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Some Transmuted Software Reliability Models

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Abstract

The Hausdorff approximation of the shifted Heaviside function $h_{t_0}(t)$ by general transmuted family of cumulative distribution functions is studied and a value for the error of the best approximation is derived in this paper.
The outcomes of numerical examples confirm theoretical conclusions and they are derived by the help of CAS Mathematica.
Real data set which is proposed by Musa in [1] using general transmuted exponential software reliability model is examined.

1. Introduction

In this article we investigate the Hausdorff approximation of the shifted Heaviside function $h_{t_0}(t)$ by quadratic and cubic transmuted exponential cumulative distribution functions, based on Owoloko et al. model [2] and Rahman et al. [3] model.
Using CAS Mathematica we illustrate the results by given by us software modules.

1.1. Preliminaries

Definition 1.1. *[3] Let T be a random variable with cumulative distribution function (c.d.f.) $C(t)$. Then a general transmuted family, called k–transmuted family is defined as:*

$$M(t) = C(t) + (1 - C(t)) \sum_{i=1}^{k} \lambda_i (C(t))^i \tag{1.1}$$

with $\lambda_i \in [-1,1]$ for $i = 1,2,\ldots,k$ and $-k \le \sum_{i=1}^{k} \lambda_i < 1$.

For the quadratic transmuted family, see Shaw et Buckley [4].

The exponential distribution is a widely used lifetime distribution.
The (c.d.f.) of exponential distribution is given by:

$$C(t) = 1 - e^{-\frac{t}{\theta}},\ t \in [0,\infty).$$

Definition 1.2. *The quadratic transmuted exponential family is defined by (see, Owoloko et al. [2]):*

$$M_1(t) = \left(1 - e^{-\frac{t}{\theta}}\right)\left(1 + \lambda e^{-\frac{t}{\theta}}\right). \tag{1.2}$$

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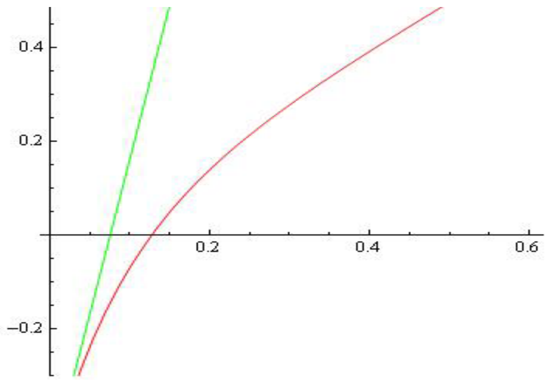

Figure 2.1: The functions $F(d)$ and $G(d)$.

Remark. From (1.1), we have

$$M(t) = C(t) + (1 - C(t))\left(\lambda_1 C(t) + \lambda_2 C^2(t)\right)$$

If $\lambda_2 = 0$ and $\lambda_1 = \lambda$ we have

$$\begin{aligned} M(t) &= C(t) + (1 - C(t))\,\lambda C(t) = \\ &= \left(1 - e^{-\frac{t}{\theta}}\right) + e^{-\frac{t}{\theta}}\lambda\left(1 - e^{-\frac{t}{\theta}}\right) = \\ &= \left(1 - e^{-\frac{t}{\theta}}\right)\left(1 + \lambda e^{-\frac{t}{\theta}}\right). \end{aligned}$$

Definition 1.3. *The (c.d.f.) of cubic transmutes exponential family is defined by:*

$$M_2(t) = (1+\lambda_1)\left(1 - e^{-\frac{t}{\theta}}\right) + (\lambda_2 - \lambda_1)\left(1 - e^{-\frac{t}{\theta}}\right)^2 - \lambda_2\left(1 - e^{-\frac{t}{\theta}}\right)^3. \tag{1.3}$$

We will note that the determination of compulsory in area of the Software Reliability Theory components, such as confidence intervals and confidence bounds, should also be accompanied by a serious analysis of the value of the best Hausdorff approximation [5] of the Heaviside function $h_{t_0}(t)$ by cumulative functions of type (1.1)–(1.2) - the subject of study in the present paper.

2. Main results

2.1. A note on the quadratic transmuted exponential family (1.2) [2]

Without loosing of generality we will look at the following (c.d.f.):

$$M_1^*(t) = \left(1 - e^{-\frac{t}{\theta}}\right)\left(1 + \lambda e^{-\frac{t}{\theta}}\right), \tag{2.1}$$

with

$$t_0 = -\theta \ln \frac{-1 + \lambda + \sqrt{1+\lambda^2}}{2\lambda}; \quad M_1^*(t_0) = \frac{1}{2}.$$

The one–sided Hausdorff distance d between the function $h_{t_0}(t)$ and the function (2.1) satisfies the relation

$$M_1^*(t_0 + d) = 1 - d. \tag{2.2}$$

The next theorem gives estimations for lower and upper bounds for d

Theorem 2.1. *Let*

$$p = -\frac{1}{2},$$

$$q = \frac{1}{2\lambda\theta}\left((1+\theta)2\lambda + (1-\lambda)^2 - (1-\lambda)\sqrt{1+\lambda^2}\right).$$

For the one–sided Hausdorff distance $d = d(\lambda,\theta)$ between $h_{t_0}(t)$ and the function (2.1) the following inequalities hold for:

$$2.1q > e^{1.05}$$

$$d_l = \frac{1}{2.1q} < d < \frac{\ln(2.1q)}{2.1q} = d_r. \tag{2.3}$$

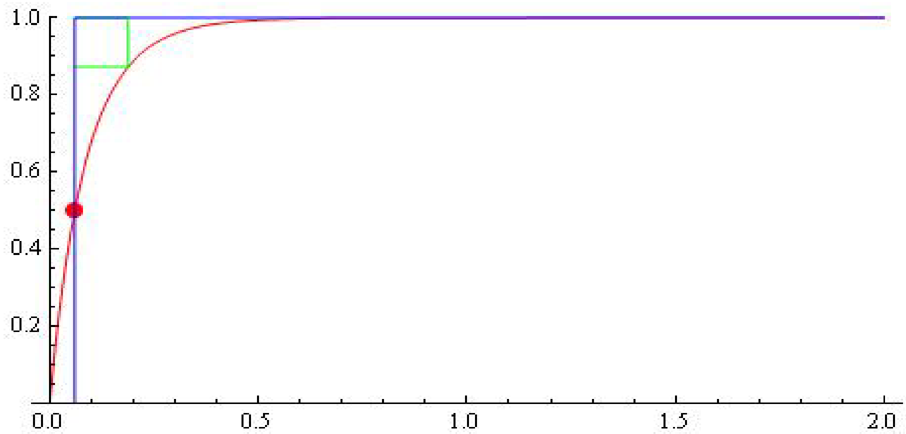

Figure 2.2: The model (2.1) for $\lambda = 0.2$, $\theta = 0.1$, $t_0 = 0.0598729$; H–distance $d = 0.127524$, $d_l = 0.0721072$, $d_r = 0.189613$.

Proof. We consider the function:

$$F(d) = M^*(t_0 + d) - 1 + d.$$

The function F is increasing because $F'(d) > 0$.
Consider the function

$$G(d) = p + qd.$$

We obtain $G(d) - F(d) = O(d^2)$ by the help of Taylor expansion.
Hence $G(d)$ approximates $F(d)$ with $d \to 0$ as $O(d^2)$ (see Fig. 2.1).
Evidently, $G'(d) > 0$.
Further, for $2.1q > e^{1.05}$ we have $G(d_l) < 0$ and $G(d_r) > 0$.

The proof of the theorem is completed. □

The model (2.1) for $\lambda = 0.2$, $\theta = 0.1$, $t_0 = 0.0598729$ is visualized on Fig. 2.2.

From the nonlinear equation (2.2) and inequalities (2.3) we have: $d = 0.127524$, $d_l = 0.0721072$, $d_r = 0.189613$.

2.2. A note on cubic transmuted exponential (c.d.f.) (1.3)

We consider the following family:

$$M_2^*(t) = (1+\lambda_1)\left(1 - e^{-\frac{t}{\theta}}\right) + (\lambda_2 - \lambda_1)\left(1 - e^{-\frac{t}{\theta}}\right)^2 - \lambda_2\left(1 - e^{-\frac{t}{\theta}}\right)^3. \tag{2.4}$$

Let t_0 is the positive root of the nonlinear equation

$$M_2^*(t_0) - \frac{1}{2} = 0.$$

The one–sided Hausdorff distance d_1 between the function $h_{t_0}(t)$ and the function (2.4) satisfies the relation

$$M_2^*(t_0 + d_1) = 1 - d_1.$$

Let

$$p_1 = e^{-\frac{3t_0}{\theta}}\left(\lambda_2 - (\lambda_1 + 2\lambda_2)e^{\frac{t_0}{\theta}} + (\lambda_1 + \lambda_2 - 1)e^{\frac{2t_0}{\theta}}\right),$$

$$q_1 = \frac{e^{-\frac{3t_0}{\theta}}}{\theta}\left(-3\lambda_2 + \theta e^{\frac{3t_0}{\theta}} + 2(\lambda_1 + 2\lambda_2)e^{\frac{t_0}{\theta}} + (1 - \lambda_1 - \lambda_2)e^{\frac{2t_0}{\theta}}\right).$$

In the next theorem lower and upper bounds for d_1 are given.

Theorem 2. For the one–sided Hausdorff distance d_1 between $h_{t_0}(t)$ and the function (2.4) the following inequalities are satisfied for:

$$2.1q_1 > e^{1.05}$$

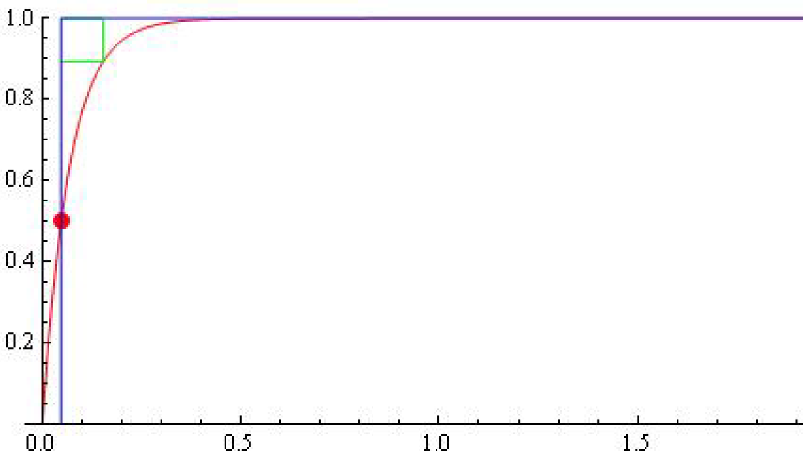

Figure 2.3: The model (2.4) for $\lambda_1 = 0.01$, $\lambda_2 = 0.05$, $\theta = 0.07$, $t_0 = 0.0473211$, $t_0 = 0.191515$; H–distance $d_1 = 0.106188$, $d_{l_1} = 0.0569506$, $d_{r_1} = 0.163197$.

$$d_{l_1} = \frac{1}{2.1q_1} < d_1 < \frac{\ln(2.1q_1)}{2.1q_1} = d_{r_1}.$$

The proof uses the ideas given here and will be skipped.

The model (2.4) for $\lambda_1 = 0.01$, $\lambda_2 = 0.05$, $\theta = 0.07$, $t_0 = 0.0473211$ is visualized on Fig. 2.3.

3. Numerical examples. Concluding remarks

Dataset, was proposed by Musa in [1]. The testing period is during the first 12 hours. The number of failures in each hour is given in Table 1.

Hour	*Number of Failures*	*Cumulative failures*
1	27	27
2	16	43
3	11	54
4	10	64
5	11	75
6	7	82
7	2	84
8	5	89
9	3	92
10	1	93
11	4	97
12	7	104

Table 1: Dataset [1]

The fitted model

$$M_2^*(t) = 104\left((1+\lambda_1)\left(1-e^{-\frac{t}{\theta}}\right) + (\lambda_2-\lambda_1)\left(1-e^{-\frac{t}{\theta}}\right)^2 - \lambda_2\left(1-e^{-\frac{t}{\theta}}\right)^3\right).$$

uses the data of Table 1 for the estimated parameters:

$$\lambda_1 = 0.207896;\ \ \lambda_2 = -0.733145;\ \ \theta = 3.44044$$

is plotted on Fig. 3.1.

In many cases it is appropriate to use the following deterministic software reliability model [1]:

$$M_3(t) = a^{b^{\frac{k_1}{t}}}.$$

The fitted model $M_3(t)$ based on the data of Table 1 for the estimated parameters:

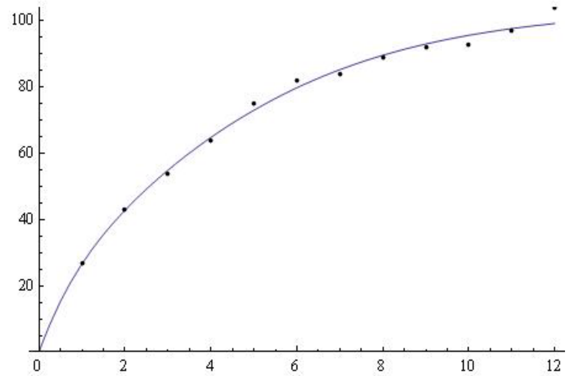

Figure 3.1: Approximation solution.

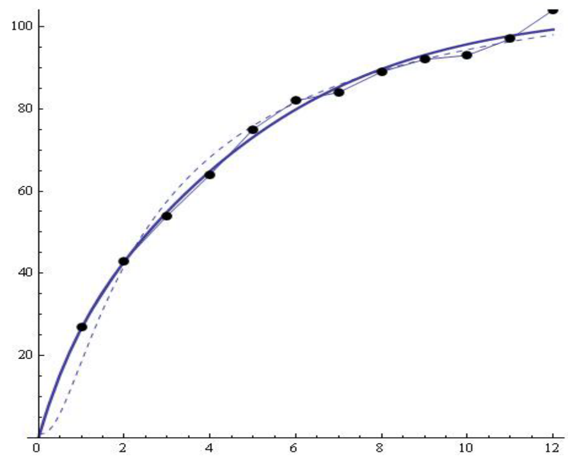

Figure 3.2: Comparison between the models: $M_2^*(t)$ – (thick) and $M_3(t)$ – (dashed).

$$a = 118.71;\ b = 0.667769;\ k_1 = 1.21843$$

is plotted on Fig. 3.2.

A good fit by the presented model $M_2^*(t)$ using for an example real data set is shown.

Obviously, studying of phenomenon ”super saturation” is mandatory element along with other important components - ”confidence bounds” and ”confidence intervals” when dealing with questions from Software Reliability Models domain.

For some software reliability models, see [6]–[47].

We hope that the results will be useful for specialists in this scientific area.

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