IJEPEM
Int. J. of Environmental Pollution & Environmental Modelling, Vol. 2(1): 20-27 (2019)
(Original Paper)

An Analysis of The Effect of Different and Equal Temperature Variation on Variable Cross-Sectioned Beams

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Abstract: In this work, the effect of equal or different temperature variations investigated for varying cross-sectioned plain element by employing matrix displacement methods. Necessary basic stiffness coefficients for stiffness matrices of frame elements and fixed end moments were obtained analytically and by using Romberg integration methods. Thus, use of table and charts is not needed. At the end of this work an example of varying cross-sectioned frame under equal and different temperature variation was solved by using a program which was prepared in BASIC language.
Keywords: non-prismatic, hunched, equal temperature, different temperature, stiffness, bars

INTRODUCTION

Elements of industrial constructions are usually considered to be as varying cross-sectional. Cross-section areas and moment of inertia are variable along the length of elements because of height and width. Stiffness matrix and fixed end forces due to the different or equal temperature which affect the element are needed for the structural analysis of the systems including variable cross-sectioned elements by using matrix displacement method. Because of the number of variation in cross-sections, tables and charts are not suitable for the computer Çakıroğlu and Tezcan[1,2].In the Literature on stiffness matrices and therefore on basic stiffness coefficients: Beam elements is divided into limited parts Tezcan and Vanderbilt and Funk et al.[2,3,4]. In linear variable cross-section elements Simpson digital integration was applied Kıral et al. [5], shear forces effect was considered Eisenberger[6], linear variable cross-sectioned elements investigated Brovn and Kosko[7,8], Elasticity modulus were regarded as variable in Fertis et al.[9], in Mezaini et al.[10] ,the linear elastic behaviour of variable sectioned frames was investigated using iso-parametric plane stress finite elements and it had been found that there are big differences between the obtain results and fixing moments in, stiffness and transport factors in the literature for the variable cross-section elements and the models of the classical frame analysis were proposed by investigating the ranks and sources of the errors. In Topçu[11] ,the basic stiffness coefficients of the variable section elements were given and the calculation of fixing moments for various methods were given using the analytical and numerical integration method. Karaduman[12], investigated the effect of heat on carrier systems consisting of variable bar sections. Behaviour of Non-prismatic beam vibration was investigated in Rute[13] . This study aimed to investigate the behaviour of non-prismatic beams with symmetrical parabolic haunches in Yüksel[14], Behaviour of reinforced concrete haunched beams subjected to cyclic shear loading was investigated in Archundia-Aranda et al.[15]. Investigation of performance of a minimum weight pre-stressed concrete beam adopting a non-prismatic section is aimed in Raju et al. [16]. Shear behaviour of non-prismatic steel reinforced concrete beams was investigated in Orr et al. [17]. Influence of the cross-section shape on the behaviour of SRG-confined prismatic unreinforced concrete specimens was investigated in Thermou et al., [18]. Ma and Chen [19] made a modal analysis of a rectangular variable cross section beam with multiple cracks under different temperatures. Also, Model Analysis of a Simply Supported Steel Beam with cracks was performed under different temperatures in Ma et al. [20]. However, any substantial investigation on the effect of equal or different temperature variation in the variable cross-sectioned beam elements was not found.

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BASIC PRINCIPLES

In this study, investigated beam element was selected to variable height and fixed width (Figure 1).

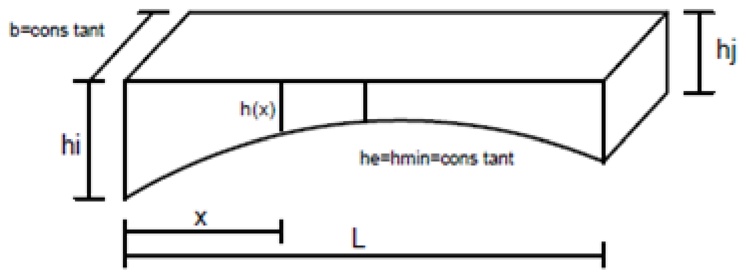

Figure 1. Variable cross-sectioned element

End displacements, d, end forces, p, and fixed end forces, f, are given in Figure 2. According to displacement method, equilibrium condition of an element in closed form is

{P}= [K]{d} + {f} (Equation 1)

K is stiffness matrix of the element and is symmetrical. Elements of k is named as stiffness coefficients.

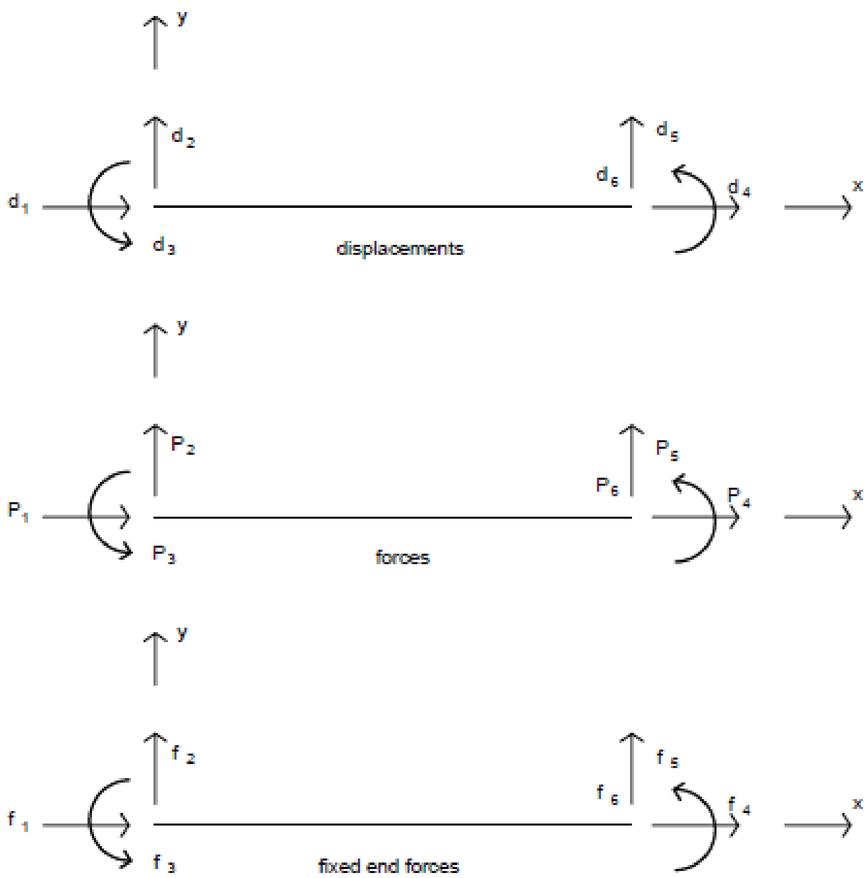

Figure 2. End displacements, end forces, and fixed end forces

These coefficients are varied with the geometry of carrier element according to the elastic parameters. The coefficients are $k_{ij} = k_{ji}$ and provide static equilibrium equations. Elements stiffness matrix K are as follows (Topçu[11]).

$$k_{11} = n_{ii}\, EA_0/L \;, \quad k_{14} = -k_{11} \;, \quad k_{22} = (m_{ii} + m_{jj} + 2m_{ij})EI_o/L^3$$
$$k_{23} = (m_{ii} + m_{ij})EI_o/L^2 \;, \quad k_{25} = -k_{22} \;, \quad k_{26} = (m_{jj} + m_{ij})EI_o/L^2$$
$$k_{33} = m_{ii}E\,I_o/L \;, \quad k_{35} = -k_{23} \;, \quad k_{36} = m_{ij}\, EI_o/L \qquad \text{(Equation 2)}$$
$$k_{44} = k_{11} \;, \quad k_{35} = -k_{22} \;, \quad k_{56} = -k_{26} \;, \quad k_{66} = m_{ij}\, EI_o/L$$
$$k_{12} = k_{13} = k_{15} = k_{16} = k_{24} = k_{34} = k_{45} = k_{46} = 0$$

E : Elastic modulus
$I_0 = I_{min}$: Minimum moment of inertia of an element
$A_0 = A_{min}$: Minimum cross-section area of an element
L : Length of an element

n_{ii}, m_{ii}, m_{ji} and m_{ij} are called as basic stiffness coefficients. When these coefficients are known, stiffness matrix of the element can be established, since all the other values are given with the size and material of the element. If the cross-section is constant along the length the n_{ii}= 1, $m_{ii} = m_{jj}$ = 4 and m_{ij} = 2

BASIC STIFFNESS COEFFICINETS

According to equation (1), 3rd column of K, f = 0 (unloaded element) and d_3 = 1 and while all the other displacements are zero, the forces at the end of the elements due to d_3 = 1 displacements are given in Figure 3.

m_{ii} and m_{ij} values, which are needed for the calculation of k_{33} and k_{63} stiffness terms, can be calculated with Mohr method (Topçu[11]).

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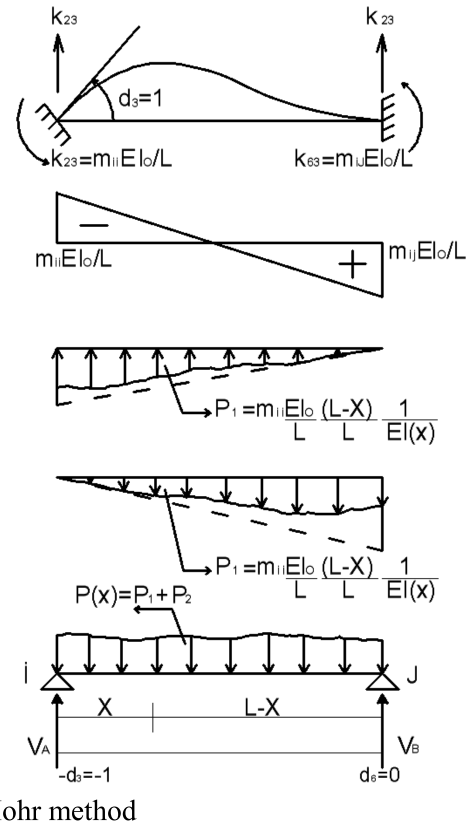

Figure 3. d_3 = 1 loading and Mohr method
[Extended load P1, needed for the <u>occurrence</u> of a unit rotation, d_3 = 1]

According to this fictitionus system

$\sum M_j = 0$

$$-V_A L - m_{ii}\frac{E.I_o}{L^2}\int_0^L \frac{(L-x)^2}{EI(x)}dx + m_{ij}\frac{E.I_o}{L^2}\int_0^L \frac{x.(L-x)}{EI(x)}dx = 0 \quad \text{(Equation 3)}$$

$\sum M_i = 0$

$$V_B L + m_{ii}\frac{E.I_o}{L^2}\int_0^L \frac{x.(L-x)}{EI(x)}dx - m_{ij}\frac{E.I_o}{L^2}\int_0^L \frac{x^2}{EI(x)}dx = 0 \quad \text{(Equation 4)}$$

integrals in equilibrium equations

$$I_1 = \int_0^L \frac{x^2}{I(x)}dx \;,\quad I_2 = \int_0^L \frac{1}{I(x)}dx \;,\quad I_3 = \int_0^L \frac{x}{I(x)}dx \quad \text{(Equation 5)}$$

if arranged as matrices

$$\begin{Bmatrix} V_A \\ V_B \end{Bmatrix} = \frac{I_o}{L^3}\begin{bmatrix} (-L^2 I_2 + 2LI_3 - I_1) & (LI_3 - I_1) \\ (-LI_3 + I_1) & (I_1) \end{bmatrix} . \begin{Bmatrix} m_{iI} \\ m_{Jj} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \quad \text{(Equation 6)}$$

is obtained. Assuming $d_6 = 1$, the equation system to obtain m_{jj} can be established

In similar manner $d_3 = 1$ and $d_6 = 1$ loading coefficient matrix, A in Mohr systems are

$$A\begin{Bmatrix} m_{ii} \\ m_{ij} \end{Bmatrix} = \begin{Bmatrix} -1 \\ 0 \end{Bmatrix} A\begin{Bmatrix} m_{iI} \\ m_{Jj} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \quad \text{(Equation 7)}$$

if these are solved

$$m_{ii} = -\frac{I_1}{\det(A)} \quad , \quad m_{ij} = -\frac{LI_3 - I_1}{\det(A)} \quad , \quad m_{jj} = -\frac{-L^2 I_2 + 2LI_3 - I_1}{\det(A)} \quad \text{(Equation 8)}$$

is obtained. Here,

$$\det(A) = -\frac{I_o}{L}(I_1 I_2 - {I_3}^2) \quad \text{(Equation 9)}$$

For the calculation of n_{ii} coefficient, f = 0 and $d_1 = 1$ and all the other displacements assumed to be zero in equation (1) then the normal force of $-n_{ii}EA_O/L$ is obtained. The extension of the element in length ($\Delta AL = d_2 - d_1 = -1$) is

$$\Delta L = -1 = -n_{ii}\frac{EA_o}{L}\int_0^L \frac{1}{EA(x)}dx \quad \text{(Equation 10)}$$

integral here is

$$I_4 = \int_0^L \frac{1}{A(x)}.dx \quad \text{(Equation 11)}$$

so

$$n_{ii} = \frac{L}{A_o}\frac{1}{I_o} \quad \text{(Equation 12)}$$

is obtained as it can be seen here, the calculation of basic stiffness coefficients n_{ii}, m_{ii}, m_{jj}, and m_{ij} reduced to solution of integrals (5) and (12).These integrals solved for constant and linear cross-section variations and for parabolic cross-section variations Romberg integration method is used and included in the computer program (Topçu[11]).

CALCULATION OF FIXED END FORCES IN THE PRESENCE OF TEMPERATURE VARIATIONS

Equal Temperature Variation

The extension of a beam under equal temperature variation can be written as

$$\Delta L_T = \alpha \,.\, L \,.\, (\Delta T) \qquad \text{(Equation 13)}$$

Here α : Temperature extension coefficient of material
L : Length of element
ΔT : Average temperature variation

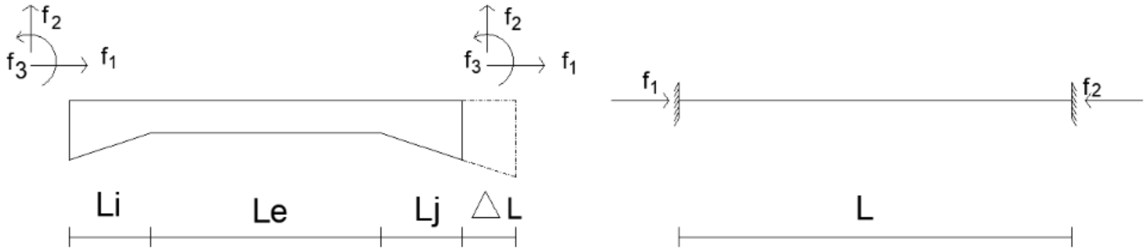

Figure 4. The state of equal temperature variation

When a variable cross-sectioned beam fixed from both end exposed to ΔT temperature variation, the extension at the both end due to axial force can be written as integral given below, since A_i (x) and A_j (x) are variable.

$$\Delta L = \frac{N}{E}\left[\int_0^u \frac{dx}{A_i(x)} + \int_u^{u+L_o} \frac{dx}{A_o} + \int_{u+L_o}^{L} \frac{dx}{A_j(x)}\right] \qquad \text{(Equation 14)}$$

Here the statement in brackets is the I_4 integral for the calculation of basic stiffness coefficient due to the normal force. Thus,

$$N = E.\alpha.(\Delta T).\frac{L}{I_4} \qquad \text{(Equation 15)}$$

and if arranged,

$$N = n_{ii}.E.A_o\alpha.(\Delta T) \qquad \text{(Equation 16)}$$

is obtained. There is no other force effecting other than this axial force the {f} vector to obtain fixed end forces due to equal temperature variation in the plane frame beam which has six degree of freedom, can be written as below

$$\{f\}_{xyz} = \begin{Bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \end{Bmatrix} = \begin{Bmatrix} n_{ii}.E.A_o.\alpha(\Delta T) \\ 0 \\ 0 \\ -n_{ii}.E.A_o.\alpha(\Delta T) \\ 0 \\ 0 \end{Bmatrix} \qquad \text{(Equation 17)}$$

Different Temperature Variation

In a free end element the deformation, due to the ΔT_1 heating in the upper face, the ΔT_2 heating in the lower face of a variable cross-sectioned frame element and forces occurred when the element fixed from both end are given in Figure 5.

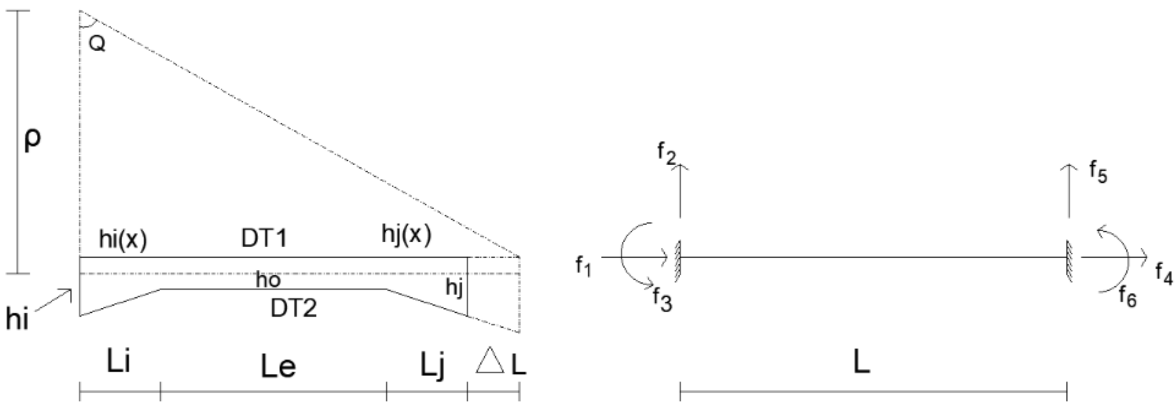

Figure 5. In different temperature variation

If the temperature variation is assumed to be linear along the height h (x) of thee element, extension in length ΔL is found as;

$$\Delta L = \alpha . L . (\Delta T_1 + \Delta T_2)/2 \qquad \text{(Equation 18)}$$

if N is drown from equation (14) and (18)

$$N = n_{ii} . E . A_o . \alpha . \frac{(\Delta T_1 + \Delta T_2)}{2} \qquad \text{(Equation 19)}$$

is obtained. From Figure 5, the cross-section rotation angle due to different heating of the upper and lower face is

$$\theta = \alpha . L . (\Delta T_2 - \Delta T_1)/h(x) \qquad \text{(Equation 20)}$$

if written in the form of,

$$M = \frac{E . I(x)}{L} . \theta \qquad \text{(Equation 21)}$$

and if the equation is arranged

$$M = \frac{E . \alpha}{L} (\Delta T_2 - \Delta T_1) \int_0^L \frac{I(x)}{h(x)} . dx \qquad \text{(Equation 22)}$$

is obtained. If the integral here is shown with S_{ii}, then it is

$$S_{ii} = \int_0^L \frac{I(x)}{h(x)} . dx = \int_0^{Li} \frac{I_i(x)}{h_i(x)} . dx + \int_{Li}^{Li+Lo} \frac{I_o}{h_o} . dx + \int_{Li+Lo}^{L} \frac{I_j(x)}{h_j(x)} . dx \qquad \text{(Equation 23)}$$

These integrals for different temperature variations are also calculated and included in the computer program.

The {f} vector which gives the fixed end forces in a fixed end beam exposed to different heating is obtained as

$$\{f\}_{xyz} \begin{Bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \end{Bmatrix} = \begin{Bmatrix} n_{ii} . E . A_o . \frac{\Delta T_1 + \Delta T_2}{2} \\ 0 \\ S_{ii} . E . \alpha . \frac{\Delta T_1 - \Delta T_2}{L} \\ -n_{ii} . E . A_o . \frac{\Delta T_1 + \Delta T_2}{2} \\ 0 \\ -S_{ii} . E . \alpha . \frac{\Delta T_1 - \Delta T_2}{L} \end{Bmatrix} \qquad \text{(Equation 24)}$$

Numerical application

Variable cross-sectioned frame given in figure 6, was solved and results are given for equal temperature variation on 20ºC and different temperature variation ΔT_1 = -5 ºC, ΔT_2= +15ºC

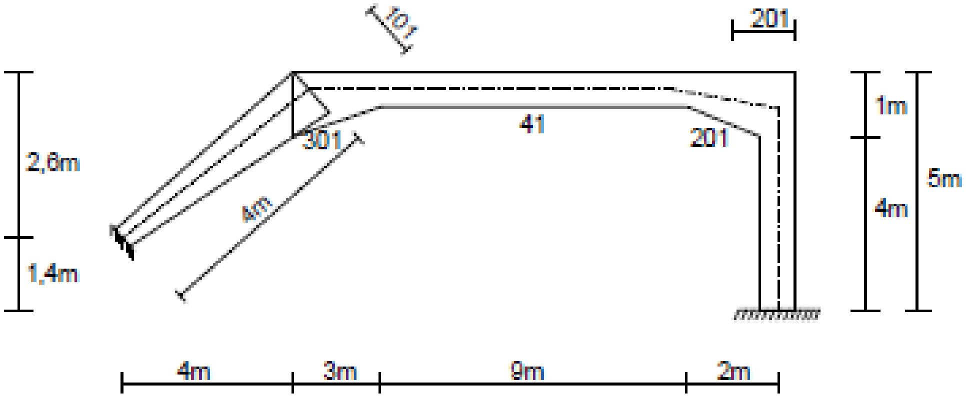

Figure 6. Example of variable cross-sectioned frame

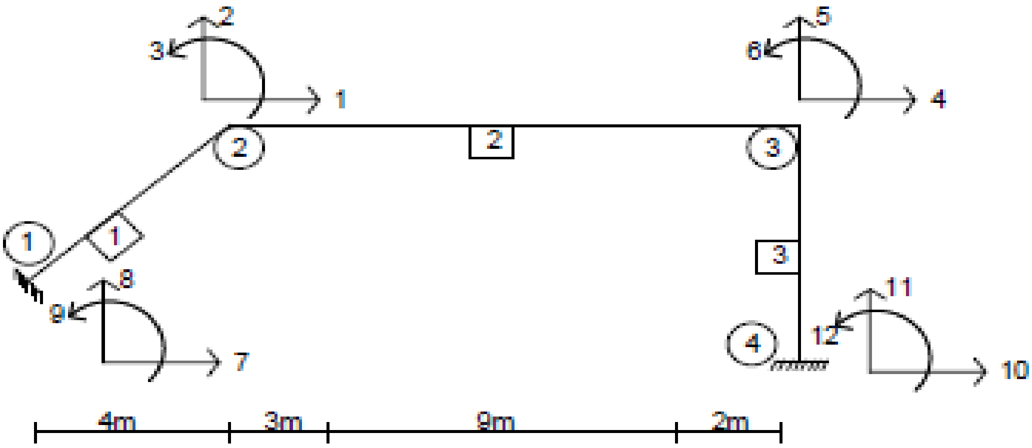

Figure 7. Koding of the example

In the solution I = 0.010 m^4, E = 2.1*10^6 t/m^2, b = 0.25 m are used.

Table1. 20 ºC Results of uniform temperature variations

Element and end forces								
No	**I**	**J**	**Ni (t)**	**Nj (t)**	**Vi (t)**	**Vj (t)**	**Mi (tm)**	**Mj (tm)**
1	1	2	-0.630	0.630	0.893	-0.893	3.518	1.837
2	2	3	-1.039	1.039	0.336	-0.336	-1.837	5.197
3	3	4	-0.336	0.336	-1.039	1.039	-5.197	0.000

Table 2. ΔT_1 = -5 ºC, ΔT_2 = +15 ºC Results of different temperature variations

Element and end forces								
No	**I**	**J**	**Ni (t)**	**Nj (t)**	**Vi (t)**	**Vj (t)**	**Mi (tm**	**Mj (tm**
1	1	2	-0.397	0.397	0.338	-0.338	-0.253	2.278
2	2	3	-0.520	0.520	0.032	-0.032	-2.278	2.600
3	3	4	-0.032	0.032	-0.520	0.520	-2.600	0.000

RESULTS

For statical solution of carrier frames consisting of variable width or height beams under the effect of equal or different temperature variation a computer program in BASIC language was prepared according to the principles presented here. The program is able to solve the frames automatically without the need for using any graphs and charts. This program prepared with matrix displacement method is able to save memory and working time of computer.

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