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Whale Optimization Algorithm for Numerical Constrained Optimization

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Abstract

Whale Optimization Algorithm (WOA), WOA is a recently developed, nature-inspired, meta-heuristic optimization algorithm. The algorithm was developed in 2016, inspired by bubble hunting strategies used by humpback whales. To determine the performance of each optimization algorithm developed, they should be tested on a different type of optimization test problems. In this paper, we aim to investigate and analyse WOA logarithm on constrained optimization the performance and accuracy of the proposed method are examined on 13 (G1-G13) constrained numerical benchmark functions, and the obtained results are compared with other meta-heuristic optimization algorithms which taken from the literature. The experimental results show that WOA has low performance on constrained optimization.

Keywords – Whale optimization algorithm, meta-heuristic algorithm, Constrained Optimization

1. INTRODUCTION

Optimization algorithms are used frequently for solving many problems [1]. The optimization is a mathematical discipline that aims to find the minimum or maximum value of an equation. For the solution of the optimization systems, heuristic methods are used along with the mathematical methods [2]. In literature, the term Meta-Heuristic is formed by the combination of the terms Meta and Heuristic and is defined as a whole of methods that solves problems through some rules and procedures [3]. Some of the heuristic techniques studied frequently in the literature are Artificial Neural Networks (ANN), Genetic Algorithm (GA), Ant Colony Optimization (ACO), Simulated Annealing (SA), Tabu Search and Particle Swarm Optimization (PSO). Some of the more recent or less studied meta-heuristics are Bee Colony Algorithm (BCA), Harmonic Search Algorithm and Kangaroo Algorithm [4]. The optimization algorithm developed needs to be tested in order to measure its performance. In the performance measurement of optimization algorithms, Traveling Salesman Problem, Unconstrained Test Problems and Constrained Test Problems are used [1]. Restricted test problems are frequently used and difficult to solve mathematical problems in applications such as engineering applications, structural optimization and allocation problems. For academics, it is very important to solve problematic problems efficiently [5]. Several different meta-heuristic methods have been proposed in the literature to solve different problem types. In the studies carried out, it is seen that meta-heuristic methods generate better results, in large-scale problems in particular. One of the recently developed meta-heuristical algorithms is the WOA. The whale optimization algorithm is a new optimization method proposed by Mirjalili and Lewia in 2016. The WOA method was tested with 29 different mathematical problems and the results of the tests showed that it could compete with other optimization algorithms [6]. This study is inception and different studies in relation to WOA are available in the literature. Experimental studies were performed on the breast cancer data set of WOA and successful results were obtained [7]. One of the data set studies where WOA is used is the problem of selecting the attributes in the datasets. In this study, the WOA was compared to the particle swarm optimization algorithm and it was demonstrated that it had performed better [8]. Saidala and Devarakonda conducted another data set study where WOA was used in 2017. Enron-Spam data set with high similarity value was selected as the data set. For the comparison, linear, quadratic, polynomial and RBF methods were used. According to all results obtained by evaluating the results concerning precision, accuracy, recall and F-values, it is seen that the values generated by WOA are competitive with other methods [9]. The WOA method was also used to aggregate the data of MR images taken for the liver. Concerning experimental results, it was confirmed that the WOA method had a higher predictive value [10]. The WOA method was used to train artificial neural networks on 20 different data sets. The method was compared to back-propagation learning algorithm and six

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evolutionary algorithms in terms of training and, it was confirmed that WOA had superior performance over existing algorithms in terms of both local optimal value avoidance and convergence rate [11]. In this study, we implemented and applied constrained optimization on WOA then analysed performance and accuracy on several constrained numerical benchmark function optimizations [12]. The rest of the paper is structured as follows. Section 2 describes the WOA algorithm in detail. Section 3 presents constrained optimization and test functions. Experimental studies are reported and discussed in Section 4. Finally, some conclusions are drawn in Section 5.

2. WHALEOPTIMISATION ALGORITHM

The whale optimization algorithm is one of the meta-heuristic algorithms developed based on the bubble-net hunting strategy of humpback whales [6]. Humpback whales are usually fed by small schools of fish near the water surface. Humpback whales create air bubbles when approaching the prey to keep their prey together as well as to allow them to approach the prey without being seen [13]. The hunting strategy of humpback whales is shown in Figure 1 [6].

Figure 1. Hunting of humpback whales

The whale optimization algorithm is fundamentally composed of 3 parts. These parts are encircling prey, Bubble Net Attacking and Search for Prey [6].

2.1. Encircling Prey

Humpback whales can recognize the location of the prey and encircle them. As the best solutions not known a priori in the optimization problems, the best solution or a point close to the best solution is considered the best solution. After the best solution is defined, the positions of the other solutions are updating accordance with the best [6]. The mathematical model of the encircling behaviour of the whales is presented in Equations 1 and 2 below [6].

$$\vec{D} = |\vec{C}.\vec{X}*(t) - \vec{X}(t)| \quad (1)$$

$$\vec{X}(t+1) = \vec{X}*(t) - \vec{A}.\vec{D} \quad (2)$$

Where represents the current iteration, $\vec{A}$ and $\vec{C}$ the coefficient vectors, and X* the best solution vectors.

Calculation of the vectors $\vec{A}$ and $\vec{C}$ are demonstrated in Equations 3 and 4 [6].

$$\vec{A} = 2\vec{a}.\vec{r} - \vec{a} \quad (3)$$

$$\vec{C} = 2.\vec{r} \quad (4)$$

Where a represents the vector linearly decreased from 2 to 0 during the iteration, and rather random vectors [6].

2.2. Bubble Net Attacking Method

The algorithm approaches, after the prey is located, the prey in two different moves, namely Shrinking Encircling Mechanism and Spiral Updating Position. Spiral Updating Position and the locations of the whales are modelling Figure 2 [6].

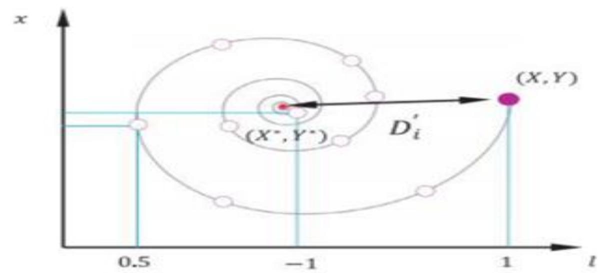

Figure 2. Spiral move

The mathematical formula for figuring out the difference between the best agent and the search agent for the spiral movement is shown in Equation 5 [6]. In Equation 6, it is demonstrated that $\vec{D}$' is the difference between the best search agent and the existing search agent [6].

$$X(t+1) = \vec{D'}.e(bl).\cos(2\pi l) + \vec{X}*(t) \quad (5)$$

$$\vec{D'} = \vec{X}*(t) - \vec{X}(t \quad (6)$$

Where b indicates the constant for logarithmic spiral, and l is a random number in the range of [-1, 1].The points around the best search agent are positioned through coefficients $\vec{A}$ and $\vec{C}$.

When whales move towards their prey, they chose either one of the linear moves or spiral moves. In WOA, which one of these linear moves or the spiral moves to be chosen is performed in a 50% probability, as shown in Equation 7 [6].

$$\vec{X}(t+1) = \begin{cases} \vec{X}^{x}(t) - \vec{A}.\vec{D} & p<0,5 \\ \vec{D}.e^{bl}.\cos(2\mu l) + \vec{X}*(t) & p=>0,5 \end{cases} \quad (7)$$

Where p is a randomly produced number between [0, 1].

2.3. Search for Prey

Whether a Global or a Local search is to be carried out is decided considering the value of the vector $\vec{A}$. In cases where $\vec{A}$>1 or $\vec{A}$<-1, it is considered a Global Search, because in these cases the points farther away from the best points can be chosen. The mathematical modelling is shown in Equations 8 and 9 [6].

$$\vec{D'} = \vec{C}.\vec{X}rand - \vec{X} \quad (8)$$

$$\vec{X}(t+1) = \vec{X}rand - \vec{A}.\vec{D} \quad (9)$$

The $\vec{X}$ rand shown in Equations 8 and 9 indicates a randomly selected search agent.

The pseudo code of the Whale Optimisation Algorithm is given in Figure 3 [6].

```
Initialize the population Xi (i = 1, 2, ..., n)
Calculate the fitness of each search agent
X*= the best search agent
while (t < Max #iterations)
for (each search agent)
Update a, A, C, l and p
       if (p < 0.5)
          if (|A| < 1)
Update the position of the current search agent in equation (1)
         else if (|A| ≥ 1)
            Select a random search agent (X_rand)
            Update the position of the current search agent in equation (9)
           end if
        else if (p ≥ 0.5)
              Update the position of the current search by the equation (5)
end if
   end for
Assign limit values for individuals getting out of boundaries
Calculate the value of the objective function
Update search agent if there is a better solution
t=t+1
end while
return X*
```

Figure 3. Pseudo code of WOA

3. CONSTRAINED OPTIMIZATION PROBLEMS

In order to observe the comparative performances of the algorithms proposed to solve real-world problems, they need to be tested with the test problems frequently used in the literature. One of them is numerically constrained optimization problems. Constrained optimization problems play an important role in the determination of challenging test performance of optimization algorithms [13]. It is extremely
The results of the empirical tests are presented in Table2.

difficult to solve such problems [1]. This difficulty originates from the attempt to find solutions within the region permitted by the constraints, due to the structure of the problems [1].
In this study, 13 constrained test functions, which are frequently used in literature, were selected and the details are shown in Table 1 [12].

Table 1. Detailed comparison of test functions

Problem	n	Type of Objective Function	p	$f(\vec{x}^*)$
G01	13	quadratic	0.0111%	-15.0000000000
G02	20	nonlinear	99.9971%	-0.8036191042
G03	10	polynomial	0.0000%	- 1.0005001000
G04	5	quadratic	51.1230%	- 30665.5386717834
G05	4	cubic	0.0000%	5126.4967140071
G06	2	cubic	0.0066%	- 6961.8138755802
G07	10	quadratic	0.0003%	24.3062090681
G08	2	nonlinear	0.8560%	- 0.0958250415
G09	7	polynomial	0.5121%	680.6300573745
G10	8	linear	0.0010%	7049.2480205286
G11	2	quadratic	0.0000%	0.7499000000
G12	3	quadratic	4.7713%	- 1.0000000000
G13	5	nonlinear	0.0000%	0.0539415140

4. EMPIRICAL TESTS

At the empirical tests of the WOA algorithm in this study, the iteration number of the G1-G13 constrained test problems was 500 and the problem coefficient (number of repeats) was 30.

Table 2. Test results of WOA. "/" indicates values for which no solution was obtained.

Problem	Optimum	Best	Worst	Average	Standard Deviation
G1	-15	-15	-6	-11.9097	2.5033852
G2	-0.803619	/	/	/	/
G3	-1.000500	0	0	0	0
G4	-30665.539	-30587.303	-29313.357	-30102.268	238.19937
G5	5126.4981	/	/	/	/
G6	-6961.81388	-6961.42989	-6945.91212	-6958.47643	3.1177653
G7	24.306	28,630	79.259	43.992	12.049
G8	-0.095825	-0.095825	-0.095825041	-0.095825041	1.17591E-10
G9	680.6300573	683.6303371	762.4505978	704.8788225	18.48857643
G10	7049.248	8977.340	1.29993E+14	1.46376E+13	3.62535E+13
G11	0,75	1	1	1	0
G12	-1	-0.988	-0.845	-0.934	0.036
G13	0.0539498	/	/	/	/

When the test results of WOA in Table 2 are analysed, it is seen that the optimum value was reached in G1 and G8, optimum value was considerably converged in G4, G7, G9 and G12, the values too far from the optimum value were obtained in G3, G6, G10 and G11, and no results were produced in G2, G5 and G13.

4.1. Penalty Function

Penalty function is a method used to solve constrained optimization problems. Constrained problems are multiplied by a penalty parameter to the extent of the violation of the restriction and added to the current value. In this way, the individual outside the boundaries is included in the problem solving again from a remote location [14]. Penalty function method pseudo code is shown in Figure 4.

```
Constraint Function
Penalty = 10^15
For i=1 to #inequality
        If inequality check result = true
                Objval = Objval + Penalty
        End If
End for
For i=1 to #equality
        If equality check result = true
                Objval = Objval + Penalty
        End If
End for
```

Figure 4. Pseudo code of Penalty Function

In the experimental studies conducted, the average test results of WOA and Non Penalty Whale Optimization Algorithm (NPWOA) were compared and shown in Table 3.
As seen in Table 3, WOA gave much better results than NPWOA. Because the penalty function multiplies the values that do not match the constraint by a high number and does not allow bad results to be included in the solution. Penalties are not used in the algorithm, bad results are resolved, preventing going to good value, and at some point, it makes the solution impossible by attaching to the local best. Based on the tests and information, we can say that it is very important to use the penalty function in constrained optimization problems.

Table 3. Comparison of Average Values of WOA and NPWOA

Pro	Opt	WOA	NPWOA
G1	-15	-11.909	-306
G2	0.803619	/	/
G3	1	0	1,98
G4	-30665.53	-30102.26	-32217.04
G5	5126.498	/	0
G6	-6961.814	-6958.476	-7973
G7	24.306	43.992	-24
G8	0.095825	-0.095825	/
G9	680.63	704.8788225	6.903
G10	7049.25	1.46376E+13	/
G11	0.75	1	0
G12	1	-0.934	/
G13	0.05395	/	/

4.2. Empirical Test Result and Compared

The test results obtained were compared with GA, ABC, PSO, FA and BA. Comparison of said algorithms with WOA in Table 4. [15] [16] [5] [17] [18].

Table 4. Comparison of the Best Values of the algorithms.

Pro	Opt	GA	ABC	PSO	FA	BA	WOA
G1	-15	-14.44	-15	-15	-12.432	-15	-15
G2	0.803619	0.796231	0.80360	0.8033	-0.803619	0.8036191	/
G3	1	0.99	1	1	-1	0.99998	0
G4	-30665.53	-30626.053	-30665.539	-30665.5	-30378.381	-30665.539	-30587.303
G5	5126.498	/	5126.484	/	5126.497	5126.498	/
G6	-6961.814	-6952.472	-6961.814	-6961.7	-6961.811	-6961.813	-6961.429
G7	24.306	31.097	24.330	24.442	24.307	24.306	28.630
G8	0.095825	0.095825	0.09583	0.09583	0.095825	0.095825	0.095825
G9	680.63	685.994	680634	680.657	680.63	680.630	683.63
G10	7049.25	9079.77	7053904	7131.01	7072.41	7049.26	8977.34
G11	0.75	0.75	0.75	0.75	0.75	0.75	1
G12	1	1	1	1	-1	1	-0.98
G13	0.05395	0.13405	0.76	/	0.054	0.0539499	/

Comparison of the best values of the algorithms given in Table 4 is presented in Table 5. The (+) sign was used to demonstrate the results with better or equal values, (-) for the worse results and (/) for no results.

Table 5. Comparison of best values of algorithms.

Pro	GA-WOA		ABC-WOA		PSO-WOA		FA-WOA		BA-WOA	
G1	-	+	+	+	+	+	-	+	+	+
G2	+	/	+	/	+	/	+	/	+	/
G3	+	-	+	-	+	-	+	+	+	-
G4	+	-	+	-	+	-	-	+	+	-
G5	/	/	+	/	/	/	+	/	+	/
G6	-	+	+	-	+	-	+	-	+	-
G7	-	+	+	-	+	-	+	-	+	-
G8	+	+	-	+	-	+	+	+	+	+
G9	-	+	+	-	+	-	+	-	+	-
G10	-	+	+	-	+	-	+	-	+	-
G11	+	-	+	-	+	-	+	-	+	-
G12	+	-	+	-	+	-	-	+	+	-
G13	+	/	+	/	/	/	+	/	+	/
Total	7	6	12	2	10	2	10	5	13	2

When data of the best test results in Table 5 are analysed, WOA has 17 better (+) results in total. This indicates that WOA has a competitive structure, however, the other algorithms result better in most of the constrained test problems.

In Table 6, a comparison of the worst values of the algorithms used for the performance evaluation of the WOA is presented.

Table 6. Comparison of the worst values of algorithms.

Pro	Optimal Value	GA [5]	ABC [15]	PSO [16]	BA [17]	WOA [18].
G1	-15	-14.015	-15	-15	-12.45309	-6
G2	0.803619	0.77914	0.7703	0.6316	0.5625529	/
G3	1	0.956	1	1	0.93641	0
G4	-30665.539	-30567.105	-30665.5390	-30665.5	-30665.539	-29313.3576
G5	5126.498	/	5374.4300	/	5181.4744	/
G6	-6961.814	-6784.255	-6961.8130	-6956.8	-6961.81388	-6945.912123
G7	24.306	38.686	24.8350	31.1843	24.30799	79.25964946
G8	0.095825	0.095723	0.0958	0.09583	0.095825	0.095825
G9	680.63	698.297	680.641	681.675	680.630	762.450
G10	7049.25	11003.533	7493.9430	8823.56	7051.7822	/
G11	0.75	0.752	0.7500	0.75	0.750000012	1
G12	1	0.999	1	1	1	-0.845586734
G13	0.05395	/	1	/	0.0539723	/

For the comparison and interpretation of the worst values of the algorithms given in Table 5, Table 6 was created. The (+) sign in Table 7 was used to demonstrate the results with better or equal values, (-) for the worse results and (/) for no results.

Table 7. Comparison of the worst values of algorithms.

Pro	GA-WOA		ABC-WOA		PSO-WOA		BA-WOA	
G1	+	-	+	-	+	-	+	-
G2	+	/	+	/	+	/	+	/
G3	+	-	+	-	+	-	+	-
G4	+	-	+	-	+	-	+	-
G5	/	/	+	/	/	/	+	/
G6	+	-	+	-	+	-	+	-
G7	+	-	+	-	+	-	+	-
G8	-	+	-	+	-	+	+	+
G9	+	-	+	-	+	-	+	-
G10	+	/	+	/	+	/	+	/
G11	+	-	+	-	+	-	+	-
G12	+	-	+	-	+	-	+	-
G13	+	/	+	/	/	/	+	/
Total	11	1	12	1	10	1	13	1

When the results of the comparison in Table 7 are analysed, WOA has 4 better (+) result values. In a comparison of the worst values, WOA obtains results less successful than other algorithms. In Table 8, a comparison of the average values of the said algorithms is presented [15] [16] [5] [17] [18].

Table 8. Comparison of average values of algorithms.

Pro	Opt	GA	ABC	PSO	FA	BA	WOA
G1	-15	-14.236	-15	-15	-10.161	-14.658	-11.909
G2	0.803619	0.788588	0.7924	0.7521	-0.782942	0.75347	/
G3	1	0,976	1	1	-1	0.98955	0
G4	-30665.539	-30590.455	-30665.5390	-30665.5	-30259.518	-30665.539	-30102.26858
G5	5126.498	/	5185.7140	/	5126.835	5129.4248	/
G6	-6961.814	-6872.204	-6961.8130	-6960.7	-6961.786	-6961.8138	-6958.476433
G7	24.306	34.98	24.473	26.71	24.311	24.306	43.992
G8	0.095825	0.095799	0.0958	0.09583	-0.095825	0.095825	-0.095825
G9	680.63	692.064	680.64	680.876	680.63	680.63	704.878
G10	7049.25	10003.225	7224.4070	7594.65	7388.856	7049.4713	1.46376E+13
G11	0.75	0.75	0.7500	0.75	0.75	0.75	1
G12	1	1	1	1	-1	1	-0.9349
G13	0.05395	/	0.9680	/	0.252	0.053952	/

For the comparison and interpretation of the average values of the algorithms given in Table 8, Table 9 was created. The (+) sign in Table 9 was used to demonstrate the results with better or equal values, (-) for the worse results and (/) for no results. With reference to the comparison value in Table 9, WOA has 9 better (+) values. In consequence of the comparison of the average values of the algorithms, it is seen that WOA results worse than other algorithms, however, has a competitive structure. As a result of the tests, it is seen that WOA obtains no results in problems G2, G5 and G13, it reaches the optimum value in problems G1 and G8, it is considerably close to the optimum value in problems G4, G6, G9 and G12, and the optimum value is reached in problems G3, G7, G10 and G11.

The best values, worst values and average values of the said algorithms were compared to that of WOA. It is seen that WOA generates 17 better results in comparison of the best values, 4 better results in comparison of the worst values and in 9 better results in comparison of the average values

Table 9. Comparison of average values of algorithms.

Pro	GA-WOA		ABC-WOA		PSO-WOA		FA-WOA		BA-WOA	
G1	+	-	+	-	+	-	-	+	+	-
G2	+	/	+	/	+	/	+	/	+	/
G3	+	-	+	-	+	-	-	+	+	-
G4	+	-	+	-	+	-	+	-	+	-
G5	/	/	+	/	/	/	+	/	+	/
G6	-	+	+	-	+	-	+	-	+	-
G7	+	-	+	-	+	-	+	-	+	-
G8	-	+	-	+	-	+	+	+	-	+
G9	+	-	+	-	+	-	+	-	+	-
G10	+	-	+	-	+	-	+	-	+	-
G11	+	-	+	-	+	-	+	-	+	-
G12	+	-	+	-	+	-	-	+	+	-
G13	/	/	+	/	/	/	+	/	+	/
TOTAL	9	2	12	1	10	1	10	4	12	1

5. CONCLUSION AND DISCUSSION

Meta-heuristic algorithms are now widely used in many fields. WOA is one of the recently proposed meta-heuristic algorithms inspired by the hunting strategy of humpback whales. In this study, we proposed a constrained WOA and applied well know 13 constrained numeric optimization test functions. The experimental results were compared with well-known similar meta-heuristic algorithms GA, ABC, PSO, FA, and BA algorithms to demonstrate the performance of WOA. The problems between the G1 and G13 were used for the comparison out of the constrained test problems.

The experimental results conducted over 13 constrained benchmark functions indicate that the OWA demonstrated has not well capabilities related to accuracy, robustness, and efficiency on constrained optimization.

Our future work will improve and modify OWA research mechanism and try again on different optimization benchmark functions.

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