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Examining the Effect of Problem Classification and Number Structures on Proportional Reasoning[1]

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ABSTRACT

The main purpose of this study is to examine sixth grade students' problem classification abilities on proportional reasoning problems and whether problem solving strategies and success rates change with problem type and number structure of the problems.

This study considered 165 randomly selected students of grade six studying in middle schools in a southern province of Turkey during academic year 2013-2014. A problem test including proportional (missing value, numerical comparison, and qualitative prediction and comparison problems) and non-proportional word problems was designed as a data collecting tool for the research. Number structures that involve within integer, between integer, both within and between integer and non-integer relations were also included in the problem test. The subjects of the study were divided into two groups. Half of the students solved the problem test first and then did the classification task (SC-condition), while the other half solved the problems in the reverse order (CS-condition). Strategies used in solving problems with different types and number structures were identified by evaluating students' answers on problems. Besides, comparisons among different categories Descriptive data analysis methods were used for this purpose.

Analysis show that the problem classification skills of sixth grade students are in average level, and problem type, and number structure of the problems affect the problem solving success.

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1. Introduction

Developing students' abilities to solve quantitative problems in daily life is one of the goals of mathematics education. In current classroom practice, word problems are predominantly used to teach and assess these abilities. The word problem is used as a vehicle to connect classroom practice with quantitative problems in real life (Hoogland, Bakker, Koning and Gravemeijer, 2012).

Students' primary experiences with mathematics are based on natural numbers. The initial years of primary education include learning of addition and subtraction, which is based on first-order relationships between countable objects. In the middle school years, students are introduced to rational numbers as well as natural numbers. During these years, students must make several major transitions in their mathematical thinking. A central change in thinking is required at this stage, which is a shift from natural number to rational numbers and from additive concepts to multiplicative concepts

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(McIntosh, 2013, p. 6). This is an important and difficult conceptual leap for students; mathematical experiences in elementary schools focus primarily on countable objects and first-order relationships. In proportional situations, students must replace additive reasoning and notions of change in absolute sense with multiplicative reasoning and notions of change in a relative sense (Baxter and Junker, 2001).

This second-order relationship is difficult for students because it requires more complicated mental structures than simple multiplication and division. Piaget considered the development of proportional reasoning to be a turning point in the development of higher order reasoning (Aleman, 2007, p. 22). In this sense, the proportional reasoning ability merits whatever time and effort that must be expended to assure its careful development (Lesh, Post, Behr, 1988; Lamon, 1993; Ben-Chaim, Fey, Fitzgerald, Benedetto, Miller, 1998; NCTM, 2000; Baykul, 2009).

Smith (2002) described the importance and complexity of proportionality in this way: "No area of elementary school mathematics is as mathematically rich, cognitively complicated, and difficult to teach as proportionality" (Johnson, 2010, p. 3). Many important concepts at the foundation level of elementary mathematics are often linked to proportional reasoning (NCTM, 2000, p. 212). Proportional reasoning is both capstone of elementary arithmetic and the cornerstone of all that is to follow. It therefore occupies a pivotal position in school mathematics programs (Lesh et al., 1988). Using proportional reasoning, students consolidate their knowledge of elementary school mathematics and build a foundation for high school mathematics. Students who fail to develop proportional reasoning are likely to encounter obstacles in understanding higher-level mathematics (Langrall and Swafford, 2000).

According to Lesh et al., (1988) proportional reasoning encompasses not only reasoning about the holistic relationship between two rational expressions but wider and more complex spectra of cognitive abilities which includes distinguishing proportional and non-proportional situations. Studies on proportional reasoning has shown that additive strategy is the most frequently used error strategy while students solve proportional problems (Karplus, Pulos, Stage, 1983; Tourniaire, 1986; Bart, Post, Behr, Lesh, 1994; Singh, 2000; Misailidou and Williams, 2003; Duatepe, Akkuş, Kayhan, 2005). Similarly, students give proportional responses to non-proportional problems (De Bock, Van Dooren, Janssens, Verschaffel, 2002; De Bock, De Bolle, Van Dooren, Janssens, Verschaffel, 2003; Duatepe et al. 2005; Van Dooren, De Bock, Vleugels, Verschaffel, 2010; Van Dooren, De Bock, Verschaffel, 2010). This shows that students have difficulty in distinguishing proportional and non-proportional problem statements. De Bock et al., (2002) point out that students focus on superficial aspects (key words and key phrases, physical configurations, etc.) and do not pay enough attention to mathematical structures while solving problems involving proportional reasoning. In line with such observations, a problem classification task with proportional and non-proportional word problems without the need to actually produce computational answers could be used to reveal students' cognitive behaviors underlying the proportional reasoning ability (Van Dooren et al., 2010).

Researchers have identified that the number structures of problems have various effects on proportional reasoning ability. Van Dooren et al., (2010) state that strategies used by students while solving problems are affected by the number structure of the problems. Steinthorsdottir (2006) states that the number structure influences the difficulty level of problems. Several studies have shown that students display a tendency to use multiplicative strategies when the presence of integer ratios and use additive strategies when the absence of integer ratios irrespective of proportional or non-proportional situations (Karplus et al., 1983; Tourniaire, Pulos, 1985; Cramer and Post, 1993; Steinthorsdottir, 2006; Van Dooren et al., 2010).

In line with such observations from available research on proportional reasoning as summarized above, it is considered that the examination of students' problem solving skills together with problem classification skills could be an alternative way to shed light on proportional reasoning. It was considered to be beneficial to study classification of proportional problems to gain broader aspect about proportional reasoning. Besides, the influence of problem type and number structure on students' use of strategies and success rate in solving proportional word problems was another area that needed to be studied. Deriving from these main aims of the study, this research laid out the problem statement as: "Examining sixth grade students' problem classification abilities on proportional reasoning problems and whether problem solving strategies and success rates change with problem type and number structure of the problems".

2. Methodology

Participants

A total of 165 (86 boys and 79 girls) students of grade six with various academic success rates from three different middle schools in a southern province of Turkey were randomly selected to participate in this study. The study was conducted during academic year 2013-2014.

Instrument

A Proportional Reasoning Problem Test (PRPT) was developed by the researcher in parallel with the objectives of renewed elementary mathematics program (MEB, 2013). The problem test contains 16 word problems with four different types of problems (4 missing value, 4 numerical comparison, 4 qualitative prediction and comparison, and 4 non-proportional). Each type of problem was designed with different number structures (1 within integer, 1 between integer, 1 both within and between integer, and 1 no integer ratios). Items were checked by two mathematics teachers, three mathematics educators and mathematicians to determine their appropriateness for the purpose of the test. These experts also checked whether the problems were correctly labeled for the appropriate type of proportional reasoning items. Two parallel problem sets with 16 problems in each were constructed, one of them to be used in problem solving task and the other to be used in problem classification task. The order of word problems appearing in the problem solving task was arranged in a way that no consecutive problem were of the same type in order to avoid establishing general judgments about the test. The number structures and the statements of these word problems are illustrated in Table 1.

Table 1. Sample Items from problem set

Problem Type	Number Structure	Statement
Missing Value	Between Integer Relation	In a car washing firm, 10 cars are washed in 2 hours. How many car can be washed in 7 hours?
Numerical Comparison	Within Integer Relation	Elvan and Elif were running around the track after school. Elvan finished 3 laps in 9 minutes. Elif finished 6 laps in 15 minutes. Which girl was running faster, or were their speeds equal? Elif, Elvan, Equal?
Non-proportional	Non-Integer Relation	Today, Burak becomes 5 years old and Serhat becomes 9 years old. When Burak is 12 years old, how old will Serhat be?
Qualitative Prediction and Comparison		Yesterday you shared some cookies with some friends. Today, you share fewer cookies with more friends. Will everyone get more, less, or the same amount as they received yesterday?

Procedure

The two tasks involving problem solving and problem classification were administered immediately after each other. Half of the pupils got the solution task before the classification task (SC-condition, n = 81). The other half got the solution task after the classification task (CS-condition, n = 84). In three schools, one class was assigned to the SC-condition and the other to the CS-condition.

In the problem solving task, students got one of the problem sets, containing 16 word problems. They were allowed as much time as necessary to complete the test and were requested justify their solutions.

In the classification task, students were given 16 word problem cards (the other problem set) and 16 envelopes. They were told that they did not need to solve the problems. Instead, they were asked to sort and group the problem cards in a way that the problems have something in common in their view.

Further, they were told to put each grouped set of problems in an envelope and to write what the word problems have in common. They could use as many envelopes as necessary. These instructions were kept somewhat vague to avoid directing students' classification.

Analysis

Problem Solving Task

Quantitative techniques were used to analyze data generated by PRPT. Responses to problems in the solution task were scored "1" if the answer was correct and "0" if the answer was not correct. Pure calculation errors (e.g., 16 × 2 = 36 instead of 32) were scored as correct when there is appropriate mathematical thinking. The highest score students could get from the PRPT was 16, since each item was scored 0 or 1. Four different proportional reasoning levels were established as following: Level 0; scores between 0 – 4, Level 1; scores between 5 – 8, Level 2; scores between 9 – 12 and Level 3; scores between 13 – 16. Descriptive statistics were used in the analysis of the quantitative data. Quantitative analysis primarily consisted of frequencies and percentages. All the analyses were done using SPSS 15.0 statistics program and are presented in tables. Strategies used for each item were coded. To check the internal consistency of the instrument, Kuder Richardson-20 co-efficient was calculated and found to be 0,786.

Problem Classification Task

Analysis of data from the problem classification task and the calculated scores for each pupil were made using the following steps:

- After the examination of all the envelopes put together by a student, the group with the largest number of missing value problems was identified and labeled as "M-group". The same procedure was applied to all four types of problems. Thus, four groups ("M-group" for missing value, "N-group" for numerical comparison, "Q-group" for qualitative prediction and comparison, and "X-group" for non-proportional) were formed for all the students.
- Each group (M, N, Q and X) got two scores, an uncorrected (Mu, Nu, Qu and Xu) score and a corrected (Mc, Nc, Qc and Xc) one. The uncorrected score for each group is the number of the problems of that type in that group. The corrected score is number of problems of that type of minus the number of other problems in that group. After calculating uncorrected and corrected scores for each group, total uncorrected (Tu) and corrected (Tc) scores were calculated for each student.
- When more than one group could be labeled as the same group, the group with the higher score was chosen.
- If any of the groups could not be distinguished, the scores for those groups were set to 0.

3. Findings

Problem Solving Task

Table 2 presents results of the proportional reasoning levels. Findings showed that the majority of the students participating in the study had average proportional reasoning (Level 1 and 2) while the rest had low (Level 0) and high (Level 3) proportional reasoning.

Table 2. Sixth Grade Students' Proportional Reasoning Levels

	F	%
Level 0	20	12,12
Level 1	45	27,27
Level 2	73	44,24
Level 3	27	16,36
Total	165	100

Table 3 presents students' performances on each type of the problems. Almost half of the students could solve 2 or 3 out of 4 problems in each type of the problems. Students showed the highest success in solving qualitative prediction and comparison problems and the lowest success in solving non-proportional problems. It can be said that students experienced more difficulty on non-proportional problems than proportional problems.

Table 3. Sixth Grade Students' Problem Solving Rates on Different Types of Problems

Problem Types	Missing Value		Numerical Comparison		Qualitative Prediction and Comparison		Non-Proportional	
Scores	f	%	f	%	f	%	f	%
0	24	14,54	17	10,30	10	6,06	21	12,72
1	27	16,36	26	15,75	13	7,87	49	29,69
2	33	20,00	58	35,15	41	24,84	59	35,75
3	46	27,87	41	24,84	43	26,06	30	18,18
4	35	21,21	23	13,93	58	35,15	6	3,63
Total	165	100	165	100	165	100	165	100

Table 4 shows the proportional reasoning levels for CS and SC conditions. These findings are similar to those with general proportional reasoning level described in Table 1. Majority of the students from both SC and CS conditions have average proportional reasoning (Level 1 and 2) while the rest have low (Level 0) and high (Level 3) proportional reasoning. Even though the performance of students in the SC-condition (=9.15) is slightly better than students in the CS-condition (=8.62), the problem solving success between these groups was not significantly different [$t(163)=.941$, $p > .05$].

Table 4. SC and CS Conditions' Proportional Reasoning Levels

	Level 0		Level 1		Level 2		Level 3		Total	
	f	%	f	%	f	%	f	%	f	%
SC-condition	6	7,41	23	28,40	40	49,38	12	14,81	81	100
CS-condition	14	16,66	22	26,19	33	39,29	15	17,86	84	100

Table 5 shows the students' performances on each type of the number structures. On an average, most of the scores were 2 out of 3 for each number structure. Students showed highest success in problems with both within and between integer ratios.

Table 5. Sixth Grade Students' Problem Solving Rates on Different Number Structures

Number Structure	Within Integer Ratio		Between Integer Ratio		Within and Between Integer Ratio		No Integer Ratio	
Scores	f	%	f	%	f	%	f	%
0	42	25,45	40	24,24	23	13,93	15	9,09
1	43	26,06	53	32,12	35	21,21	55	33,33
2	49	29,69	55	33,33	78	47,27	57	34,54
3	31	18,78	17	10,30	29	17,57	38	23,03
Total	165	100	165	100	165	100	165	100

Table 6 and 7 show the strategies used by the students to solve proportional (missing value and numerical comparison) problems and Table 8 shows the strategies used by students to solve non-proportional problems. Analysis of responses showed that students used five distinct solution strategies in missing value problems, six distinct solution strategies in numerical comparison problems, and six distinct solution strategies in non-proportional problems. Factor of change is the most frequently used strategy to solve missing value and numerical comparison problems; whereas the additive strategy is the most frequently used strategy to solve non-proportional problems. Findings also revealed the existence of additive strategy use in solving proportional problems and the existence of multiplicative strategy use in solving non-proportional problems.

Table 6. Strategies Used for Missing Value Problems

	f	%
Factor of Change	148	22,42
Unit Rate	85	12,88
Build-up	71	10,76
Cross Multiplication	8	1,21
Evidence of Proportional Reasoning	73	11,06

Table 7. Strategies Used for Numerical Comparison Problems

	f	%
Factor of Change	105	15,90
Build-up	8	1,21
Additive	50	7,57
Unit Rate	38	5,75
Common Factor or Multiple	25	3,78
Evidence of Proportional Reasoning	18	2,72

Table 8. Strategies Used for Non-Proportional Problems

	f	%
Additive	196	29,70
Evidence of Additive	33	5,00
Linear	66	10,00
Evidence of Linear	27	4,09
Multiplicative	154	23,33
Constant	38	5,76

Problem Classification Task

Table 9. Mean Uncorrected (Mu, Nu, Qu, Xu, Tu) and Corrected (Mc, Nc, Qc, Xc, Tc) Scores for the Classification Task

	Mu	Mc	Nu	Nc	Qu	Qc	Xu	Xc	Tu	Tc
SC-condition	1,90	0,81	1,73	1,12	2,32	1,37	1,74	1,27	7,69	4,58
CS-condition	2,05	0,54	1,76	0,96	2,95	1,26	1,65	0,98	8,42	3,74
Total	1,98	0,67	1,75	1,04	2,64	1,32	1,70	1,12	8,06	4,15

Table 9 provides an overview of the different scores on the classification task. First, this table reveals a mean Tu-score of 8,06 (out of a total of 16). Most pupils could correctly group 8 problems. In contrast with the high Tu score, the mean Tc-score is only 4,15, indicating that pupils frequently also included some other types of problems in their groups. Figure 1 shows the distribution of the total uncorrected and corrected scores. Most of the uncorrected scores concentrate between 6 and 12.

For the proportional problems, the uncorrected scores (Mu, Nu and Qu) are 1,98, 1,75 and 2,64, respectively. Students could group the qualitative prediction and comparison problems best. This situation could be related to the structure of this type of problem since these type of problems do not include any numerical values in their statements. For the non-proportional problems, the uncorrected score (Xu) is 1,70. These scores are lower than those for the proportional problems, which indicates that students not only have difficulty in solving non-proportional problems but also in classifying them.

For all four types problems the corrected scores (Mc, Nc, Qc and Xc) are lower than the uncorrected ones (Mu, Nu, Qu and Xu). This also shows that students included some other types of problems together with their correctly classified problems.

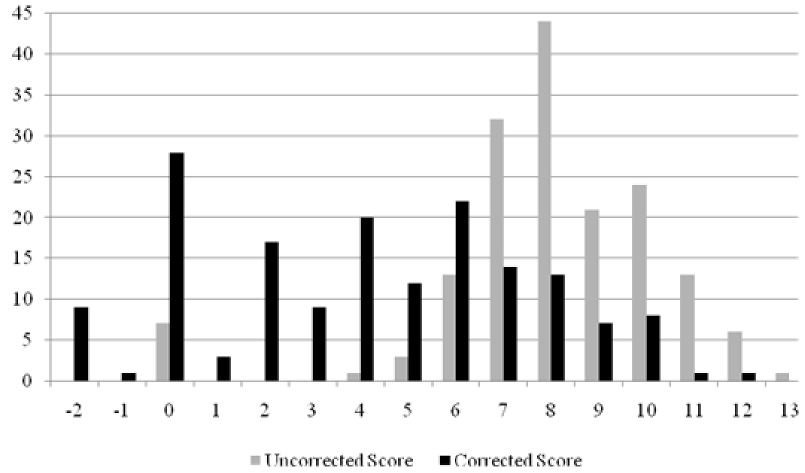

Figure 1. Frequency histogram of the total uncorrected and corrected scores

4. Discussion and Conclusion

This study was carried out to examine sixth grade students' problem classification abilities on proportional reasoning problems and whether problem solving strategies and success rates change with problem type and number structure of the problems.

The findings of the study revealed that the majority of the sixth grade students demonstrated average problem solving success on PRPT. Besides, the students showed better success on proportional (missing value, numerical comparison and qualitative comparison and prediction) word problems than non-proportional word problems.

Qualitative prediction and comparison word problems do not include numerical values. Thus, solution of this type of word problems requires comparisons that are independent of specific numerical values. Thinking qualitatively allows students to check the feasibility of answers and to establish appropriate parameters for problem situations. Therefore, this type of problems encourages students to use such approaches and improves their calculations and problem solving skills (Cramer and Post, 1993). Proportional reasoning encompasses not only quantitative reasoning but also qualitative reasoning. In this study, students showed the highest success rate in qualitative prediction and comparison word problems. This could be explained by Cramer and Post's (1993) views about qualitative reasoning.

It was seen that the success rate in the solution of missing value problems was lower than in other proportional problem types in this study. Structural similarity between non-proportional and missing value problems could be the reason for this situation. This similarity could mislead students not to notice the multiplicative nature of the missing value problems. As a result, students could use the same solution strategies that they used in non-proportional problems. In their study, Van Dooren et al., (2010) stated that students usually give non-multiplicative answers to non-integer missing value problems. It can be said that the results of this study are congruent with the results of the aforementioned.

When the high success rates in missing value and numerical comparison problems are compared, it is seen that students showed better performance in missing value problems than in numerical comparison problems. This observation is congruent with an earlier study, wherein, the students had more difficulty with numerical comparison problems than missing value problems (Singh, 2000).

The results of the study indicate that sixth grade students showed the lowest success rate in solving non-proportional problems. This result is similar to the findings of other studies in the available literature (Singh, 2000; De Bock et al., 2002; De Bock et al., 2003 Aladağ, 2009; Van Dooren et al., 2010; Van Dooren, et al., 2010).

The findings of the problem classification task could be interpreted as satisfactory performance by students' even though they were not familiar with this kind of task in their classroom environment. Besides, it can also be said that students could realized the mathematical structures underlying some problems and could classify these problems appropriately. It is seen that the corrected scores were lower than the uncorrected scores. This situation could be explained by the fact that students could classify some problems appropriately but could not realize the mathematical structures underlying some others. Similar situation is seen in the results of Van Dooren et al.'s (2010) study.

Students showed the highest performance in classifying qualitative comparison and prediction problems with respect to the proportional problems. This can be explained by the structure of this type of problems. Since this type of problems does not include numerical values, students could distinguish them from other problems. Students showed the lowest performance in classifying numerical comparison problems with respect to proportional problems. In this respect, students' problem solving and problem classifying skills show similarity.

Sixth grade students who participated in the study showed lower performance in classifying non-proportional problems than classifying proportional problems. Similarly, students had the lowest success rate in solving non-proportional problems in this study. Thus it can be concluded that the structure of this type of problems is the most difficult type of problem for students in terms of classifying and solving.

In the problem solving task, students who received the classification task after the problem solving task performed slightly better than those who started with the classification task. However, the problem solving success between these groups was not significantly different. This result is congruent with Silver's (1979) work but contradicts with that of Van Dooren et al.'s (2010). In this study, it can be said that the problem classification task had no effect on the problem solving performance. Problem classification is an unfamiliar task for students at this level and therefore it could be studied deeper in further research.

The findings of the study revealed that the number structure of problems affect the difficulty level of problems. With regard to proportional (missing value and numerical comparison) problems, it was seen that students could easily solve problems with integer relationships. However, they had difficulties in solving problems with non-integer relationships. These results are congruent with that of Steinthorsdottir (2006). The author stated that problems with integer relationships are the easiest problems for students and problems with non-integer relationships are the most difficult ones.

Analysis of the responses showed that the most frequently used strategy in solving missing value problems is factor of change. Even though cross multiplication is the most frequently used strategy in the studies in related literature (Cramer and Post, 1993; Duatep et al., 2005), it was used by very few students in this study. The result of this study is different in this sense.

Cross multiplication is commonly used in the solution of proportional reasoning problems in mathematics textbooks (Baykul, 2009, p. 342). Using this algorithm is not sufficient to reason proportionally. Proportional reasoning encompasses wider and more complex spectra of cognitive abilities, which include both mathematical and psychological dimensions (Lesh et al., 1988). The rare usage of cross multiplication in this study could be interpreted as a positive circumstance. Students who used this algorithm applied it to an entire problem set without actually comprehending the mathematical structures underlying the problems. As a result, using this rote algorithm led them to false answers. Singh, (2000) stated that the usage of memorized strategies in different types of problem situations has negative effects on proportional reasoning.

Results of the study further showed that the most frequently used strategy in non-proportional problems is the additive strategy. This result is congruent with Duatepe et al. (2005). It was also seen that students used multiplicative strategies to solve non-proportional problems. Structural similarity between non-proportional and missing value problems could cause students to give multiplicative answers to non-proportional problems. Similarly, Van Dooren, De Bock, Hessels, Janssens and, Verschaffel (2005), found that students are inclined to give proportional answers to non-proportional problems because they tend to focus on the superficial features of problems.

In summary, the research findings revealed the existence of additive strategy use in proportional problems and the existence of multiplicative strategy use in non-proportional problems. This shows that students have difficulty in distinguishing proportional and non-proportional problem statements. Students should be encouraged to realize the mathematical structures underlying problems so that they can be more successful in distinguishing proportional and non-proportional problems and develop better conceptual understandings. In this sense, students should be faced to both proportional and non-proportional problems with various number structures in order to overcome the overuse of proportionality and erroneous strategies depending on number structure and type of the problems. For further studies, it is recommended that clinical interviews with students should be conducted in order to explore why and how they make solution strategy choices. In order to develop problem solving and proportional reasoning skills, students should be faced with problem classification tasks in their classroom environment so that they can realize the mathematical structures underlying the problems. The problems mentioned above should be designed as rich tasks so that students can use multiple solution strategies.

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