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Dynamical Behaviors of Separated Homotopy Method Defined by Conformable Operator

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Abstract

In this paper, we consider some linear/nonlinear differential equations (DEs) containing conformable derivative operator (CDO). We obtain approximate solutions of these mentioned DEs in the form of infinite series which converges swiftly to its exact value by using separated homotopy method (SHM). Using the conformable operator in solutions of different types of DEs makes the solution steps are computable easily. As well as some theoretical results of the conformable operator, it has been used in modelling the DEs and describing certain problems such as engineering, material sciences, economic and other areas of application. In this context, the aim of this study is to apply the mentioned method to some illustrative linear/nonlinear problems and to solve them as mathematically. In addition, comparing the exact solutions with the obtained solutions is considered by the presentation of some plots. Therefore, the results of this study show the reliability and simplicity of the methods constructed with the conformable operator.

Keywords: *Conformable operator; separated homotopy method; approximate solution; nonlinear equation*
2010 Mathematics Subject Classification: *26A33; 35F31*

1. Introduction

In the past decades, scientists at various branches have devoted considerable effort to find robust and stable approximate methods for solving fractional differential equations (FPDEs) of physical, mathematical, economical, etc. interests. These illustrative studies and developments in applied sciences have found out that fractional calculus has a great importance in mathematical modelling due to memory effect. For this reason, approaching the real-life problems in terms of fractional calculus has facilitated to model and solve them easily and accurately. Therefore, it has been applied to a very wide area of science. For example, to physical and chemical problems [1, 2, 3, 4, 5, 6, 7, 8, 9], engineering sciences [10, 11, 12, 13], financial instruments [14, 16, 15], geosciences [17, 18], epidemic models [19, 20, 22, 25], analysis of biological models [21, 23, 24] etc. In this context, some important theoretical aspects of fractional calculus have been found out by some researchers [26, 27, 28]. Moreover, the mentioned approximate methods have been applied extensively to real-life problems by taking these theoretical aspects into consideration. For instance, approximate-analytical methods include homotopy analysis method (HAM) [29, 30, 31, 32], Adomian decomposition method (ADM) [33, 34, 35, 36], multivariate Padé approximation method [37, 38], reduced differential transform method (RDTM) [39], homotopy perturbation method (HPM) [40, 41, 42], modified homotopy perturbation method (MHPM) [43, 44, 45, 46] have been used.
In this article, SHM which is a separated version of the HPM has been considered to obtain solutions of FPDEs and the CDO has been used as fractional derivative operator. Khalil et al. [47] defined the CDO in 2014 and many authors have applied it to some FPDEs [48, 49, 50, 51, 52, 53, 54, 55, 56, 57].

2. Conformable Derivative Operator

Definition 2.1. *Given a function* $\upsilon : [0,\infty) \to \mathrm{R}$*. Then the conformable derivative of* υ *order* $\alpha \in (0,1]$ *and for all* $t>0$*, is defined by [47]*

$$\Pi_{*t}^{\alpha}(\upsilon)(t) = \lim_{\rho \to 0} \frac{\upsilon\left(t+\rho t^{1-\alpha}\right)-\upsilon(t)}{\rho}.$$

Definition 2.2. *Let* υ *be an* $n-$*times differentiable at* t*. Then the conformable derivative of* υ *order* α *is defined as [47, 58]:*

$$\Pi_{*t}^{\alpha}(\upsilon(t)) = \lim_{\rho \to 0} \frac{\upsilon^{(\lceil\alpha\rceil-1)}\left(t+\rho t^{(\lceil\alpha\rceil-\alpha)}\right)-\upsilon^{(\lceil\alpha\rceil-1)}(t)}{\rho},$$

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for all $t>0,\ \ \alpha\in(n,n+1]$.

Lemma 2.3. *Let* υ *be an* $n-$*times differentiable at* t. *Then* $\Pi_{*t}^{\alpha}(\upsilon(t))=t^{\lceil\alpha\rceil-\alpha}\upsilon^{\lceil\alpha\rceil}(t)$ *for all* $t>0,\ \alpha\in(n,n+1]$ *[47].*

3. Investigation of the Conformable Separated Homotopy Method

In this section we illustrate the solution strategies that are generated by separated homotopy method in conformable-type derivative operator (CSHM). Now we introduce a solution algorithm in an effective way for the general nonlinear PDEs. In this regard, we take the following nonlinear equation:

$$\Pi_{*t}^{\alpha}u(x,t)+L(u,u_x,u_{xx})+N(u,u_x,u_{xx})=\upsilon(x,t),\quad t>0, \tag{3.1}$$

where L represents a linear operator, N is a nonlinear operator, υ is a known function and Π_{*t}^{α}, $(n-1<\alpha\leq n)$ shows the conformable derivative of order α. We also have the initial conditions

$$u^{k}(x,0)=g_k(x),\quad k=0,1,\ldots,n-1. \tag{3.2}$$

In consideration of the SHM, we generate the following homotopy:

$$(1-\omega)\Pi_{*t}^{\alpha}u(x,t)+\omega\left[\Pi_{*t}^{\alpha}u(x,t)+L(u,u_x,u_{xx})+N(u,u_x,u_{xx})-\upsilon(x,t)\right]=0, \tag{3.3}$$

or

$$\Pi_{*t}^{\alpha}u(x,t)+\omega\left[L(u,u_x,u_{xx})+N(u,u_x,u_{xx})-\upsilon(x,t)\right]=0, \tag{3.4}$$

The separated form of the HPM proposed by Odibat and Momani [43] can be installed based on the supposition that the function $\upsilon(x,t)$ in Eq. (3.1) can be divided into parts,

$$\upsilon(x,t)=\sum_{n=0}^{\infty}\upsilon_n(x,t). \tag{3.5}$$

If we substitute (3.5) into (3.4), we have:

$$\Pi_{*t}^{\alpha}u(x,t)+\omega\left[L(u,u_x,u_{xx})+N(u,u_x,u_{xx})\right]=\sum_{n=0}^{\infty}\omega^n\upsilon_n(x,t), \tag{3.6}$$

where $\omega\in[0,1]$, which always changes from 0 to 1. The basic hypothesis is that the solution of Eq. (3.6) can be thought as a power series in ω:

$$u=u_0+\omega u_1+\omega^2u_2+\omega^3u_3+\cdots. \tag{3.7}$$

Therefore, we obtain

$$\begin{array}{ll}\omega^0:\Pi_{*t}^{\alpha}u_0=\upsilon_0(x,t), & u_0^{(k)}(x,0)=g_k(x),\\ \omega^1:\Pi_{*t}^{\alpha}u_1=-L(u_0)-N(u_0)+\upsilon_1(x,t), & u_1^{(k)}(x,0)=0,\\ \omega^2:\Pi_{*t}^{\alpha}u_2=-L(u_1)-N(u_0,u_1)+\upsilon_2(x,t), & u_2^{(k)}(x,0)=0,\\ \omega^3:\Pi_{*t}^{\alpha}u_3=-L(u_2)-N(u_0,u_1,u_2)+\upsilon_3(x,t), & u_3^{(k)}(x,0)=0,\\ \vdots & \end{array} \tag{3.8}$$

Then, by applying the operator I_{*t}^{α} on both side of Eq. (3.8), the first few terms of the CSHM solution can be given by

$$\begin{array}{l}u_0=\sum_{k=0}^{m-1}u^{(k)}(x,0)\frac{t^k}{k!}+I_{*t}^{\alpha}\left[\upsilon_0(x,t)\right],\\ u_1=-I_{*t}^{\alpha}\left[L(u_0)\right]-I_{*t}^{\alpha}\left[N(u_0)\right]+I_{*t}^{\alpha}\left[\upsilon_1(x,t)\right],\\ u_2=-I_{*t}^{\alpha}\left[L(u_1)\right]-I_{*t}^{\alpha}\left[N(u_0,u_1)\right]+I_{*t}^{\alpha}\left[\upsilon_2(x,t)\right],\\ u_3=-I_{*t}^{\alpha}\left[L(u_2)\right]-I_{*t}^{\alpha}\left[N(u_0,u_1,u_2)\right]+I_{*t}^{\alpha}\left[\upsilon_3(x,t)\right],\\ \vdots\end{array} \tag{3.9}$$

As the final step of the solution, we get:

$$u(x,t)=\sum_{n=0}^{\infty}u_n(x,t).$$

4. Implementation of the Method

In this section of the study, we show the effectiveness and practicableness of the suggested method by applying it to two different problems.

Example 4.1. *We consider the following linear differential equation [59]*

$$\Pi_{*t}^{\alpha}u+u_x=\frac{t^{1-\alpha}}{\Gamma(2-\alpha)}\sin(x)+t\cos(x),\ \ t>0,\ 0\leq x\leq 1,\ 0<\alpha\leq 1, \tag{4.1}$$

with the initial condition

$$u(x,0)=0, \tag{4.2}$$

and the boundary conditions

$$u(0,t)=0,\quad u(1,t)=t\sin(1). \tag{4.3}$$

In order to solve the Eq. (4.1) by using the CSHM, we firstly consider the initial conditions Eq. (4.2) and according to the homotopy Eq. (3.6) and setting $\upsilon_0(x,t)=\frac{t^{1-\alpha}}{\Gamma(2-\alpha)}\sin(x)$, $\upsilon_1(x,t)=t\cos(x)$ and $\upsilon_n(x,t)=0$, $n\geq 2$. The form of homotopy in Eq. (3.6) allows us to obtain the individual terms $u_0,u_1,u_2,\ldots$ in Eq. (3.7). Substituting (3.7) in (3.6) and collecting the terms with the same powers of ω, we have

$$\begin{array}{ll}\omega^0:\Pi_{*t}^{\alpha}u_0=\upsilon_0(x,t), & u_0(x,0)=0,\\ \omega^1:\Pi_{*t}^{\alpha}u_1=-(u_0)_x+\upsilon_1(x,t), & u_1(x,0)=0,\\ \omega^2:\Pi_{*t}^{\alpha}u_2=-(u_1)_x, & u_2(x,0)=0,\\ \omega^3:\Pi_{*t}^{\alpha}u_3=-(u_2)_x, & u_3(x,0)=0,\\ \vdots & \end{array} \tag{4.4}$$

Then, by applying the operator I_{*t}^{α} on both side of Eq. (4.4), the first few terms of the SHM solution can be given by

$$u_0(x,t)=\frac{t\sin(x)}{\Gamma(2-\alpha)},$$

$$u_1(x,t)=\frac{t^{\alpha+1}\cos(x)}{(\alpha+1)}-\frac{t^{\alpha+1}\cos(x)}{(\alpha+1)\Gamma(2-\alpha)},$$

$$u_2(x,t)=\frac{t^{2\alpha+1}\sin(x)}{(\alpha+1)(2\alpha+1)}-\frac{t^{2\alpha+1}\sin(x)}{(\alpha+1)(2\alpha+1)\Gamma(2-\alpha)},$$

$$u_3(x,t)=-\frac{t^{3\alpha+1}\cos(x)}{(\alpha+1)(2\alpha+1)(3\alpha+1)}+\frac{t^{3\alpha+1}\cos(x)}{(\alpha+1)(2\alpha+1)(3\alpha+1)\Gamma(2-\alpha)},$$

$$\vdots$$

In this tempo, the remainder of the components of the series can be achieved. Then the approximate solution of Eq. (4.1) is given by

$$u(x,t)=\frac{t\sin(x)}{\Gamma(2-\alpha)}+\frac{t^{\alpha+1}\cos(x)}{(\alpha+1)}-\frac{t^{\alpha+1}\cos(x)}{(\alpha+1)\Gamma(2-\alpha)}+\frac{t^{2\alpha+1}\sin(x)}{(\alpha+1)(2\alpha+1)}-\frac{t^{2\alpha+1}\sin(x)}{(\alpha+1)(2\alpha+1)\Gamma(2-\alpha)}$$

$$-\frac{t^{3\alpha+1}\cos(x)}{(\alpha+1)(2\alpha+1)(3\alpha+1)}+\frac{t^{3\alpha+1}\cos(x)}{(\alpha+1)(2\alpha+1)(3\alpha+1)\Gamma(2-\alpha)}+\cdots$$

Then the exact solution of the Eq. (4.1) subject to the initial condition Eq. (4.2) for $\alpha=1$, is obtained with the CSHM as

$$u(x,t)=t\sin(x).$$

Following Figure 4.1 shows the solutions obtained with the SHM and the exact solution for different cases of α.

Example 4.2. *Now let us consider the following nonlinear wave differential equation [60]*

$$\Pi_{*t}^{\alpha}u-uu_{tt}=1-\frac{1}{2}\left(x^2+t^2\right),\quad 0\leq x,t\leq 1,\quad 1<\alpha\leq 2, \tag{4.5}$$

with the initial conditions

$$u(0,t)=\frac{t^2}{2},\quad u_x(0,t)=0. \tag{4.6}$$

We solve the Eq. (4.5) by using the CSHM. If we consider Eq. (4.6) with the homotopy in Eq. (3.6) and setting $\upsilon_0(x,t)=1$, $\upsilon_1(x,t)=-\frac{1}{2}\left(x^2+t^2\right)$ and $\upsilon_n(x,t)=0$, $n\geq 2$. The form of homotopy Eq. (3.6) allows us to obtain the individual terms $u_0,u_1,u_2,\ldots$ in Eq. (3.7). Substituting (3.7) in (3.6) and by compiling the terms with the same powers of ω, we get

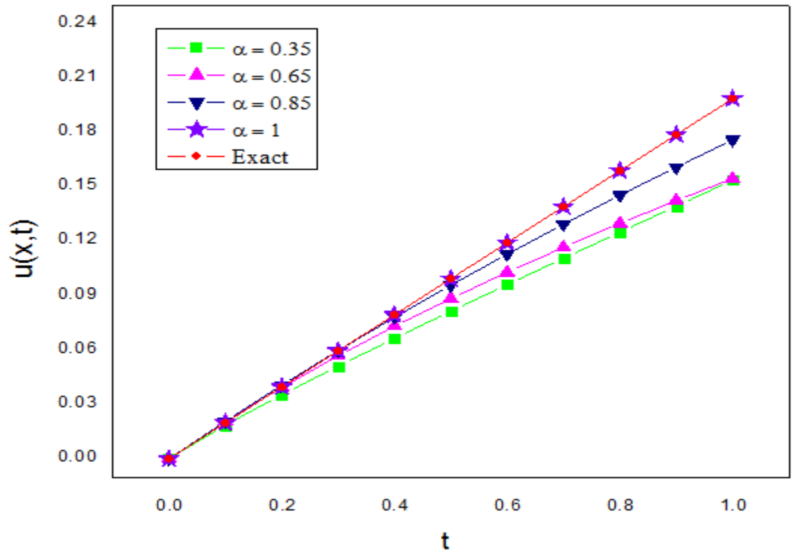

Figure 4.1: Comparison the CSHM and the exact solutions at $x = 0.2$ for values of α.

$$\begin{aligned}
&\omega^0 : \Pi^\alpha_{*t} u_0 = \upsilon_0(x,t), && u_0(x,0) = \tfrac{t^2}{2},\ (u_0)_x(x,0) = 0\\
&\omega^1 : \Pi^\alpha_{*t} u_1 = u_0(u_0)_{tt} + \upsilon_1(x,t), && u_1(x,0) = 0,\quad (u_1)_x(x,0) = 0\\
&\omega^2 : \Pi^\alpha_{*t} u_2 = u_0(u_1)_{tt} + u_1(u_0)_{tt}, && u_2(x,0) = 0,\quad (u_2)_x(x,0) = 0\\
&\vdots
\end{aligned} \tag{4.7}$$

By applying the steps as mentioned in Example 4.1 to both sides of Eq. (4.7), we have

$$\begin{aligned}
u_0(x,t) &= \tfrac{x^\alpha}{\alpha(\alpha-1)} + \tfrac{t^2}{2},\\
u_1(x,t) &= \tfrac{x^{2\alpha}}{2\alpha^2(\alpha-1)(2\alpha-1)} - \tfrac{x^{\alpha+2}}{2(\alpha+1)(\alpha+2)},\\
u_2(x,t) &= \tfrac{x^{3\alpha}}{6\alpha^3(\alpha-1)(2\alpha-1)(3\alpha-1)} - \tfrac{x^{2\alpha+2}}{2(\alpha+1)(\alpha+2)(2\alpha+1)(2\alpha+2)},\\
&\vdots
\end{aligned}$$

Then the approximate solution of Eq. (4.5) is given by

$$\begin{aligned}
u(x,t) &= \tfrac{x^\alpha}{\alpha(\alpha-1)} + \tfrac{t^2}{2} + \tfrac{x^{2\alpha}}{2\alpha^2(\alpha-1)(2\alpha-1)} - \tfrac{x^{\alpha+2}}{2(\alpha+1)(\alpha+2)} + \tfrac{x^{3\alpha}}{6\alpha^3(\alpha-1)(2\alpha-1)(3\alpha-1)}\\
&- \tfrac{x^{2\alpha+2}}{2(\alpha+1)(\alpha+2)(2\alpha+1)(2\alpha+2)} + \cdots
\end{aligned}$$

The exact solution of the Eq. (4.5) for $\alpha = 2$, is obtained with the CSHM as

$$u(x,t) = \frac{(x^2+t^2)}{2}.$$

In Figures 4.2 and 4.3, we represent the special case solutions of the problem (4.5) obtained with the suggested method and we explain the comparison between founded solutions with the exact solution. Also, the change of solution function can be seen in these figures.

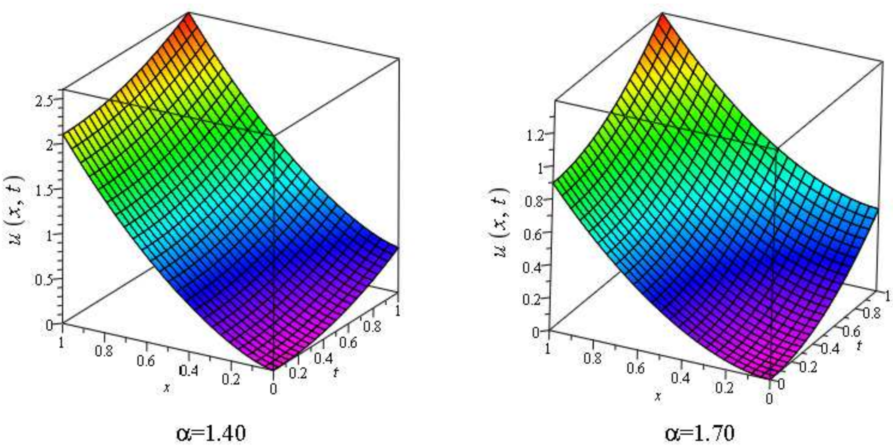

Figure 4.2: CSHM solutions with $\alpha = 1.4$ and $\alpha = 1.7$ for Example 4.2

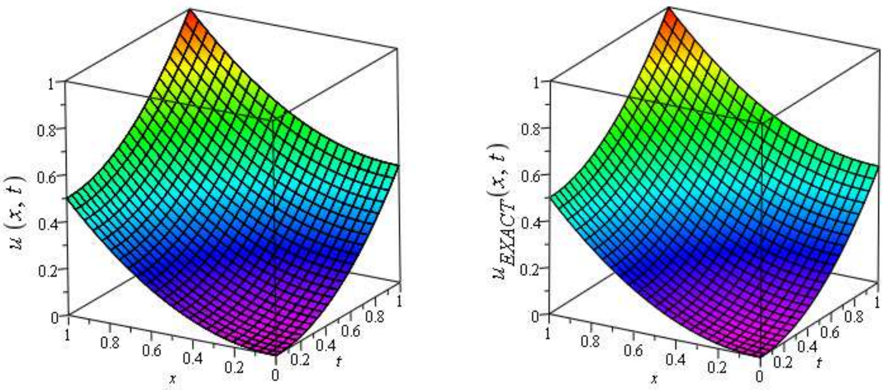

Figure 4.3: CSHM solution with $\alpha = 2$ and exact solution for Example 4.2

5. Conclusion

In this work, approximate-analytical solutions of some linear/nonlinear PDEs are obtained by using the SHM method considering the conformable derivative operator. The fundamental solutions for linear and nonlinear wave differential equation have been investigated by applying the suggested method. The results of numerical computations have been illustrated by the figures under the variation of order α, time value t and distance term x. Moreover, figures have been used to show the behaviours of the problem under the variation of problem parameters. By this study, it can be determined that the CSHM is an effective and accurate method which can compute the series easily in short time even for nonlinear PDEs and the results verified the validity, reliability, and accurateness of this mentioned method.

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