omega_c^2}\sqrt{1+T_2^2\omega_c^2}\tan\left(\phi_m + L\omega_c + \arctan\left(\frac{K\left(T_1+T_2\right)\omega_c}{K-KT_1T_2\omega_c^2}\right)\right)}{K\sqrt{1+\tan\left(\phi_m + L\omega_c + \arctan\left(\frac{K\left(T_1+T_2\right)\omega_c}{K-KT_1T_2\omega_c^2}\right)\right)^2}}\,. \tag{18}$$

Parameters of the PI controller in Eq. 17 and Eq. 18 can be used to obtain the PI controller to satisfy given gain and phase margin specifications denoted in Eq. 13 and Eq. 14. It would be useful to explain the results in illustrative examples.

IV. ILLUSTRATIVE EXAMPLES

This section gives two examples to clarify the given procedure.

Example 1: Consider the following SOPTD plant provided from [9].

$$P_1(s) = \frac{0.35}{(0.67s+1)(5s+1)} e^{-1.28s} \tag{19}$$

Desired phase crossover frequency is $\omega_c = 0.05 rad/\sec$ and the phase margin is $\phi_m = 45^o$. Replacing the unknown variables in Eq. 17 and Eq. 18, following PI controller is obtained.

$$C_1(s) = 7.94726 + \frac{0.81648}{s} \tag{20}$$

Bode diagram of the system $G_1(s) = C_1(s)P_1(s)$ is illustrated in Figure 3. It is clearly seen in the figure that the phase crossover frequency is tuned to be $\omega_c = 0.05 rad/\sec$ and the phase margin is $\phi_m = 45^o$. Thus, the proposed method is successfully implemented.

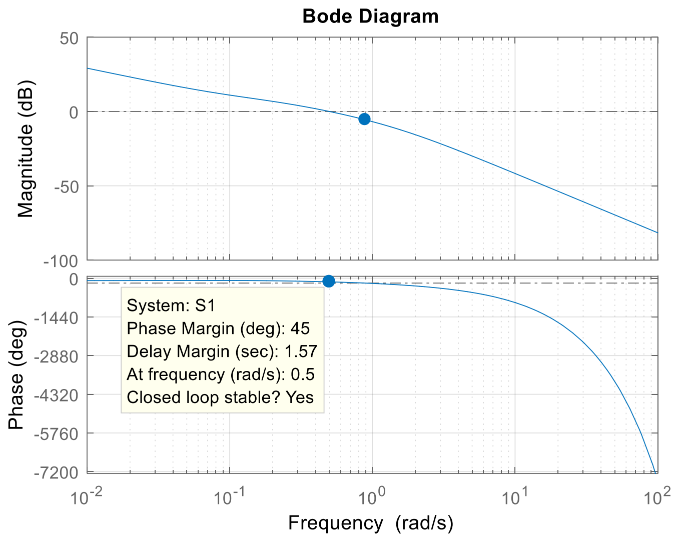

Figure 3. *Bode diagram of the system* $G_1(s) = C_1(s)P_1(s)$.

We can check the stability of the system with the step response of the closed-loop system given in Figure 4.

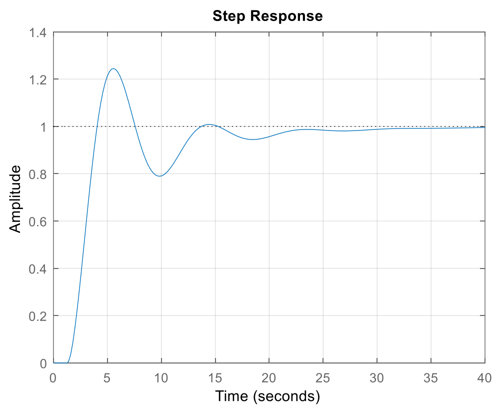

Figure 4. *Step response of the closed loop system in Example 1.*

The method can also be evaluated by considering varying phase margin values. Table 1 lists the parameters of the PI controller found for $\phi_m \in [30^o, 50^o]$ with increment steps of 5^o at $\omega_c = 10 rad/\sec$.

Table 1. *Parameters of the PI controller found for* $\phi_m \in [30^o, 50^o]$.

ϕ_m	k_p	k_i
30°	7.253823256105445	1.817113741189143
35°	7.542964064167476	1.494092897979227
40°	7.774698361134965	1.159701105657732
45°	7.947262509082995	0.816483287675509
50°	8.059343190648601	0.467051538867987

Bode diagrams of the systems with $P_1(s)$ and the 5 controllers listed in Table 1 are given in Figure 5.

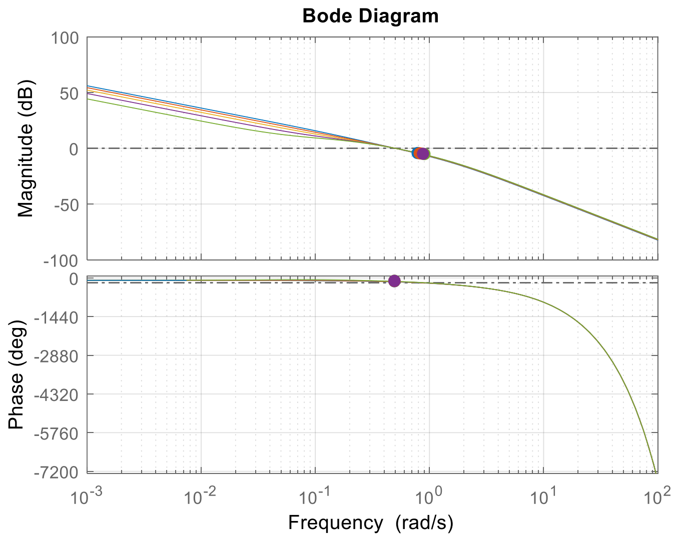

Figure 5. *Bode diagrams of the systems with* $P_1(s)$ *and the 5 controllers listed in Table 1.*

Similarly, stability of the systems with $P_1(s)$ and the 5 controllers can be checked with the step responses in Figure 6.

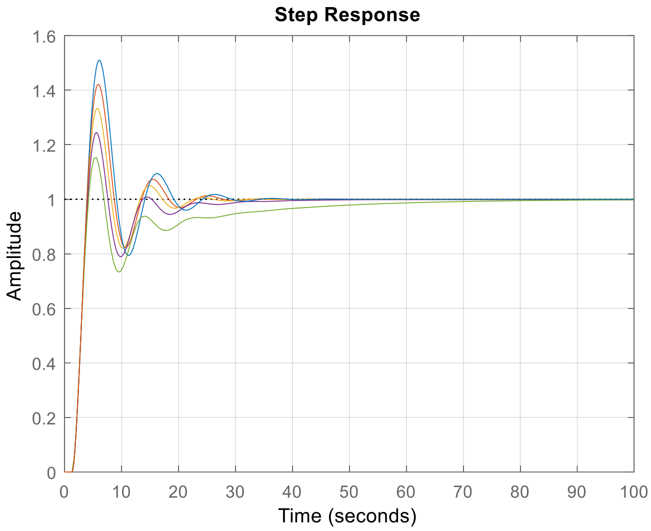

Figure 6. *Step responses of the systems with* $P_1(s)$ *and the 5 controllers.*

From this example, efficiency of the proposed method is clearly shown. It would be advantageous to apply the proposed method in another example.

Example 2: Consider the following SOPTD plant [9].

$$P_2(s) = \frac{-0.45}{(2.9889s+1)(5.7011s+1)}e^{-s} \tag{21}$$

Phase crossover frequency for this example is desired to be $\omega_c = 0.2 rad/\sec$. Phase margin is assumed to change in the interval $\phi_m \in [30^o, 60^o]$ with increment steps of 5^o. Table 2 shows the parameters of the PI controller obtained for this case.

Table 2. *Parameters of the PI controller found for* $\phi_m \in [30^o, 60^o]$.

ϕ_m	k_p	k_i
30°	-2.026862490010131	-0.672583721683918
35°	-2.312247335425856	-0.634693796411493
40°	-2.580034582445876	-0.591973468109785
45°	-2.828186208426009	-0.544747864272408
50°	-3.054813629654498	-0.493376400260164
55°	-3.258192074614415	-0.438250043933125
60°	-3.436773710536352	-0.379788340149143

Figure 7 shows the Bode diagrams and Figure 8 gives the step responses of the systems with $P_2(s)$ and the 7 controllers in Table 2.

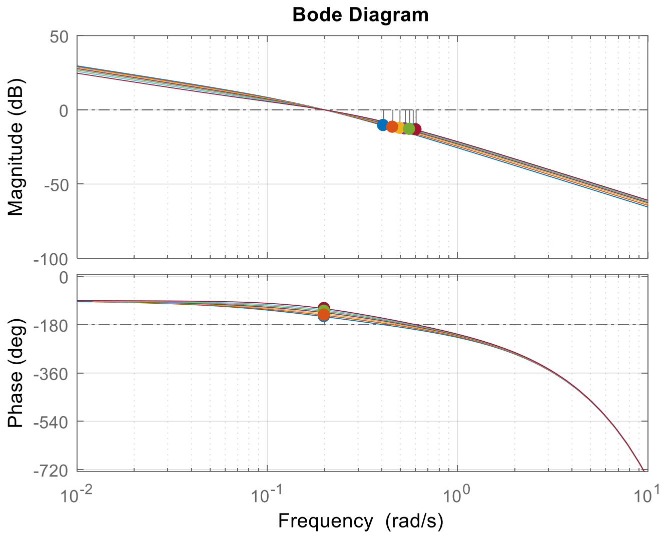

Figure 7. *Bode diagrams of the systems with* $P_2(s)$ *and the 7 controllers in Table 2.*

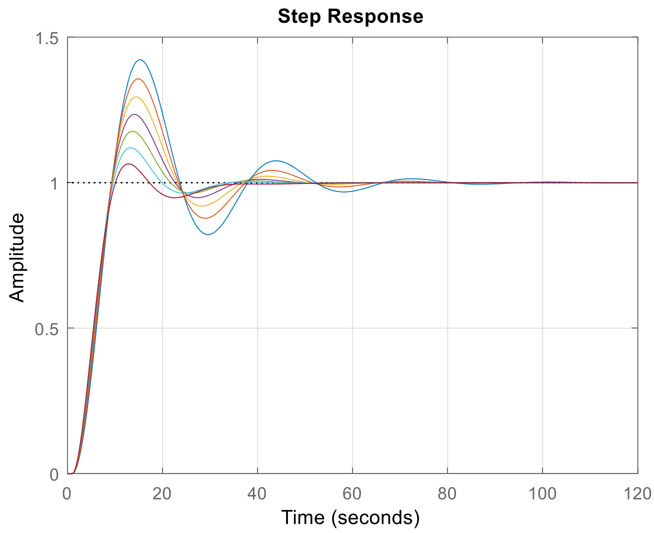

Figure 8. *Step responses of the systems with* $P_2(s)$ *and the 7 controllers in Table 2.*

Thus, the method is proved illustratively.

IV. CONCLUSION

This paper proposes a design scheme of proportional integral controllers for the stability and performance of second order plus time delay plants. Two specifications of gain and phase are considered in the paper. The aim is to achieve desired phase margin value by equalizing the gain of the system at the gain crossover frequency to 1. Gain and phase specifications for the system are inspired from an ideal system. The method analytically obtains general computation equations for mentioned systems. Proposed equations are applied on two different time-delayed plants in the second order and it is observed that the method successfully provides desired phase specifications. The method can be extended to obtain phase and gain margin properties simultaneously in future studies.

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