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RESEARCH ARTICLE

Interval type-2 fuzzy c-Control charts using ranking methods

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Abstract

Control charts are important for process or product because they provide information about the control situation of process and product. Because of this feature, control charts are used in many fields. Information about the product and/or process, which is under control or not, can be provided by looking the control charts. Fuzzy numbers are used to reduce information losses in operations with crisp numbers. In control charts applications, especially for qualitative control charts, the fuzzy set theory reduces the information losses and provide more flexible decision-making process. In the literature, there are some fuzzy control charts with type-1 fuzzy sets but there are few studies about fuzzy control charts regarding the cases where the data are expressed by type-2 fuzzy sets. The purpose of the study is to create an innovation using the ranking methods, which has not used for control charts in accessible literature, for the fuzzy control charts with interval type-2 fuzzy sets. The fuzzy results are compared with the crisp results. This study introduces ranking methods as new approach to generate interval type-2 fuzzy control charts, which is a different field.

Keywords: Interval type-2 trapezoidal fuzzy sets, Fuzzy control charts, c-Control charts, Nonconformities, Ranking methods

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1. Introduction

Control charts are one of the statistical process control methods that inform the process or product according to the upper control limit (UCL) and lower control limit (LCL) determined by the data. It allows to take precautions by recognizing abnormal conditions of the product or process. Similarly, the product or process indicates that the system is suitable when normal conditions occur. Beside these, the control charts are easy to understand because of their visual representation.

Control charts applied in various areas were first used in Bell Laboratories [19].

The control charts vary according to the fault type they are interested in. In general, it is divided qualitatively and quantitatively in the general sense. Since quantitative data are measurable, which are used for variable control charts, the data collection is much simpler than quantification. On the other hand, qualitative data are more relevant, which are used for attribute control charts, the data collection is more difficult and subjective.

In many areas, the fuzzy set theory, which was developed by Zadeh was first used in 1965, reduces the subjectivity and data loss of data collection. Since fuzzy numbers are more flexible than crisp numbers, are used in many areas and thought to be beneficial because they can transform linguistic expressions into numbers. There are some studies about control charts which are considered as one of these areas [8, 6, 17, 1, 5, 7, 23, 15, 10].

It is the work of Wang and Raz, and Raz and Wang [23, 15], who first mentioned linguistic definitions for quality character. Then Kanagawa et al. Wang and Raz's work and talked about fuzzy probability and fuzzy membership approaches [10].

In his work, Asai mentioned that the control charts generated by categorical data and the fuzzy logic for them can be used [2]. Other works mention that fuzzy control charts can be created with the categorical data, Laviolette et al., and Woodall et al. [12, 24].

In the literature, Gülbay et al., Gülbay and Kahraman, Şentürk and Erginel have constructed control graphs for type-1 fuzzy numbers using the α-cut method [9, 7, 17].

Some studies in the literature are related to $\overline{X}$ fuzzy control charts. Faraz and Shapiro have studied $\overline{X} - S$ fuzzy control charts using LR fuzzy numbers [6]. Shu and Wu use triangular fuzzy numbers. With these numbers, the fuzzy $\overline{X} - R$ graphics are created separately and investigated whether the process is under control or not [20]. Similarly, Alaeddini et al. generate $\overline{X}$ fuzzy control charts using triangular fuzzy numbers [1].

Cheng has drawn control charts with distance to possibility and control charts with distance to necessity [5].

On the other hand, Gülbay and Kahraman defined fuzzy control limits and calculated fuzzy control limits for type-1 fuzzy numbers. This study allows for more flexible decision making such as "rather in control" and "rather out of control" for fuzzy control charts [8].

Finally, in the accessible literature, it can be mentioned that type-1 fuzzy numbers are used for process capability analysis. While Kaya and Kahraman use trapezoidal and triangular fuzzy numbers for process capability analysis, Senvar and Kahraman intend to provide flexibility for the process capability indices and thus use type-1 fuzzy sets [11, 18].

After talking about the type-1 fuzzy control charts, the type-2 control charts have gradually begun to enter the literature. Şentürk and Antucheviciene draw interval type-2 fuzzy control charts with using defuzzification method [16]. Our previous work has used defuzzification method for interval type-2 fuzzy control charts. In addition to defuzzification method, the likelihood method for interval type-2 fuzzy sets is used to create c-control charts [21]. Similarly, we use different ranking methods for interval type-2 fuzzy sets to generate fuzzy control charts [22].

In this study, ranking methods are used to draw interval type-2 fuzzy control charts based on interval type-2 trapezoidal fuzzy sets. Regarding the accessible literature, this is the first study to use the ranking methods for fuzzy control charts.

The study has been prepared in the following draft. The operations required to calculate the interval type-2 fuzzy control limits are described in Section 2. In Section 3, interval type-2 ranking

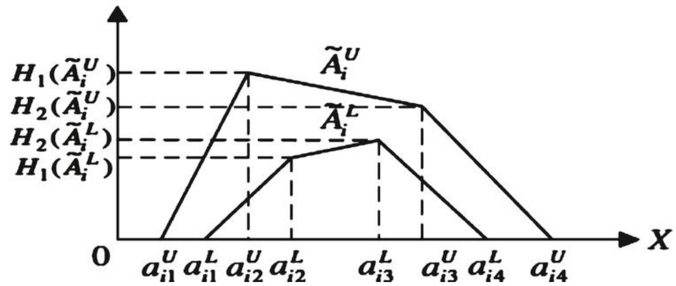

Figure 1. Illustration of interval type-2 trapezoidal fuzzy sets

methods to compare data are mentioned. In Section 4, the ranking methods which are mentioned will be adapted for interval type-2 fuzzy control charts. A numerical example will be given in Section 5. Section 6 is about conclusion and future research.

2. Interval Type-2 Fuzzy Sets

Zadeh has generated type-2 fuzzy numbers which means fuzzifing membership degrees. Therefore, he has provided more realistic data [25]. Mendel, in his work, has mentioned that type-2 fuzzy numbers are more useful in defining some linguistic expressions [13].

The general representation of type-2 fuzzy numbers is $\tilde{\tilde{A}} = \{(x,u), \mu_{\tilde{\tilde{A}}}(x,u) \mid \forall x \in X, \forall u \in J_x \subseteq [0,1], 0 \le \mu_{\tilde{\tilde{A}}}(x,u) \le 1\}$ where J_x is in interval $[0,1]$. When all $\mu_{\tilde{\tilde{A}}}(x,u) = 1$, $\tilde{\tilde{A}}$ is called an interval type-2 fuzzy set [3].

In this study, we use interval type-2 trapezoidal fuzzy sets as given below:

$$\widetilde{\widetilde{A}}_i = \left(\left(a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i4}^U; H_1\left(\widetilde{A_i^U}\right), H_2\left(\widetilde{A_i^U}\right)\right), \left(a_{i1}^L, a_{i2}^L, a_{i3}^L, a_{i4}^L; H_1\left(\widetilde{A_i^L}\right), H_2\left(\widetilde{A_i^L}\right)\right)\right)$$

where a_{ik}^m is the reference point of the interval type-2 fuzzy set $\widetilde{\widetilde{A}}_i, k = 1,2,3,4, m = U, L$ (U for upper membership function and L for lower membership function) and 1≤ i ≤ n. $H_j\left(\widetilde{A_i^m}\right) \in [0,1]$: denotes the membership value of the element $a_{i(j+1)}^m$, $j = 1,2, m = U, L$ and 1≤ i ≤ n. Figure 1 shows the illustration of trapezoidal interval type-2 fuzzy sets.

Addition, subtraction, multiplication, and multiplication with a scaler number required for control charts are given below. Eqs. 2.1-2.4 show these operations.

$$\begin{aligned}\widetilde{\widetilde{A_1}} + \widetilde{\widetilde{A_2}} = ((&a_{11}^U + a_{21}^U, a_{12}^U + a_{22}^U, a_{13}^U + a_{23}^U, a_{14}^U + a_{24}^U; \min(H_1(\widetilde{A_1^U}); H_1(\widetilde{A_2^U})),\\ &\min(H_2(\widetilde{A_1^U}); H_2(\widetilde{A_2^U}))), (a_{11}^L + a_{21}^L, a_{12}^L + a_{22}^L, a_{13}^L + a_{23}^L, a_{14}^L + a_{24}^L;\\ &\min(H_2(\widetilde{A_1^L}); H_2(\widetilde{A_2^L}))))\end{aligned} \tag{2.1}$$

$$\begin{aligned}\widetilde{\widetilde{A_1}} - \widetilde{\widetilde{A_2}} = ((&a_{11}^U - a_{24}^U, a_{12}^U - a_{23}^U, a_{13}^U - a_{22}^U, a_{14}^U - a_{21}^U; \min(H_1(\widetilde{A_1^U}); H_1(\widetilde{A_2^U})),\\ &\min(H_2(\widetilde{A_1^U}); H_2(\widetilde{A_2^U}))), (a_{11}^L - a_{24}^L, a_{12}^L - a_{23}^L, a_{13}^L - a_{22}^L, a_{14}^L - a_{21}^L;\\ &\min(H_2(\widetilde{A_1^L}); H_2(\widetilde{A_2^L}))))\end{aligned} \tag{2.2}$$

$$\begin{aligned}\widetilde{\widetilde{A_1}} * \widetilde{\widetilde{A_2}} = ((&a_{11}^U * a_{21}^U, a_{12}^U * a_{22}^U, a_{13}^U * a_{23}^U, a_{14}^U * a_{24}^U; \min(H_1(\widetilde{A_1^U}); H_1(\widetilde{A_2^U})),\\ &\min(H_2(\widetilde{A_1^U}); H_2(\widetilde{A_2^U}))), (a_{11}^L * a_{21}^L, a_{12}^L * a_{22}^L, a_{13}^L * a_{23}^L, a_{14}^L * a_{24}^L;\\ &\min(H_2(\widetilde{A_1^L}); H_2(\widetilde{A_2^L}))))\end{aligned} \tag{2.3}$$

$$k * \widetilde{\widetilde{A}}_i = ((k * a_{i1}^U, k * a_{i2}^U, k * a_{i3}^U, k * a_{i4}^U; H_1(\widetilde{A_i^U}), H_2(\widetilde{A_i^U})), (k * a_{i1}^L, k * a_{i2}^L, k * a_{i3}^L, k * a_{i4}^L; H_1(\widetilde{A_i^L}), H_2(\widetilde{A_i^L}))) \tag{2.4}$$

3. Ranking Methods for Interval Type-2 Fuzzy Sets

Since type-2 fuzzy sets are somehow difficult to calculate, usually interval type-2 fuzzy sets are preferred. Different methods are being developed for comparing fuzzy sets. Various comparison methods for interval type-2 fuzzy sets are available in the literature. One of these methods is ranking method.

In this study, two ranking methods are used; Chen et al.'s ranking method and Qin and Liu's ranking method, respectively.

3.1. Chen et al.'s ranking method. Chen et al. proposed ranking method for interval type-2 trapezoidal fuzzy sets. Ranking of A is shown in Eq.(3.1) [4].

$$\begin{aligned} RV\left(\widetilde{\widetilde{A}}_i\right) = &\left[\frac{(a_{i1}^U + K_i) + (a_{i4}^U + K_i)}{2} + \frac{\left(H_1\left(A_i^U\right) + H_2\left(A_i^U\right) + H_1\left(A_i^L\right) + H_2\left(A_i^L\right)\right)}{4}\right] \\ &* \left\{\left[\frac{(a_{i1}^U + K_i) + (a_{i2}^U + K_i) + (a_{i3}^U + K_i) + (a_{i4}^U + K_i)}{8}\right]\right. \\ &+ \left.\left[\frac{(a_{i1}^L + K_i) + (a_{i2}^L + K_i) + (a_{i3}^L + K_i) + (a_{i4}^L + K_i)}{8}\right]\right\} \end{aligned} \tag{3.1}$$

K_i value is the value that makes the numbers positive, The K_i value will be evaluated as 0, since there will be no negative data for the control charts, in this study.

3.2. Qin and Liu's ranking method. Qin and Liu proposed ranking method for type-2 fuzzy sets. Ranking of A is shown in Eq.(3.2) [14].

$$\begin{aligned} \text{Rank}(A) = &\sum_{i=1}^{3}(M_i(A^U) + M_i(A^L)) - \frac{1}{4}\sum_{i=1}^{3}(S_i(A^U) + S_i(A^L)) \\ &+ \sum_{i=1}^{2}(H_i(A^U) + H_i(A^L)) \end{aligned} \tag{3.2}$$

where $M_i\left(A^j\right) = \left(a_{1p}^j + a_{1p+1}^j\right)/2$ and $S_i\left(A^j\right) = \sqrt{\frac{1}{2}\sum_{k=i}^{i+1}\left(\left(a_{1i}^j - \frac{1}{2}\sum_{k=i}^{i+1}a_{1i}^j\right)\right)^2}$, i=1,2,3.

4. C-Control Charts with Interval Type-2 Fuzzy Sets

Linguistic data can be represented by fuzzy sets. For this reason, there are lots of applications in many areas. Control charts that can be regarded as one of these fields. It is suitable for control charts, especially attribute control charts, because of the data are linguistic and categorical.

The attribute control charts are separated by the fraction rejected as nonconforming to the specifications, number of nonconforming items, number of nonconformities and number of nonconformities per unit. In this study, we have been working on the fuzzifing of control charts dealing with the number of nonconformities referred to as c control charts. For classical c control charts, control limit are calculated as given below (see Eqs. (4.1)-(4.3)).

(4.1) $CL = \bar{c}$

(4.2) $LCL = \bar{c} - 3\sqrt{\bar{c}}$

(4.3) $UCL = \bar{c} + 3\sqrt{\bar{c}}$

where $\bar{c}$ is the mean of the nonconformities.

In this study, each sample point is expressed as a interval type-2 trapezoidal fuzzy numbers $\left(\left(a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i4}^U; H_1\left(\widetilde{A_i^U}\right), H_2\left(\widetilde{A_i^U}\right)\right), \left(a_{i1}^L, a_{i2}^L, a_{i3}^L, a_{i4}^L; H_1\left(\widetilde{A_i^L}\right), H_2\left(\widetilde{A_i^L}\right)\right)\right)$

The fuzzy control limits are then calculated using the operations of interval type-2 trapezoidal fuzzy sets. These equations are shown in Eqs. (4.4)-(4.6).

$$\begin{aligned}\widetilde{CL} &= \Big((\overline{a_1^U}, \overline{a_2^U}, \overline{a_3^U}, \overline{a_4^U}; \min(H_1(\widetilde{A_i^U})), \min(H_2(\widetilde{A_i^U}))),\\ &\quad (\overline{a_1^L}, \overline{a_2^L}, \overline{a_3^L}, \overline{a_4^L}; \min(H_1(\widetilde{A_i^L})), \min(H_2(\widetilde{A_i^L})))\Big)\\ &= \Bigg(\Big(\frac{\sum_{i=1}^m a_{i1}^U}{m}, \frac{\sum_{i=1}^m a_{i2}^U}{m}, \frac{\sum_{i=1}^m a_{i3}^U}{m}, \frac{\sum_{i=1}^m a_{i4}^U}{m}; \min(H_1(\widetilde{A_i^U})), \min(H_2(\widetilde{A_i^U}))\Big),\\ &\quad \Big(\frac{\sum_{i=1}^m a_{i1}^L}{m}, \frac{\sum_{i=1}^m a_{i2}^L}{m}, \frac{\sum_{i=1}^m a_{i3}^L}{m}, \frac{\sum_{i=1}^m a_{i4}^L}{m}; \min(H_1(\widetilde{A_i^L})), \min(H_2(\widetilde{A_i^L}))\Big)\Bigg)\end{aligned} \tag{4.4}$$

$$\begin{aligned}\widetilde{UCL} &= \Bigg(\Big(\overline{a_1^U} + 3\sqrt{\overline{a_1^U}}, \overline{a_2^U} + 3\sqrt{\overline{a_2^U}}, \overline{a_3^U} + 3\sqrt{\overline{a_3^U}}, \overline{a_4^U} + 3\sqrt{\overline{a_4^U}}; \min(H_1(\widetilde{A_i^U})), \min(H_2(\widetilde{A_i^U}))\Big),\\ &\quad \Big(\overline{a_1^L} + 3\sqrt{\overline{a_1^L}}, \overline{a_2^L} + 3\sqrt{\overline{a_2^L}}, \overline{a_3^L} + 3\sqrt{\overline{a_3^L}}, \overline{a_4^L} + 3\sqrt{\overline{a_4^L}}; \min(H_1(\widetilde{A_i^L})), \min(H_2(\widetilde{A_i^L}))\Big)\Bigg)\end{aligned} \tag{4.5}$$

$$\begin{aligned}\widetilde{LCL} &= \Bigg(\Big(\overline{a_1^U} - 3\sqrt{\overline{a_4^U}}, \overline{a_2^U} - 3\sqrt{\overline{a_3^U}}, \overline{a_3^U} - 3\sqrt{\overline{a_2^U}}, \overline{a_4^U} - 3\sqrt{\overline{a_1^U}}; \min(H_1(\widetilde{A_i^U})), \min(H_2(\widetilde{A_i^U}))\Big),\\ &\quad \Big(\overline{a_1^L} - 3\sqrt{\overline{a_4^L}}, \overline{a_2^L} - 3\sqrt{\overline{a_3^L}}, \overline{a_3^L} - 3\sqrt{\overline{a_2^L}}, \overline{a_4^L} - 3\sqrt{\overline{a_1^L}}; \min(H_1(\widetilde{A_i^L})), \min(H_2(\widetilde{A_i^L}))\Big)\Bigg)\end{aligned} \tag{4.6}$$

After calculating interval type-2 control limits, the ranking methods mentioned in the previous section are used to compare limits based on the data.

5. Numerical Example

In this section, numerical example is given so that the methods can be better understood. Data for nonconformities are shown in Table 1, which shows crisp value of data, and Table 2, which shows linguistic values of data. Interval type-2 trapezoidal fuzzy sets are transformed from linguistic data and control limits are obtained as interval type-2 trapezoidal numbers using Eqs. (4.4)-(4.6).

$\widetilde{CL}$, $\widetilde{LCL}$ and $\widetilde{UCL}$ are calculated as interval type-2 trapezoidal fuzzy sets and these are given below.

$$\widetilde{CL} = ((18.13, 22.67, 26.93, 32.07; 0.63, 0.59), (19.37, 23.67, 26.00, 30.30; 0.48, 0.45))$$
$$\widetilde{LCL} = ((1.14, 7.10, 12.65, 19.29; 0.63, 0.59), (32.57, 38.26, 41.30, 46.81; 0.48, 0.45))$$
$$\widetilde{UCL} = ((30.91, 36.95, 42.50, 49.05; 0.63, 0.59), (32.57, 38.26, 41.30, 46.81; 0.48, 0.45))$$

Table 1. Crisp values for numerical example

Sample No	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Crisp Value	30	25	9	6	38	22	6	40	13	12	6	32	13	51	40
Sample No	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Crisp Value	40	41	39	18	28	34	18	30	25	36	18	10	32	23	8

Table 2. Linguistic values for numerical example

Sample No	Between	Approximately	Sample No	Between	Approximately
1		30	16		40
2	20-30		17	32-50	
3	5-12		18		39
4		6	19	15-21	
5		38	20		28
6	20-24		21	32-35	
7	4-8		22	10-25	
8	36-44		23		30
9	11-15		24		25
10	10-13		25	31-41	
11		6	26	10-25	
12		32	27	5-14	
13		13	28	28-35	
14	50-52		29	20-25	
15	38-41		30		8

5.1. Solving with Chen et al.'s ranking method. In this study, ranking methods are used to generate control charts. One of these ranking methods is proposed by Chen et al. [4]. We refer to this method as shown in Eq. (3.1) in Section 3.1.

Table 3 shows ranking values using Chen et al.'s ranking method for numerical example.

The ranking values are calculated for the control limits regarding with Chen et. al's method. These values are obtained as 638.16, 1612.44, and 107.44 for CL, UCL, and LCL, respectively. Based on the calculations, the control chart is drawn using Chen et al.'s method in Figure 2.

Referring to Figure 2 and Table 3, it can be said that sample points of 3, 4, 7, 11, 14, 17 and 30 are out of control, remainings are in control.

5.2. Solving with Qin and Liu's ranking method. The other ranking method, used in this study, is proposed by Qin and Liu [14]. We refer to this method as shown in Eq. (3.2) in Section 3.2.

Table 3. Ranking values using Chen et al.'s method

Sample No	Data	Sample No	Data
1	925.05	16	1568.48
2	639.04	17	1897.13
3	88.06	18	1486.46
4	37.42	19	556.2
5	1509.29	20	874.34
6	485.26	21	1205.72
7	55.98	22	554.95
8	1525.07	23	875.08
9	201	24	675.04
10	134.95	25	1284.57
11	46.54	26	309.87
12	1063.24	27	125.23
13	176.84	28	956.04
14	2357.84	29	539.74
15	1400.68	30	100.69

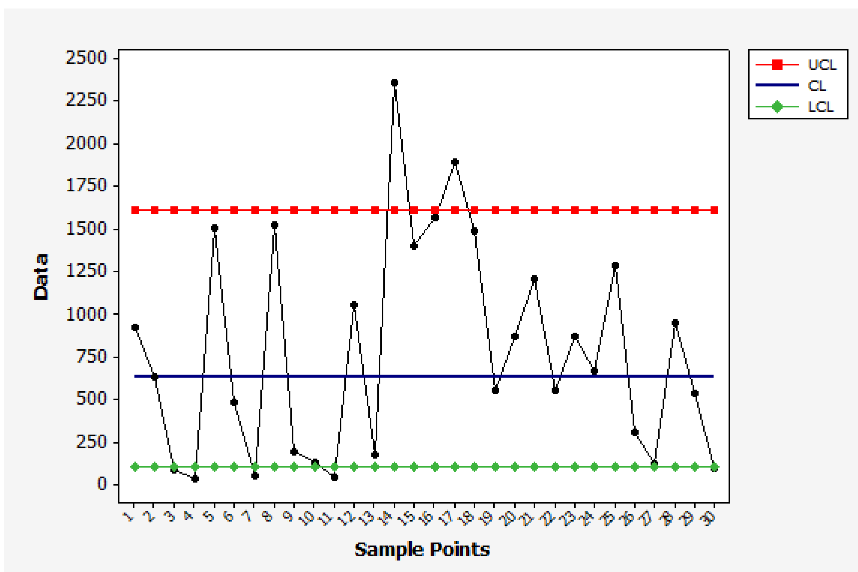

Figure 2. Control charts with Chen et al.'s method

Table 4 shows ranking values using Qin and Liu's ranking method for numerical example.

The ranking values are calculated for the control limits using Qin and Liu's method. These values are obtained as 148.24, 236.78, and 57.82 for CL, UCL, and LCL, respectively. The control chart is depicted using Qin and Liu's method in Figure 3.

Table 4. Ranking values using Qin and Liu's method

Sample No	Data	Sample No	Data
1	182.07	16	242.47
2	147.64	17	246.58
3	50.32	18	232.07
4	38.11	19	119.26
5	230.95	20	172.75
6	128.53	21	200.65
7	40.82	22	134.79
8	232.05	23	178.54
9	78.18	24	155.61
10	71.44	25	210.99
11	40.33	26	99.91
12	196.86	27	58.12
13	79.57	28	183.62
14	300	29	136.78
15	227.54	30	55.54

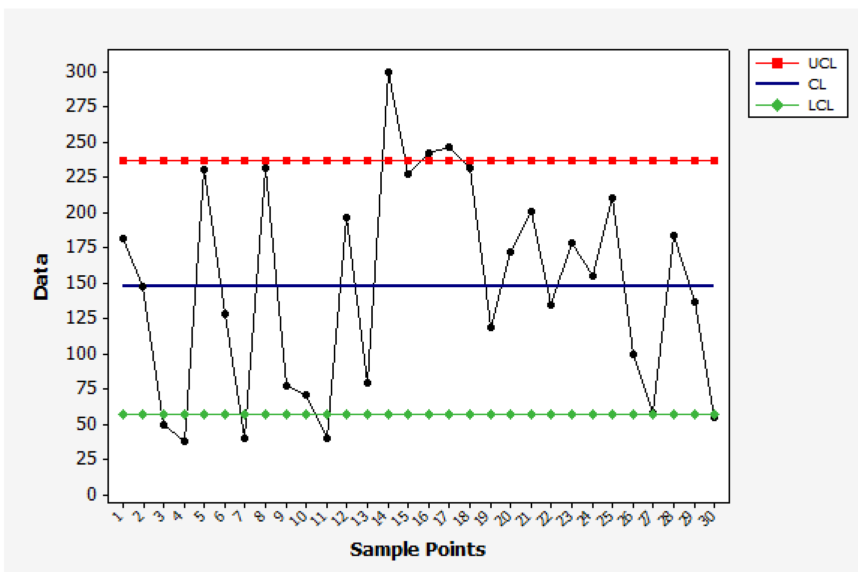

Figure 3. Control charts with Qin and Liu's method

The sample points of 3, 4, 7, 11, 14, 16, 17 and 30 are out of control, the remaining ones are in control when looking at the Table 4 and Figure 3.

Finally, the control charts generated with the crisp data are compared with the ranking methods' results. Classical c-control chart is drawn with using Minitab and shown in Figure 4.

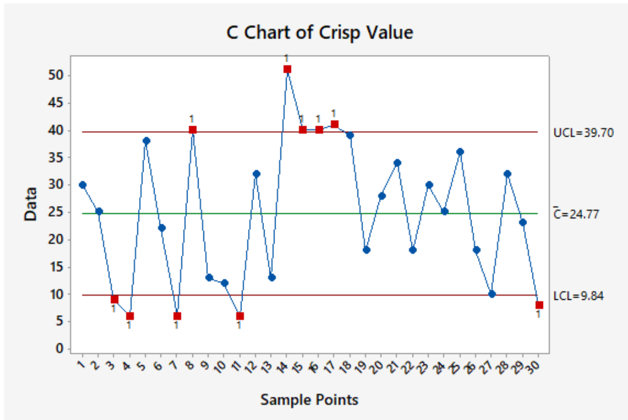

Figure 4. Control charts with classical method

When looking Fig. 4, the sample points of 3, 4, 7, 8, 11, 14-17 and 30 are out of control, while the others are in control.

The last stage of study is the comparison of methods with each other. Table 5 is about comparison of all methods. Chen et al.'s ranking methods' results are 90% similar to the results obtained from classical control chart while Qin and Liu's ranking methods' results are 93.9% similar to the results obtained from classical control chart.

Table 5. Comparisons of classical control chart with ranking methods

Sample No	Classical Control Chart	Chen et al.'s Ranking Method	Qin&Liu's Ranking Method	Sample No	Classical Control Chart	Chen et al.'s Ranking Method	Qin&Liu's Ranking Method
1	IC	IC	IC	16	OC	IC	OC
2	IC	IC	IC	17	OC	OC	OC
3	OC	OC	OC	18	IC	IC	IC
4	OC	OC	OC	19	IC	IC	IC
5	IC	IC	IC	20	IC	IC	IC
6	IC	IC	IC	21	IC	IC	IC
7	OC	OC	OC	22	IC	IC	IC
8	OC	IC	IC	23	IC	IC	IC
9	IC	IC	IC	24	IC	IC	IC
10	IC	IC	IC	25	IC	IC	IC
11	OC	OC	OC	26	IC	IC	IC
12	IC	IC	IC	27	IC	IC	IC
13	IC	IC	IC	28	IC	IC	IC
14	OC	OC	OC	29	IC	IC	IC
15	OC	IC	IC	30	OC	OC	OC

6. Conclusions

This article differs from the studies appeared in accessible literature in regarding with the fuzzy control charts considering the ranking methods. Also, there are very limited studies on interval type-2 fuzzy sets in the accessible literature. From these two perspectives, this study, for the first time, seeks to obtain c-control charts using ranking methods for interval type-2 fuzzy sets.

In the study, firstly, interval type-2 fuzzy control limits are set. After that, control charts are created with two different ranking methods. Then, the results obtained from ranking methods' are compared with the classical c-control chart, and the consistency of the results are investigated.

An important contribution of this study is that not only the control limits are calculated as interval type-2 fuzzy sets but also the ranking values. The other important point of this study is that it is the first study that tests ranking methods for fuzzy control charts.

For further studies, this research can be extended to include effects of different ranking methods for interval type-2 fuzzy sets.

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