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A Note on (m,n)-Γ-Ideals of Ordered LA-Γ-Semigroups

Abul Basar[1*]

[1]*Department of Mathematics, Jamia Millia Islamia, New Delhi, India*
[*]*Corresponding author E-mail: basar.jmi@gmail.com*

Abstract

In this paper, we investigate the notion of (m,n)-ideals in a non-associative algebraic structure, which we call an ordered LA-Γ-semigroup. We prove that if $(S,\Gamma,\cdot,\leq)$ is a unitary ordered LA-Γ-semigroup with zero and S has the condition that it contains no non-zero nilpotent (m,n)-ideals and if $R(L)$ is a 0-minimal right (left) ideal of S, then either $(R\Gamma L] = \{0\}$ or $(R\Gamma L]$ is a 0-minimal (m,n)-ideal of S. Also, we prove that if $(S,\Gamma,\cdot,\leq)$ is a unitary ordered LA-Γ-semigroup; A is an (m,n)-ideal of S and B is an (m,n)-ideal of A such that B is idempotent, then B is an (m,n)-ideal of S.

Keywords: *LA-semigroups, (m,n)-ideals, ordered LA-Γ-semigroups*
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1. Introduction

The notion of a left almost semigroup (LA-semigroup) was introduced by M. A. Kazim and M. Naseeruddin [8]. Interestingly, LA-semigroups have been given different names like ”left invertive groupoid” and ”Abel-Grassmann’s groupoid” (AG-groupoid) by different algebraists [10] and [5].

The concept of an LA-Γ-semigroup (Γ-AG-groupoid) was introduced by T. Shah and I. Rehman [18]. These objects are, in fact, 2-sorted non-associative algebraic structures with one ternary operation subjected to a sort of axiom. More precisely, they are ordered triplets $(S,\Gamma,\cdot)$ consisting of two sets S and Γ and a ternary operation $S\times\Gamma\times S\to S$ with the property that $(x\cdot\alpha\cdot y)\cdot\beta\cdot z = (z\cdot\alpha\cdot y)\cdot\beta\cdot x$ for all $x,y,z\in S$ and all $\alpha,\beta\in\Gamma$. Note that every plain LA-semigroup S can be considered as an LA-Γ-semigroup by taking Γ as a singeleton $\{1\}$, where 1 is the identity element of S, when S has a such element, or it is a symbol not representing an element of S, and the Γ-multiplication in S is defined by $a1b = ab$, where ab is the usual product in plain LA-semigroup S.

Various types of ideals, rough ideals, prime (m, n)-ideals have been studied in different algebraic structures by many algebraists [1], [2], [3], [4], [6], [7], [9], [10], [11], [12], [13], [14], [15], [17], [19] and [20]. All the results of this paper can be obtained for LA-semigroups without order and without Γ.

Definition 1.1. *An LA-semigroup $(S,\cdot)$ together with a partial order $\leq$ on S that is compatible with LA-semigroup operation such that for all $x,y,z\in S$ and $\alpha,\beta\in\Gamma$, we have*

$$x\leq y\Rightarrow z\alpha x\leq z\beta y \text{ and } x\alpha z\leq y\beta z,$$

is called an ordered LA-Γ-semigroup.

For subsets A,B of an LA-Γ-semigroup S, the product set AB of the pair (A,B) relative to S is defined as $A\Gamma B = \{a\gamma b : a\in A, b\in B \text{ and } \gamma\in\Gamma\}$ and for $A\subseteq S$, the product set AA relative to S is defined as $A^2 = AA = A\Gamma A$. Note that A^0 acts as an identity operator. That is, $A^0\Gamma S = S = S\Gamma A^0$. Also, $(A] = \{s\in S : s\leq a \text{ for some a} \in \text{A}\}$. Let $(S,\Gamma,\cdot,\leq)$ be an ordered LA-Γ-semigroup and let A,B be nonempty subsets of S, then we easily have the following:

(i) $A\subseteq (A]$;
(ii) If $A\subseteq B$, then $(A]\subseteq (B]$;
(iii) $(A]\Gamma(B]\subseteq (A\Gamma B]$;
(iv) $(A] = ((A]]$;
(v) $((A]\Gamma(B]] = (A\Gamma B]$;
(vi) For every left (resp. right) ideal T of S, $(T] = T$.

Email addresses: **basar.jmi@gmail.com (Abul Basar)**

Definition 1.2. *Suppose $(S,\Gamma,\cdot,\leq)$ is an ordered LA-Γ-semigroup and m,n are non-negative integers. An LA-sub-semigroup A of S is called an (m,n)-Γ-ideal of S if:*

(i) $A^m\Gamma S\Gamma A^n \subseteq A$;
(ii) for any $a\in A$ and $s\in S$, $s\leq a$ implies $s\in A$.

Equivalently: an ordered LA-Γ-semigroup A of S is called an (m,n)-Γ-ideal of S if

$$(A^m\Gamma S\Gamma A^n] \subseteq A.$$

If A is an (m,n)-ideal of an ordered LA-Γ-semigroup $(S,\Gamma,\cdot,\leq)$, then $(A]=A$.

The purpose of this paper is to investigate (m,n)-Γ-ideals in ordered LA-Γ-semigroups as an extension of the results in [7]. Also, the results of this paper can be obtained for a locally associative ordered LA-semigroup which will generalize and extend the notion of a locally associative LA-semigroup [16].

2. (m,n)-Γ-ideals in ordered LA-Γ-semigroups

We start with the following example:

Example 2.1. *Suppose $S=\{x,y,z,w,e\}$ with a left identity w. Let $x\cdot\gamma\cdot y = x\cdot y$. The following multiplication table and order show that $(S,\Gamma,\cdot,\leq)$ is a unitary ordered LA-Γ-semigroup with a zero elemmaent x:*

$\cdot$	x	y	z	w	e
x	x	x	x	x	x
y	x	e	e	z	e
z	x	e	e	y	e
w	x	y	z	w	e
e	x	e	e	e	e

Lemma 2.2. *Suppose R and L are respectively the right and the left ideals of a unitary ordered LA-Γ-semigroup $(S,\Gamma,\cdot,\leq)$, then $(R\Gamma L]$is an (m,n)-ideal of S.*

Proof. Suppose R and L are the right and the left ideals of S respectively, then we have the following:

$$\begin{aligned}
(((R\Gamma L)^m]\Gamma S\Gamma((R\Gamma L)^n]] &\subseteq (((R\Gamma L)^m]\Gamma(S]\Gamma((R\Gamma L)^n]] \\
&\subseteq ((R\Gamma L)^m\Gamma S\Gamma((R\Gamma L)^n]] \\
&= ((R^m\Gamma L^m\Gamma S)\Gamma(R^n\Gamma L^n)] \\
&= ((R^m\Gamma L^m\Gamma R^n)\Gamma(S\Gamma L^n)] \\
&= ((L^m\Gamma R^m\Gamma R^n)\Gamma(S\Gamma L^n] \\
&= ((R^n\Gamma R^m\Gamma L^m)\Gamma(S\Gamma L^n)] \\
&= ((R^m\Gamma R^n\Gamma L^m)\Gamma(S\Gamma L^n)] \\
&= ((R^{m+n}\Gamma L^m)\Gamma(S\Gamma L^n)] \\
&= (S\Gamma(R^{m+n}\Gamma L^m\Gamma L^n] \\
&= (S\Gamma(L^n\Gamma L^m\Gamma R^{m+n})] \\
&= ((S\Gamma S]\Gamma L^{m+n}\Gamma R^{m+n}] \\
&\subseteq (S\Gamma S\Gamma L^{m+n}\Gamma R^{m+n}] \\
&= (S\Gamma L^{m+n}\Gamma S\Gamma R^{m+n}] \\
&= (R^{m+n}\Gamma S\Gamma L^{m+n}\Gamma S] \\
&= ((R^m\Gamma R^n\Gamma(S\Gamma S])\Gamma(L^m\Gamma L^n\Gamma(S\Gamma S])] \\
&\subseteq (((R^m\Gamma R^n]\Gamma(S\Gamma S])\Gamma((L^m\Gamma L^n]\Gamma(S\Gamma S])] \\
&\subseteq ((R^m\Gamma R^n\Gamma S\Gamma S)\Gamma(L^m\Gamma L^n\Gamma S\Gamma S)] \\
&= ((S\Gamma S\Gamma R^n\Gamma R^m)\Gamma(S\Gamma S\Gamma L^n\Gamma L^m)] \\
&\subseteq (((S\Gamma S]\Gamma R^n\Gamma R^m)\Gamma((S\Gamma S]\Gamma L^n\Gamma L^m)] \\
&= (S\Gamma R^{m+n}\Gamma S\Gamma L^{m+n}].
\end{aligned}$$

Therefore,

$$\begin{aligned}(S\Gamma R^{m+n}\Gamma S\Gamma L^{m+n}] &= ((S\Gamma R^{m+n-1}\Gamma R)\Gamma(S\Gamma L^{m+n-1}\Gamma L)]\\ &= ((S\Gamma(R^{m+n-2}\Gamma R\Gamma R))\Gamma(S\Gamma(S\Gamma(L^{m+n-2}\Gamma L\Gamma L))]\\ &= (S\Gamma(R\Gamma R\Gamma R^{m+n-2}))\Gamma(S\Gamma(L\Gamma L\Gamma L^{m+n-2}))\\ &\subseteq ((S\Gamma S\Gamma R\Gamma R^{m+n-2})\Gamma(S\Gamma S\Gamma L\Gamma L^{m+n-2})]\\ &\subseteq ((S\Gamma R\Gamma S\Gamma R^{m+n-2})\Gamma(S\Gamma L\Gamma S\Gamma L^{m+n-2})]\\ &\subseteq ((R^{m+n-2}\Gamma S\Gamma R\Gamma S)\Gamma(L\Gamma S\Gamma L^{m+n-2})]\\ &\subseteq ((R^{m+n-2}\Gamma S\Gamma(R\Gamma S])\Gamma(L\Gamma S\Gamma L^{m+n-2})]\\ &\subseteq ((R^{m+n-2}\Gamma S\Gamma R)\Gamma(S\Gamma L\Gamma L^{m+n-2})]\\ &\subseteq (((R\Gamma S]\Gamma R^{m+n-2})\Gamma(S\Gamma L^{m+n-1})]\\ &\subseteq (R\Gamma R^{m+n-2}\Gamma S\Gamma L^{m+n-1}]\\ &\subseteq (S\Gamma R^{m+n-1}\Gamma S\Gamma L^{m+n-1}].\end{aligned}$$

So,

$$\begin{aligned}(((R\Gamma L)^m]\Gamma S\Gamma((R\Gamma L)^n]] &\subseteq (S\Gamma R^{m+n}\Gamma S\Gamma L^{m+n}]\\ &\subseteq (S\Gamma R^{m+n-1}\Gamma S\Gamma L^{m+n-1}]\\ &\subseteq \cdots\\ &\subseteq (S\Gamma R\Gamma S\Gamma L] \subseteq (S\Gamma R\Gamma(S\Gamma L]]\\ &\subseteq (S\Gamma R\Gamma L] \subseteq ((S\Gamma S\Gamma R)\Gamma L]\\ &= ((R\Gamma S\Gamma S)\Gamma L] \subseteq (((R\Gamma S]\Gamma S)\Gamma L] \subseteq (R\Gamma L].\end{aligned}$$

Furthermore,

$$\begin{aligned}(R\Gamma L]\Gamma(R\Gamma L] &\subseteq (R\Gamma L\Gamma R\Gamma L] = ((L\Gamma R\Gamma L\Gamma R)\Gamma L]\\ &= ((R\Gamma R\Gamma L)\Gamma L] = ((R\Gamma R\Gamma L)\Gamma L] \subseteq (((R\Gamma S]\Gamma S)\Gamma L] \subseteq (R\Gamma L].\end{aligned}$$

This proves that $(R\Gamma L]$ is an (m,n)-ideal of S. □

Theorem 2.3. *Suppose $(S,\Gamma,\cdot,\leq)$ is a unitary ordered LA-Γ-semigroup with zero. If S has the property that it contains no non-zero nilpotent (m,n)-ideals and if $R(L)$ is a 0-minimal right(left) ideal of S, then either $(R\Gamma L] = \{0\}$ or $(R\Gamma L]$ is a 0-minimal (m,n)-ideal of S.*

Proof. Let $R(L)$ is a 0-minimal right (left) ideal of S such that $(R\Gamma L] \neq \{0\}$, then by lemmama 2.1, $(R\Gamma L]$ is an (m,n)-ideal of S. Now we prove that $(R\Gamma L]$ is a 0-minimal (m,n)-ideal of S. Suppose $\{0\} \neq M \subseteq (R\Gamma L]$ is an (m,n)-ideal of S. We see that as $(R\Gamma L] \subseteq R\cap L$, we obtain $M \subseteq R\cap L$. Therefore, $M \subseteq R$ and $M \subseteq L$. By the assumption, $M^m \neq \{0\}$ and $M^n \neq \{0\}$. As $\{0\} \neq (S\Gamma M^m] = (M^m\Gamma S]$, so

$$\begin{aligned}\{0\} \neq (M^m\Gamma S] &\subseteq (R^m\Gamma S] = (R^{m-1}\Gamma R\Gamma S] = (S\Gamma R\Gamma R^{m-1}]\\ &= (S\Gamma R\Gamma R^{m-2}\Gamma R] \subseteq (R\Gamma R^{m-2}\Gamma(R\Gamma S]]\\ &\subseteq (R\Gamma R^{m-2}\Gamma R] = (R^m],\end{aligned}$$

and

$$\begin{aligned}(R^m] &\subseteq S\Gamma(R^m] \subseteq (S\Gamma R^m] \subseteq (S\Gamma S\Gamma R\Gamma R^{m-1}]\\ &\subseteq (R^{m-1}\Gamma R\Gamma S] = ((R^{m-2}\Gamma R\Gamma R)\Gamma S]\\ &= ((R\Gamma R\Gamma R^{m-2})\Gamma S] \subseteq (S\Gamma R^{m-2}\Gamma(R\Gamma S]]\\ &\subseteq (S\Gamma R^{m-2}\Gamma R] \subseteq ((S\Gamma S\Gamma R^{m-3}\Gamma R)\Gamma R]\\ &= ((R\Gamma R^{m-3}\Gamma S\Gamma S)\Gamma R] \subseteq (((R\Gamma S]\Gamma R^{m-3}\Gamma S)\Gamma R]\\ &\subseteq (R\Gamma R^{m-3}\Gamma S)\Gamma R] \subseteq ((R^{m-3}\Gamma(R\Gamma S])\Gamma R]\\ &\subseteq (R^{m-3}\Gamma R\Gamma R] = (R^{m-1}],\end{aligned}$$

so, $\{0\} \neq (M^m\Gamma S] \subseteq (R^m] \subseteq (R^{m-1}] \subseteq \cdots \subseteq (R] = R$. It is obvious to see that $(M^m\Gamma S]$ is a right ideal of S. Therefore, $(M^m\Gamma S] = R$ as R is o-minimal. Moreover,

$$\begin{aligned}\{0\} &\neq (S\Gamma M^n] \subseteq (S\Gamma L^n] = (S\Gamma L^{n-1}\Gamma L]\\ &\subseteq (L^{n-1}\Gamma(S\Gamma L]] \subseteq (L^{n-1}\Gamma L] = (L^n],\end{aligned}$$

and

$$\begin{aligned}(L^n] &\subseteq (S\Gamma L^n] \subseteq (S\Gamma S\Gamma L\Gamma L^{n-1}] \subseteq (L^{n-1}\Gamma L\Gamma S]\\ &= ((L^{n-2}\Gamma L\Gamma L)\Gamma S] \subseteq ((S\Gamma L]\Gamma L^{n-2}\Gamma L]\\ &\subseteq (L\Gamma L^{n-2}\Gamma L] \subseteq (L^{n-2}\Gamma S\Gamma L]\\ &\subseteq (L^{n-2}\Gamma L] = (L^{n-1}] \subseteq \cdots \subseteq (L],\end{aligned}$$

so, $\{0\} \neq (S\Gamma M^n] \subseteq (L^n] \subseteq (L^{n-1}] \subseteq \cdots \subseteq (L] = L$. It is obvious to see that $(S\Gamma M^n]$ is a left ideal of S. Therefore, $(S\Gamma M^n] = L$ as L is 0-minimal. So,

$$\begin{aligned} M &\subseteq (R\Gamma L] = ((M^m\Gamma S]\Gamma(S\Gamma M^n]] = (M^n\Gamma S\Gamma S\Gamma M^m] \\ &= ((S\Gamma M^m\Gamma S)\Gamma M^n] \subseteq ((S\Gamma M^m\Gamma S\Gamma S)\Gamma M^n] \\ &\subseteq ((S\Gamma M^m\Gamma S)\Gamma M^n] = ((M^m\Gamma S\Gamma S)\Gamma M^n] \\ &\subseteq (M^m\Gamma S\Gamma M^n] \subseteq M. \end{aligned}$$

Therefore, $M = (R\Gamma L]$. It implies that $(R\Gamma L]$ is a 0-minimal (m,n)-ideal of S. □

It is easy to see that if $(S,\Gamma,\cdot,\leq)$ is a unitary ordered LA-Γ-semigroup and $M \subseteq S$, then $(S\Gamma M^2]$ and $(S\Gamma M]$ are the left and the right ideals of S respectively.

Theorem 2.4. *Suppose $(S,\Gamma,\cdot,\leq)$ is a unitary ordered LA-Γ-semigroup with zero 0. If $R(L)$ is a 0-minimal right (left) ideal of S, then either $(R^m\Gamma L^n] = \{0\}$ or $(R^m\Gamma L^n]$ is a 0-minimal (m,n)-ideal of S.*

Proof. Let $R(L)$ is a 0-minimal right (left) ideal of S such that $(R^m\Gamma L^n] \neq \{0\}$, then $R^m \neq \{0\}$ and $L^n \neq \{0\}$. Hence $\{0\} \neq R^m \subseteq R$ and $\{0\} \neq L^n \subseteq L$, which proves that $R^m = R$ and $L^n = L$ as $R(L)$ is a 0-minimal right (left) ideal of S. So by Lemma 2.1, $(R^m\Gamma L^n] = (R\Gamma L]$ is an (m,n)-ideal of S. Now we prove that $(R^m\Gamma L^n]$ is a 0-minimal (m,n)-ideal of S. Suppose $\{0\} \neq M \subseteq (R^m\Gamma L^n] = (R\Gamma L] \subseteq R\cap L$ is an (m,n)-ideal of S. Therefore,

$$\{0\} \neq (S\Gamma M^2] \subseteq (M\Gamma M\Gamma S\Gamma S] = (M\Gamma S\Gamma M\Gamma S] \subseteq ((R\Gamma S]\Gamma(R\Gamma S]] \subseteq R,$$

and

$$\{0\} \neq (S\Gamma M] \subseteq (S\Gamma L] \subseteq L.$$

Therefore, $R = (S\Gamma M^2]$ and $(S\Gamma M] = L$ as $R(L)$ is a 0-minimal right (left) ideal of S. As

$$(S\Gamma M^2] \subseteq (M\Gamma M\Gamma S\Gamma S] = (S\Gamma M\Gamma M] \subseteq (S\Gamma M],$$

Thus,

$$\begin{aligned} M &\subseteq (R^m\Gamma L^n] \subseteq (((S\Gamma M)^m]\Gamma((S\Gamma M)^n]] = ((S\Gamma M)^m\Gamma(S\Gamma M)^n] \\ &= (S^m\Gamma M^m\Gamma S^n\Gamma M^n] = (S\Gamma S\Gamma M^m\Gamma M^n] \subseteq (M^n\Gamma M^m\Gamma S] \\ &\subseteq ((S\Gamma S)\Gamma(M^{m-1}\Gamma M)\Gamma M^n] = ((M\Gamma M^{m-1})\Gamma(S\Gamma S)\Gamma M^n] \\ &\subseteq (M^m\Gamma S\Gamma M^n] \subseteq M, \end{aligned}$$

So $M = (R^m\Gamma L^n]$, which implies that $(R^m\Gamma L^n]$ is a 0-minimal (m,n)-ideal of S. □

Theorem 2.5. *Suppose $(S,\Gamma,\cdot,\leq)$ is a unitary ordered LA-Γ-semigroup. Suppose A is an (m,n)-ideal of S and B is an (m,n)-ideal of A such that B is idempotent. Then B is an (m,n)-ideal of S.*

Proof. It is easy to see that B is an LA-sub-semigroup of S. Furthermore, as $(A^m\Gamma S\Gamma A^n] \subseteq A$ and $(B^m\Gamma A\Gamma B^n] \subseteq B$, then

$$\begin{aligned} (B^m\Gamma S\Gamma B^n] &\subseteq ((B^m\Gamma B^m\Gamma S)\Gamma(B^n\Gamma B^n)] = ((B^n\Gamma B^n)\Gamma(S\Gamma B^m\Gamma B^m)] \\ &= (((S\Gamma B^m\Gamma B^m)\Gamma B^n)\Gamma B^n] \subseteq (((B^n\Gamma B^m\Gamma B^m)\Gamma(S\Gamma S))\Gamma B^n] \\ &= (((B^m\Gamma B^n\Gamma B^m)\Gamma(S\Gamma S))\Gamma B^n] = ((S\Gamma(B^n\Gamma B^m\Gamma B^m))\Gamma B^n] \\ &= ((S\Gamma(B^n\Gamma B^m\Gamma B^{m-1}\Gamma B))\Gamma B^n] = ((S\Gamma(B\Gamma B^{m-1}\Gamma B^m\Gamma B^n))\Gamma B^n] \\ &= ((S\Gamma(B^m\Gamma B^m\Gamma B^n))\Gamma B^n] \subseteq ((B^m\Gamma(S\Gamma S\Gamma B^m\Gamma B^n))\Gamma B^n] \\ &= ((B^m\Gamma(B^n\Gamma B^m\Gamma S\Gamma))\Gamma B^n] \subseteq ((B^m\Gamma(S\Gamma B^m\Gamma B^n))\Gamma B^n] \\ &\subseteq ((B^m((S\Gamma S\Gamma B^{m-1}\Gamma B)\Gamma B^n))\Gamma B^n] \subseteq ((B^m\Gamma(B^m\Gamma S\Gamma B^n))\Gamma B^n] \\ &\subseteq ((B^m(A^m\Gamma S\Gamma A^n])\Gamma B^n] \subseteq (B^m\Gamma A\Gamma B^n] \subseteq B, \end{aligned}$$

which implies that B is an (m,n)-ideal of S. □

Lemma 2.6. *Suppose $(S,\Gamma,\cdot,\leq)$ is a unitary ordered LA-Γ-semigroup. Then $<s>_{(m,n)} = (s^m\Gamma S\Gamma s^n]$ is an (m,n)-ideal of S.*

Proof. Let S be a unitary ordered LA-Γ-semigroup. It is obvious to see that $(<s>_{(m,n)})^n \subseteq <s>_{(m,n)}$. Now

$$\begin{aligned} (((<s>_{(m,n)})^m\Gamma S)\Gamma(<s>_{(m,n)})^n] &= (((((s^m\Gamma S)\Gamma s^n))^m]\Gamma S\Gamma(((s^m\Gamma S)\Gamma s^n)^n]] \\ &\subseteq (((s^m\Gamma S)\Gamma s^n)^m\Gamma S\Gamma((s^m\Gamma S)\Gamma s^n)^n] \\ &= (((s^{mm}\Gamma S^m)\Gamma s^{mn})\Gamma S\Gamma(s^{mn}\Gamma S^n)\Gamma s^{nn}] \\ &= (s^{nn}\Gamma(s^{nn}\Gamma S^n)\Gamma S\Gamma((s^{mm}\Gamma S^m)\Gamma s^{mn}] \\ &= ((S\Gamma((s^{mm}\Gamma S^m)\Gamma s^{mn})\Gamma s^{mn}\Gamma S^n)\Gamma s^{nn}] \\ &= ((s^{mn}\Gamma(S\Gamma((s^{mm}\Gamma S^m)\Gamma s^{mn}))\Gamma S^n)\Gamma s^{nn}] \\ &\subseteq (s^{mn}\Gamma S\Gamma s^{nn}] \subseteq (s^{mn}\Gamma S^n\Gamma s^{nn}] \\ &= ((s^m\Gamma S\Gamma s^n)^n] \subseteq (((s^m\Gamma S\Gamma s^n)]^n] \\ &= ((<s>_{(m,n)})^n] \subseteq (<s>_{(m,n)}], \end{aligned}$$

which implies that $<s>_{(m,n)}$ is an (m,n)-ideal of S. □

Theorem 2.7. *Suppose $(S,\Gamma,\cdot,\leq)$ is a unitary ordered LA-Γ-semigroup and $<s>_{(m,n)}$ is an (m,n)-ideal of S. Then the following assertions hold:*

(i) $((<s>_{(1,0)})^m\Gamma S] = (s^m\Gamma S]$;
(ii) $(S\Gamma(<s>_{(0,1)})^n] = (S\Gamma s^n]$;
(iii) $((<s>_{(1,0)})^m\Gamma S\Gamma(<s>_{(0,1)})^n] = (s^m\Gamma S\Gamma s^n]$.

Proof. (i) Since $<s>_{(1,0)} = (s\Gamma S]$, we obtain

$$\begin{aligned}((<s>_{(1,0)})^m\Gamma S] &= (((s\Gamma S])^m\Gamma S] \subseteq (((s\Gamma S)^m]\Gamma S] \subseteq ((s\Gamma S)^m\Gamma S]\\ &= ((s\Gamma S)^{m-1}\Gamma(s\Gamma S)\Gamma S] = (S\Gamma(s\Gamma S)\Gamma(s\Gamma S)^{m-1}]\\ &\subseteq ((s\Gamma S)\Gamma(s\Gamma S)^{m-1}] = ((s\Gamma S)\Gamma(s\Gamma S)^{m-2}\Gamma(s\Gamma S)]\\ &= ((s\Gamma S)^{m-2}\Gamma(s\Gamma S\Gamma s\Gamma S)] = ((s\Gamma S)^{m-2}\Gamma(s^2\Gamma S)]\\ &= \cdots = ((s\Gamma S)^{m-(m-1)}\Gamma(s^{m-1}\Gamma S)] \text{ if m is odd}\\ &= \cdots = ((s^{m-1}\Gamma S)\Gamma(s\Gamma S)^{m-(m-1)}] \text{ if m is even.}\\ &= (s^m\Gamma S].\end{aligned}$$

(ii) and (iii) can be proved similarly. □

Conclusion: The notion of LA-Γ-semigroups has been widely studied algebraic structures and it is a very good field of study for future research work. In this paper, we studied the notion of (m,n)-Γ-ideals in LA-Γ-semigroups. We obtained that if $(S,\Gamma,\cdot,\leq)$ is a unitary ordered LA-Γ-semigroup with zero 0 and S satisfies the condition that it contains no non-zero nilpotent (m,n)-Γ-ideals and if $R(L)$ is a 0-minimal right (left) Γ-ideal of S, then either $(R\Gamma L] = \{0\}$ or $(R\Gamma L]$ is a 0-minimal (m,n)-Γ-ideal of S. Also, we showed that if $(S,\Gamma,\cdot,\leq)$ is a unitary ordered LA-Γ-semigroup; A is an (m,n)-Γ-ideal of S and B is an (m,n)-Γ-ideal of A such that B is idempotent, then B is an (m,n)-Γ-ideal of S.

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