Eğitim Bilimleri Araştırmaları Dergisi
ULUSLARARASI E-DERGİ
Cilt:5 Sayı:1 Nisan 2015

EBAD - JESR

Journal of Educational Sciences Research
INTERNATIONAL E-JOURNAL
Vol:5 No:1 April 2015

Visualization of Relationship between a Function and Its Derivative[1]

Karmelita PJANIĆ[2], Edin LIĐAN[3] & Admir KURTANOVIĆ[4]

ABSTRACT

The first and second derivatives of a function provide an enormous amount of useful information about function itself as well as of the shape of the graph of the function. Mathematics curriculum in Bosnia and Herzegovina emphasises algebraic representation of a function ant its derivatives. That implies that concept of a derivative of a function is only partialy developed. On the other hand, an important skill to develop is that of producing the graph of the derivative of a function, given the graph of the function and conversely, to producing the graph of a function, given the graph of its derivative. In this paper we describe one possibility of enhancing pupils understanding of relationship between function and its derivative using specially designed Wolfram Mathematica applet. Preliminary results of implementation of the applet during the topic Examination of functions using its derivatives, indicate that visualization support better understanding of concept of function and its derivative.

Key Words: Function, Derivative, Graphical representation

crossref DOI Number: http://dx.doi.org/10.12973/jesr. 2015.51.12

[1] Paper presented on 8th International Computer & Instructional Technologies Symposium, Trakya Universiy Edirne, September 18-20, 2014.

[2] University of Bihać, Bosnia and Herzegovina - kpjanic@gmail.com
[3] University of Bihać, Bosnia and Herzegovina - lidjan_edin@hotmail.com
[4] University of Bihać, Bosnia and Herzegovina - kule21@hotmail.com

INTRODUCTION

Mathematical description and presentation of various phenomena and relationships is done by using the function. Therefore, we can reasonably say that the concept of function is one of the fundamental concepts in mathematics. Comprehension of this concept, as well as the strategies of its teaching and learning, is insatiable, important and interesting topic for study not only for mathematicians (Sfard, 1992; Sierpinska, 1992; Vinner & Dreyfus, 1989; Evangelidu at all, 2004; Pjanić, 2011; Pjanić & Nesimović, 2012). The concept of function can be viewed from various aspects such as the creation of a mental image (Vinner & Dreyfus, 1989), concept representations and connections between representations (Hitt Espinosa, 1998; Pjanić, 2011; Pjanić & Nesimović, 2012), correlation to physics (Mikelsen, 2005; Hadžibegović & Pjanić, 2011).

Representation and comprehension of concept of function and its derivative

On the importance of the issue of understanding the concept of function pointed Hit Espinosa (1998). On the one hand, different representations of the notion of function (graphics, charts, text, algebraic formula) provide a multitude of opportunities to better understanding the concept, but at the same time make the process of understanding more complicated. The most common tasks regarding functions in regular classes of elementary and secondary schools in Bosnia and Herzegovina, is drawing the graph of a function which is given by formula. Pupil is required one-directional knowledge of translating function from algebraic to graphical representation. Forcing only one manner of translating between just two representations leads to inadequate and poor formation of the notion of function (Pjanić, 2011). With this approach pupils' knowledge becomes fragmented, and pupils are not able to pass from one representation of function to another neither are able to interpret different representations, i.e. pupils are not able to interpret functions given graphically. With this, mathematical thinking is losing in its flexibility. Sierpinska (1992) indicated importance of making connections between different representations of function. In addition to the problem of establishing connections among different representations of function she stressed the issue of the interpretation of the graphical and symbolic representations of functions.

It is impossible to develop a concept of derivative without understanding the concept of function. When dealing with derivatives, then we effectively exploit the idea of infinitely small things. The importance of the calculus is huge and is reflected in the fact that it can describe physical laws underlying our entire world. Derivative is a key concept in understanding higher level mathematics. Students in Bosnia and Herzegovina meet the concept of derivation for the first time in the fourth (final) year of general secondary school. The mathematics program calls for learning the rules of derivation and application of the derivation in the examination of function. In the regular classroom pupils usually are asked to examine and draw the graph of the function given by formula.

The concept of derivative has to be built upon the previously constructed concepts. Derivation can be "seen" as a function, as the number (if determined in a point), the limit of the sequence of coefficients of secants' directions and as the rate of change. The range of potential determinations of derivative offers various possibilities in representing this concept (symbolic, algebraic, graphical,...). As previously stated, the initial understanding of differentiation involves a solid understanding of the notion of function. At the higher level of understanding of differentiation, it is necessary to understand the concept of the curve, i.e. to understand that each curve cannot be presented as a function (circle, ellipse, etc.). Thus, the derivation and differentiation, as advanced concepts in mathematics, "rely" to "simpler" concepts and cannot be understood without a solid, and in some cases, specific understanding of those "simpler" concepts.

Advanced concepts carry with them an internal complexity. Further on, even more advanced concepts such as differential equations are formed based on notions of derivative and differentiation. Those more advanced concepts that rely on concept of derivative will not be understood or explained without comprehension of differentiation either in conceptual and in procedural form - as a technique of differentiation.

Visualization in mathematics

Contemporary literature offers different approaches to studying the role of visualization in teaching mathematics. We will point out some of them. View on visualization in mathematics as "making the invisible and visible images" in the sense of "to be able to imagine the possible and impossible", gave Mason (1992). Zimmerman and Cunningham (1991) insist that mathematical visua1isation is not merely 'math appreciation through pictures' - a superficial substitute for understanding. Rather they maintain that visualisation supplies depth and meaning to understanding, serving as a reliable guide to problem solving, and inspiring creative discoveries. In order to achieve this understanding, however, they propose that visualization cannot be isolated from the rest of mathematics, implying that symbolical, numerical and visual representations of ideas must be formulated and connected. This project is conceptualised on the basis that visual thinking and graphical representation must be linked to other modes of mathematical thinking and other forms of representation (Tall, 1989). Presmeg (1995) see visualization as 'the relationship between images' - 'in order to visualize there is a need to create many images to construct relationships that will facilitate visualization and reasoning'. Hitt Espinosa (1997) suggests that visualization of mathematical concepts is 'not a trivial cognitive activity - to visualize is not the same as to see'. Being able to visualize is the 'ability to create rich, mental images which the individual can manipulate in his mind, rehearse different representations of the concept and, if necessary, use paper or a computer screen to express the idea in question'.

Visualization requires non-sequential, parallel information processing, and as such represents a major cognitive challenge for pupils which, step by step, is leading to sequential algorithmic reasoning (Eisenberg & Dreyfus, 1991).

ICT promotes visualization in learning and teaching mathematics. Main advantage of well-designed computer learning/teaching materials in mathematics is based on animations and interactive applets, which have a positive impact to the understanding of the presented material and also providing learning through research. A whole range of computer programs and tools with the intention of helping pupils and students in the formation of mathematical images, are available. There is great potential in ICT usage in promotion and encouraging visualization.

Research Questions

The aim of research is to explore if repetition and systematization of knowledge about functions and derivations with support of designed applet in Wolfram Mathematica influence pupils' performance in interpreting algebraic, numerical and graphical representations of function and its derivative. The main focus of the research has been laid on task of drawing a graph of the function given the graph of its derivative. This problem is reversed to one that pupils usually solve in regular classes. Accordingly to research's aim we formulated research questions:

- If and how much will pupils' performance in interpreting algebraic representation of function (argument, rate of change, monotony intervals) and its derivative change with regards to usage of Mathematica applet as support tool during repetition lessons.
- If and how much pupils' performance in translating function from graphical to algebraic and numerical representations will change with regards to usage of Mathematica applet as support tool during repetition lessons.
- If and how much pupils' performance in obtaining graphical representation of derivative of given function will change with regards to usage of Mathematica applet as support tool during repetition lessons.
- If and how much pupils' performance in obtaining graph of function knowing graph of its derivative will change with regards to usage of Mathematica applet as support tool during repetition lessons.

STUDY DESIGN

In order to examine the possibility of improving the understanding of transitioning representations of concept of function from one to another, interpreting functions' graphs and the relationship between the graphical representation of the function and its derivatives, an Wolfram Mathematica applet was created. The applet is created dynamically with the specific design, which enables parallel display of graph of function and graph of its derivative. Such design allows pupils to comparatively explore graphs and features of elementary functions and theirs derivatives. Figure 1 shows one frame of applet.

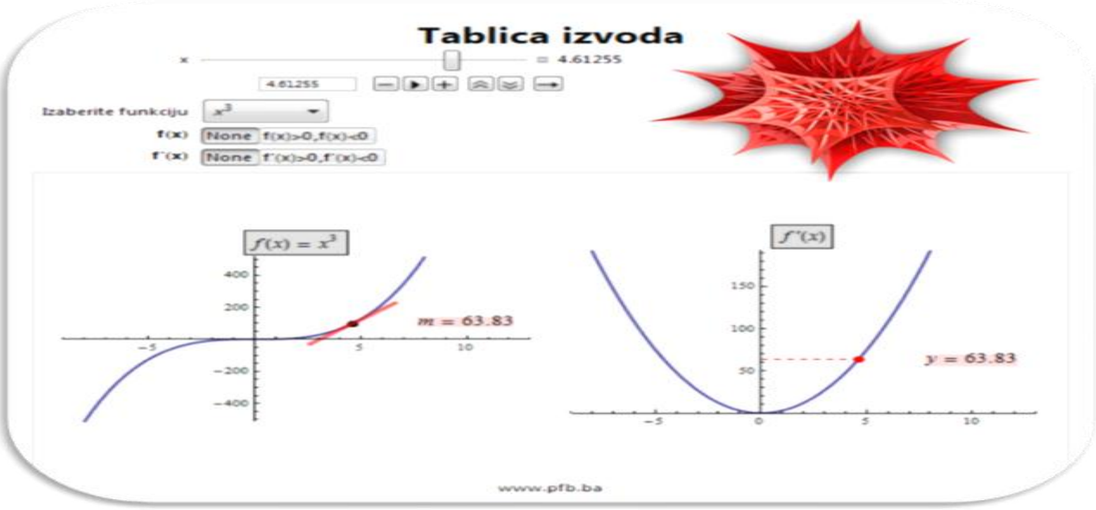

Figure 1. *One frame of applet*

The study included 132 high school seniors from western part of Bosnia and Herzegovina. The preliminary test that was intended to check the previously acquired knowledge about the concept of functions, derivation and application of the rules of derivation was given to all pupils. Based on the obtained results, the pupils were allocated into experimental and control group, comparable in performance in pretest. Repetition lessons in control group were conducted traditionaly: pen and pencil solving tasks of finding out derivative of given function and drawing graph of given function using derivatives. Mathematica applet was presented to experimental group within repetition lessons. Beside solving traditional tasks, pupils in experimental group had opportunity to explore connections of graphs of function and its derivative. Mathematica applet served as supporting tool, and was demonstrated in addition to traditional pen and pencil tasks. After repetition lessons post-test was given to both groups. Post-test asked pupils to interpret graphs of functions regarding to basic features, to identify changes in growth or decline of a function and to draw the graph of *f(x)* when given the graph of its derivative. Problem of drawing graph of function when given graph of its derivative usually is unfamiliar to pupils in Bosnia and Herzegovina. It should be noted that this type of problem was not presented to control and experimental group during repetition lessons. The graphs of functions included in post-test were not identical to those presented in Mathematica applet. Furthermore, due to testing pupils' performance in translating function in algebraic and graphical representation to numerical, linear, quadratic and cubic functions were selected in order to minimize mistakes in calculations. Answers in posttest were classified in three cathegories: no answer, incorrect answer and correct answer. Pre - test and post-test results have been processed in PASW 18.

RESULTS AND DISCUSSION

In the post-test focus is on identifying and establishing links between different representations of functions and its derivatives. The obtained results we present in the sequel.

In order to check differences in control and experimental group performance in interpreting algebraic representation of function (argument, rate of change, monotony intervals) and its derivative three functions (linear, quadratic and cubic) presented by formulae had been given to pupuls in each group. Based on given formula and domain, pupils were supposed to interprete change of argument and change of function value translating algebraic representation to numerical. Furthermore, based on given formula and obtained numerical values they had to determine monotony intervals in given domain and to determine sign of derivative of each given function.

Obtained results shows that performance in interpreting algebraic representation of given function is correlated to gropus. Chi-square test (Table 1) confirmed that performance in interpretation of algebraic representation of linear, quadratic and cubic functions depends on way how repetition clasess had been organised. In the most of cases, intensity of such dependance is strong as results of Cramer's V test show out.

Table 1. *Chi-square test: Dependance of interpretation of algebraic representation of function to usage of Matematica applet as support tool*

	Change of argument			*Change of function*			*Sign of derivation*		
	linear	*quadratic*	*cubic*	*linear*	*quadratic*	*cubic*	*linear*	*quadratic*	*cubic*
χ^2	38,382	21,425	26,552	18,957	35,596	33,086	23,743	23,350	35,096
df	2	2	2	2	2	2	2	2	2
Asymp. Sig	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
Cramer's V	0,539	0,403	0,449	0,379	0,519	0,501	0,424	0,421	0,516
Sig.	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000

0 cells (0%) have expected count less than 5. Sign level 0.01

By analysing crosstabulations we found out that number of correct answers that pupils who used Matematica applet as supporting tool during repetition classes had offered, dominantly contributed to statistical results in Table 1.

In order to answer to the second and the third research questions, pupils had been asked to interprete graph of given function in terms of numerical values of function in given domain and upon that to conclude if function increase or decline in given domain. Additionaly, pupils were asked to recognise graph of derivative of given function among several graphs offered. Insight in pupils' answers shows that there were 89,7% of pupils from control group and 95,3% pupils from experimental group who accurately translated graphical representation of given function to numerical. Also, there is no significant diference between control and experimental group in obtaining graphical representation of derivative of given function (χ^2= 4,751, df = 2, Asymp.Sig = 0,093). Frequences of pupils' answers related to translation of graphical representation of function to numerical as well as of visualization of derivative of given function can be compared in Figure 2.

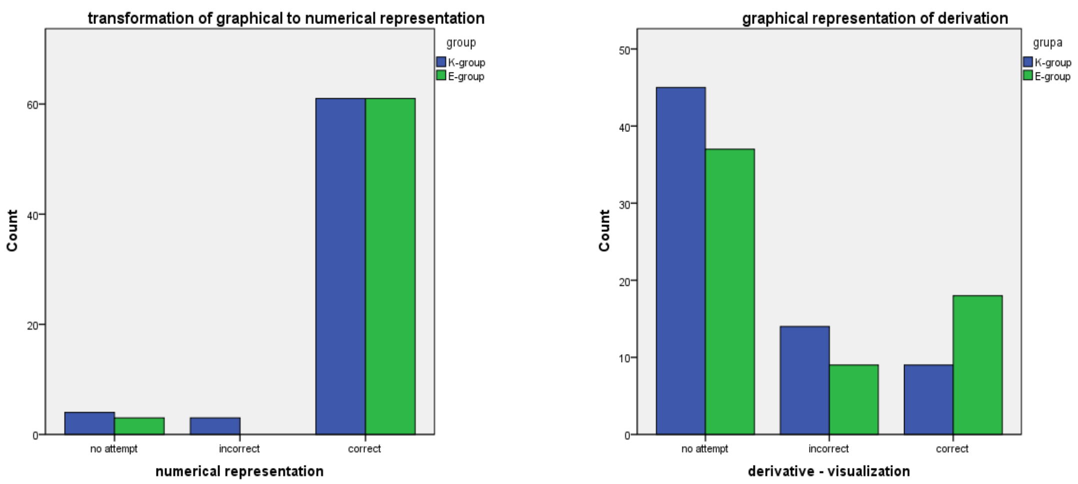

Figure 2. *Transformation between graphical and numerical representations*

On the other hand, usage of Mathematica applet as suporting tool in repetition clases had weak influence in pupils' interpretation of numerical representation of function (χ^2= 6,253, df = 2, Asymp.Sig = 0,044, Cramer's V = 0,218, A.Sig = 0,044). Frequences of pupils' answers related to second and third research questions can be compared in Figure 2.

Final task in post-test was related to making connections in between graphical representations of derivative and function. Frequences of pupils' answers to this task are shown in Figure 3.

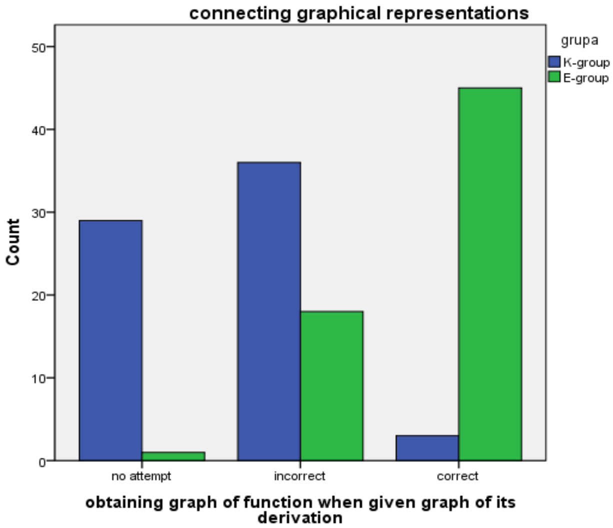

Figure 3. *Pupils' performance in obtaining graph of function given the graph of its derivative*

Distribution of pupils answers indicate that there is significant relationship between performance in obtaining graph of function given graph of its derivative and usage of Mathematica applet as support tool in repetition classes (χ^2 = 68,825, df = 2, Asymp.Sig = 0,000, Cramer's V = 0,722, A.Sig = 0,000). Usage of Mathematica applet during repetition classes strongly promote pupils' performance in this task.

CONCLUSION

Animations that are present in electronic materials for learning can be extremely useful and meaningful for students when it comes to the adoption of concepts of function and derivative.

Usage of computer applets can be beneficial for pupils. The fact that computer is sketching graphs and carrying out manipulations frees pupils to concentrate on the concepts.

One possibility of enhancing pupils' understanding of relationship between a function and its derivative using specially designed applet aimed to improve understanding of connections between graphical representations of function and its derivative. Preliminary results of implementation of the Wolfram Mathematica applet during the topic Examination of functions using derivatives indicate that visualization supports better understanding of the concept of function and its derivative. Results of post-test show that pupils who had used Mathematica applet as supporting tool during repetition classes on derivatives are more confident in establishing connections between derivative and function, as well as in using graphical representation of function and derivative.

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Please cite as:

Pjanić, K., Liđan, E. & Kurtanović, A. (2015). Visualization of relationship between a function and its derivative. *Eğitim Bilimleri Araştırmaları Dergisi - Journal of Educational Sciences Research, 5* (1), 205–213. http://ebad-jesr.com/