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(G'/G)-expansion Method for the Conformable space-time Fractional Jimbo-Miwa and Burger-like Equations

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Abstract
In this work, new analytic solutions for the nonlinear space-time fractional (3 + 1)-dimensional Jimbo-Miwa equation and Burger-like equation including conformable derivative are obtained by using the G'/G expansion method. The obtained traveling wave solutions are represented by the hyperbolic, trigonometric and rational functions. Simulations of the obtained solutions are presented at the end of the paper.

Keywords: *The space time fractional (3 + 1)-dimensional Jimbo-Miwa equation; The space time fractional Burger-like equation; Conformable derivative; G'/G expansion method; Traveling wave solutions.*

AMS Subject Classification (2010): *35R11 ; 35C07; 34K37.*

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1. Introduction

Fractional partial differential equations play an important role in various fields of science and engineering. Recently, scientists have developed various methods to solve such equations [1–12].

Jimbo and Miwa [13] first studied the (3 + 1)-dimensional Jimbo-Miwa equation as the second equation in the well known Kadomtsev-Petviashvili hierarchy of integrable systems. Certain interesting (3+1)-dimensional waves in physics but not pass any of the conventional integrability tests are modelled by the Jimbo-Miwa equation [14]. Furthermore, it plays an important role in modeling of the three-dimensional waves in plasma and optics. Hirota?s bilinear method, Kudryashov method, sub equation method, Exp-function method and generalized three-wave method have been applied to the (3 + 1)-dimensional Jimbo-Miwa equation in [15–19], respectively. Space-time fractional (3 + 1)-dimensional Jimbo-Miwa equation in the sense of Jumarie?s modified Riemann-Liouville derivative has been solved by using the generalized Bernoulli equation method and the $(G'/G, 1/G)$-expansion method in [20, 21], respectively. Conformable time fractional (3+1)-dimensional Jimbo-Miwa equation has been solved by using the modified form of the Kudryashov method [22].

Burgers equation is related to applications in acoustic phenomena and have been used to model turbulence and certain steady-state viscous flows. The Burgers equation has an important place in various areas of applied sciences and physical applications, such as modeling of fluid mechanics and financial mathematics, astrophysics [23]. Different varieties of Burger equation are available in literature such as inviscid Burgers? equation, viscous Burgers? equation, Burgers-like equation and coupled Burger?s equations. Improved tanh function method, sub-equation method, $\tan(F(\frac{\xi}{2}))$-expansion method have been applied to the Burgers-like equations in [24–26]. The space and time fractional Burgers-like equations with Caputo fractional derivative by the variational iteration method have been solved in [27].

(3 + 1)-dimensional Jimbo-Miwa equation and Burger-like equation have been widely studied in the literature. But there is not much work for the conformable fractional derivative case. To our knowledge, only time-fractional

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(3 + 1)-dimensional Jimbo-Miwa equation has been investigated [22]. In this study, we consider conformable space-time fractional (3 + 1)-dimensional Jimbo-Miwa equation and Burger-like equation. We establish new exact solitary wave solutions to these equations by G'/G expansion method.

2. Description of conformable fractional derivative and its properties

For a function $f:(0,\infty)\to R$, the conformable fractional derivative of f of order $0<\alpha<1$ is defined as (see, for example, [28])

$$T_t^{\alpha} f(t)=\lim_{\varepsilon\to 0}\frac{f(t+\varepsilon t^{1-\alpha})-f(t)}{\varepsilon}. \tag{2.1}$$

Some important properties of the conformable fractional derivative are as follows:

$$\begin{aligned} T_t^{\alpha}(af+bg)(t) &= aT_t^{\alpha}f(t)+bT_t^{\alpha}g(t),\ \forall a,b\in R, && (2.2)\\ T_t^{\alpha}(t^{\mu}) &= \mu t^{\mu-\alpha}, && (2.3)\\ T_t^{\alpha}(f(g(t))) &= t^{1-\alpha}g^{'}(t)f^{'}(g(t)). && (2.4)\end{aligned}$$

3. Analytic solutions to the conformable space-time fractional Jimbo-Miwa Equation equation

The conformable space-time fractional (3 + 1)-dimensional Jimbo-Miwa equation is given as follows (see, for example, [15–19])

$$\begin{aligned}&T_x^{\beta}T_x^{\beta}T_x^{\beta}T_y^{\gamma}u+3T_x^{\beta}T_y^{\gamma}uT_x^{\beta}u+3T_y^{\gamma}uT_x^{\beta}T_x^{\beta}u+2T_y^{\gamma}T_t^{\alpha}u-3T_x^{\beta}T_z^{\theta}=0,\\ &0<\alpha\le 1,\ 0<\beta\le 1,\ 0<\gamma\le 1,\ 0<\theta\le 1.\end{aligned} \tag{3.1}$$

Let us consider the following transformation

$$u(x,y,z,t)=U(\xi),\ \xi=k\frac{t^{\alpha}}{\alpha}+m\frac{x^{\beta}}{\beta}+n\frac{y^{\gamma}}{\gamma}+p\frac{z^{\theta}}{\theta}, \tag{3.2}$$

where k, m, n, p are constants. Substituting (3.2) into Eq.(3.1) we obtain the following ordinary differential equation (ODE)

$$m^3nU^{(4)}+6m^2nU'U''+(2kn-3mp)U''=0. \tag{3.3}$$

Integrating of Eq.(3.3) with zero constant of integration, we have

$$m^3nU'''+3m^2n(U')^2+(2kn-3mp)U'=0. \tag{3.4}$$

Let us suppose that the solution of Eq.(3.4) can be expressed in the following form:

$$U(\xi)=\sum_{i=0}^{N}a_i(\frac{G(\xi)'}{G(\xi)})^i, \tag{3.5}$$

where $G=G(\xi)$ satisfies the linear ODE in the form

$$G''+\lambda G'+\mu G=0,\ \lambda\neq 0,\mu\neq 0, \tag{3.6}$$

where a_i, λ and μ are constants to be determined.

Substituting Eq.(3.5) into Eq.(3.4) and then by balancing the highest order derivative term and nonlinear term in result equation, the value of N can be determined as 1. Therefore, Eq.(3.5) reduces to

$$U(\xi)=a_0+a_1\Big(\frac{G'}{G}\Big). \tag{3.7}$$

Substituting Eq.(3.7) into Eq.(3.4), collecting all the terms with the same power of $\frac{G'}{G}$, we can obtain a set of algebraic equations for the unknowns a_0, a_1, λ, μ, k,m,n,p:

$$\begin{aligned}
&3na_1^2m^2 - 6na_1m^3 = 0,\\
&6\lambda na_1^2m^2 - 12\lambda na_1m^3 = 0,\\
&3a_1mp - 2a_1kn - m^3n(a_1\lambda^2 + 2a_1\mu) + 3a_1^2m^2n(\lambda^2 + 2\mu) - 6a_1m^3\mu n - 6a_1\lambda^2m^3n = 0,\\
&3a_1\lambda mp - 2a_1k\lambda n - \lambda m^3n(a_1\lambda^2 + 2a_1\mu) + 6a_1^2\lambda m^2\mu n - 6a_1\lambda m^3\mu n = 0,\\
&3a_1m\mu p - 2a_1k\mu n - m^3\mu n(a_1\lambda^2 + 2a_1\mu) + 3a_1^2m^2\mu^2n = 0.
\end{aligned}$$

Solving the algebraic equations in Mathematica, we obtain the following set of solutions: $k = -\frac{\lambda^2m^3n-4m^3\mu n-3mp}{2n}$, $a_1 = 2m$.

When $\lambda^2 - 4\mu > 0$,

$$u_1(x,y,z,t) = a_0 + 2m\Big(-\frac{\lambda}{2} + \frac{\sqrt{\lambda^2-4\mu}}{2}\Big(\frac{C_1\sinh(\frac{\sqrt{\lambda^2-4\mu}\xi}{2}) + C_2\cosh(\frac{\sqrt{\lambda^2-4\mu}\xi}{2})}{C_1\cosh(\frac{\sqrt{\lambda^2-4\mu}\xi}{2}) + C_2\sinh(\frac{\sqrt{\lambda^2-4\mu}\xi}{2})}\Big)\Big).$$

When $\lambda^2 - 4\mu < 0$,

$$u_2(x,y,z,t) = a_0 + 2m\Big(-\frac{\lambda}{2} + \frac{\sqrt{4\mu-\lambda^2}}{2}\Big(\frac{-C_1\sin(\frac{\sqrt{4\mu-\lambda^2}\xi}{2}) + C_2\cos(\frac{\sqrt{4\mu-\lambda^2}\xi}{2})}{C_1\cos(\frac{\sqrt{4\mu-\lambda^2}\xi}{2}) + C_2\sin(\frac{\sqrt{4\mu-\lambda^2}\xi}{2})}\Big)\Big).$$

When $\lambda^2 - 4\mu = 0$,

$$u_3(x,y,z,t) = a_0 + 2m\Big(-\frac{\lambda}{2} + \frac{C_2}{C_1 + C_2\xi}\Big). \tag{3.8}$$

Here $\xi = (-\frac{\lambda^2m^3n-4m^3\mu n-3mp}{2n}\frac{t^\alpha}{\alpha} + m\frac{x^\beta}{\beta} + n\frac{y^\gamma}{\gamma} + p\frac{z^\theta}{\theta})$. In Fig.1 and Fig.2, we obtain simulation of the periodic solution (3.8). Fig.1 shows 3D plot of the traveling wave solution $u_2(x, 0.03, 0.03, t)$ of Eq.(3.1) for $\alpha = 0.75,\ \beta = 1$, $\gamma = 0.5,\ \theta = 0.75$, $m = 1,\ n = 5,\ p = -2,\ a_0 = 10$, $\lambda = 0.02,\ \mu = 0.3$, $C_1 = -3,\ C_2 = 4$, $-10 < x < 10$ and $0 < t < 10$. Fig.2 shows 2D plot of the traveling wave solution $u_2(x, 0.03, 0.03, 0.5)$ of Eq.(3.1) for $\alpha = 0.75,\ \beta = 1$, $\gamma = 0.5,\ \theta = 0.75$, $m = 1,\ n = 5,\ p = -2,\ a_0 = 10$, $\lambda = 0.02,\ \mu = 0.3$, $C_1 = -3,\ C_2 = 4$, $-10 < x < 10$ at $t = 0.5$. Note that the 3D graph gives the action of u in space x at time t and illustrates the change of amplitude and shape for each obtained solitary wave solutions. 2D graph shows the action of u in space x at fixed time $t = 0.5$.

4. Analytic solutions to the conformable space-time fractional Burgers-like equation

We consider conformable space-time fractional Burger-like equation (see, for example, [24–26])

$$T_t^\alpha u + T_x^\beta u + uT_x^\beta u + \frac{1}{2}T_x^\beta T_x^\beta u = 0,\ \ 0 < \alpha \le 1,\ \ 0 < \beta \le 1. \tag{4.1}$$

Let us consider the following transformation

$$u(x,t) = U(\xi),\ \ \xi = k\frac{t^\alpha}{\alpha} + m\frac{x^\beta}{\beta}, \tag{4.2}$$

where k, m are constants. Substituting (4.2) into Eq.(4.1) we obtain the following differential equations

$$(k+m)U' + mUU' + \frac{m^2}{2}U'' = 0. \tag{4.3}$$

Integrating of (4.3) with zero constant of integration, we have

$$(k+m)U + \frac{m}{2}U^2 + \frac{m^2}{2}U' = 0. \tag{4.4}$$

Let us suppose that the solution of Eq.(4.4) can be expressed in the form Eq.(3.5). Substituting Eq.(3.5) into Eq.(4.4) and then by balancing the highest order derivative term and nonlinear term in result equation, the value of N can be determined as 1. Therefore, Eq.(3.5) reduces to

$$U(\xi) = a_0 + a_1\Big(\frac{G'}{G}\Big). \tag{4.5}$$

Substituting Eq.(4.5) into Eq.(4.4), collecting all the terms with the same power of $\frac{G'}{G}$, we can obtain a set of algebraic equations for the unknowns a_0, a_1, λ, μ, k,m:

$$\begin{aligned}&(ma_1^2)/2 - (m^2 a_1)/2 = 0,\\ &a_1(k+m) + a_0 a_1 m - (a_1 \lambda m^2)/2 = 0,\\ &(ma_0^2)/2 + (k+m)a_0 - (a_1 m^2 \mu)/2 = 0.\end{aligned}$$

Solving the algebraic equations in Mathematica, we obtain the following set of solutions:

Case 1: $a_0 = \frac{m}{2}\Big(\lambda + \sqrt{\lambda^2-4\mu}\Big), a_1 = m, k = \frac{m}{2}\Big(-2 - m\sqrt{\lambda^2-4\mu}\Big)$:

When $\lambda^2 - 4\mu > 0$,

$$u_1(x,y,t) = \frac{m}{2}\Big(\lambda + \sqrt{\lambda^2-4\mu}\Big) + m\Big(-\frac{\lambda}{2} + \frac{\sqrt{\lambda^2-4\mu}}{2}\Big(\frac{C_1\sinh(\frac{\sqrt{\lambda^2-4\mu}\xi}{2}) + C_2\cosh(\frac{\sqrt{\lambda^2-4\mu}\xi}{2})}{C_1\cosh(\frac{\sqrt{\lambda^2-4\mu}\xi}{2}) + C_2\sinh(\frac{\sqrt{\lambda^2-4\mu}\xi}{2})}\Big)\Big).$$

When $\lambda^2 - 4\mu < 0$,

$$u_2(x,y,t) = \frac{m}{2}\Big(\lambda + \sqrt{\lambda^2-4\mu}\Big) + m\Big(-\frac{\lambda}{2} + \frac{\sqrt{4\mu-\lambda^2}}{2}\Big(\frac{-C_1\sin(\frac{\sqrt{4\mu-\lambda^2}\xi}{2}) + C_2\cos(\frac{\sqrt{4\mu-\lambda^2}\xi}{2})}{C_1\cos(\frac{\sqrt{4\mu-\lambda^2}\xi}{2}) + C_2\sin(\frac{\sqrt{4\mu-\lambda^2}\xi}{2})}\Big)\Big).$$

When $\lambda^2 - 4\mu = 0$,

$$u_3(x,y,t) = \frac{m}{2}\Big(\lambda + \sqrt{\lambda^2-4\mu}\Big) + m\Big(-\frac{\lambda}{2} + \frac{C_2}{C_1 + C_2\xi}\Big). \tag{4.6}$$

Here $\xi = \frac{m}{2}\Big(-2 - m\sqrt{\lambda^2-4\mu}\Big)\frac{t^\alpha}{\alpha} + m\frac{x^\beta}{\beta}$.

Case 2: $a_0 = \frac{m}{2}\Big(\lambda - \sqrt{\lambda^2-4\mu}\Big), a_1 = m, k = \frac{m}{2}\Big(-2 + m\sqrt{\lambda^2-4\mu}\Big)$:

When $\lambda^2 - 4\mu > 0$,

$$u_4(x,y,t) = \frac{m}{2}\Big(\lambda - \sqrt{\lambda^2-4\mu}\Big) + m\Big(-\frac{\lambda}{2} + \frac{\sqrt{\lambda^2-4\mu}}{2}\Big(\frac{C_1\sinh(\frac{\sqrt{\lambda^2-4\mu}\xi}{2}) + C_2\cosh(\frac{\sqrt{\lambda^2-4\mu}\xi}{2})}{C_1\cosh(\frac{\sqrt{\lambda^2-4\mu}\xi}{2}) + C_2\sinh(\frac{\sqrt{\lambda^2-4\mu}\xi}{2})}\Big)\Big).$$

When $\lambda^2 - 4\mu < 0$,

$$u_5(x,y,t) = \frac{m}{2}\Big(\lambda - \sqrt{\lambda^2-4\mu}\Big) + m\Big(-\frac{\lambda}{2} + \frac{\sqrt{4\mu-\lambda^2}}{2}\Big(\frac{-C_1\sin(\frac{\sqrt{4\mu-\lambda^2}\xi}{2}) + C_2\cos(\frac{\sqrt{4\mu-\lambda^2}\xi}{2})}{C_1\cos(\frac{\sqrt{4\mu-\lambda^2}\xi}{2}) + C_2\sin(\frac{\sqrt{4\mu-\lambda^2}\xi}{2})}\Big)\Big).$$

When $\lambda^2 - 4\mu = 0$,

$$u_6(x,y,t) = \frac{m}{2}\Big(\lambda - \sqrt{\lambda^2-4\mu}\Big) + m\Big(-\frac{\lambda}{2} + \frac{C_2}{C_1 + C_2\xi}\Big).$$

Here $\xi = \frac{m}{2}\Big(-2 + m\sqrt{\lambda^2-4\mu}\Big)\frac{t^\alpha}{\alpha} + m\frac{x^\beta}{\beta}$.

In Fig.3, we obtain simulation of the solitary wave solution (4.6). Fig.3 is simulation of the kink-shaped soliton solution. Kink wave is traveling wave which rises or descends from one asymptotic state to another. The kink solution approaches a constant at infinity. Fig.3 presents 3D plot of the traveling wave solution $u_1(x,1,t)$ of Eq.(4.1) for $\alpha = 0.5,\ \ \beta = 1, \theta = 0.75, m = 1,\ \ n = 0.5, \lambda = 3,\ \ \mu = 2, C_1 = 0.5\ \ C_2 = -0.25, -5 < x < 15, 0 < t < 15$.

5. Conclusion

In the present paper, the G'/G expansion method is applied to solve the space-time fractional (3 + 1)-dimensional Jimbo-Miwa equation and Burger-like equation including conformable derivative. The obtained traveling wave solutions are represented by the hyperbolic, trigonometric and rational functions. The obtained solutions by using presented method are new and have not been expressed in the literature so far. The method can also be applied to the other conformable nonlinear fractional differential equations.

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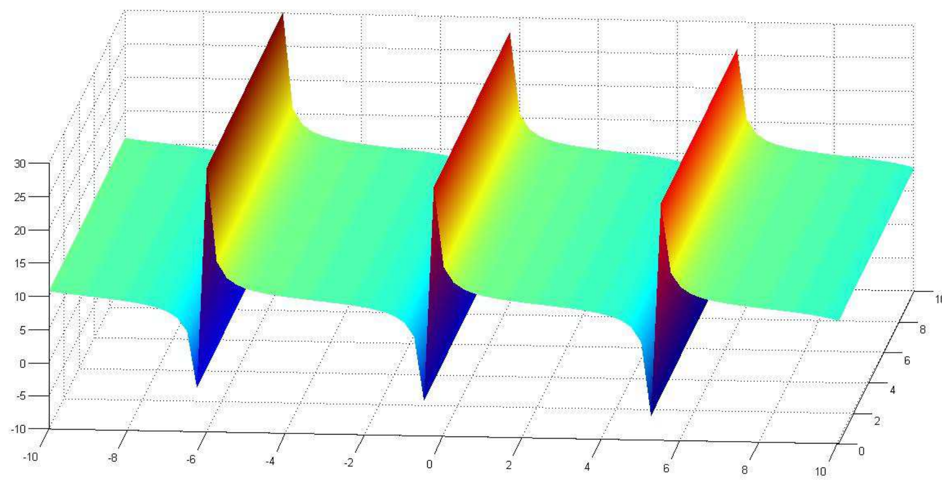

Figure 1. 3D plot of the periodic wave solution $u_2(x, 0.03, 0.03, t)$ of Eq.(3.1) .

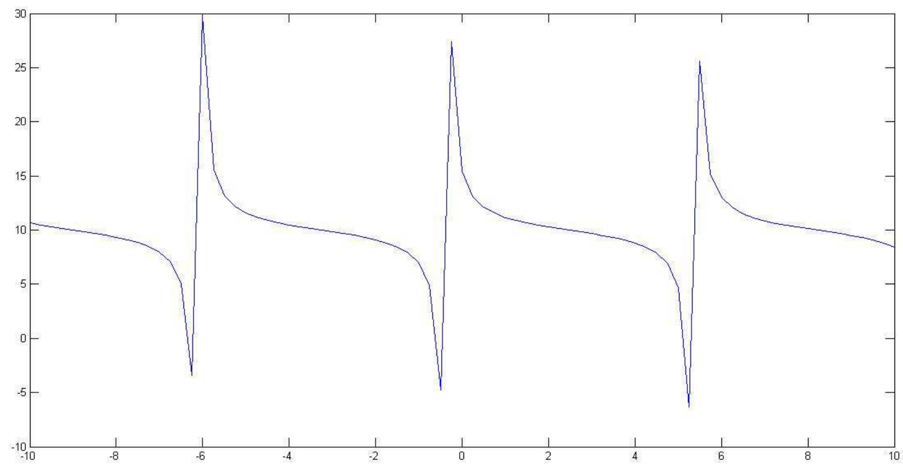

Figure 2. 2D plot of the periodic wave solution $u_2(x, 0.03, 0.03, 0.5)$ of Eq.(3.1).

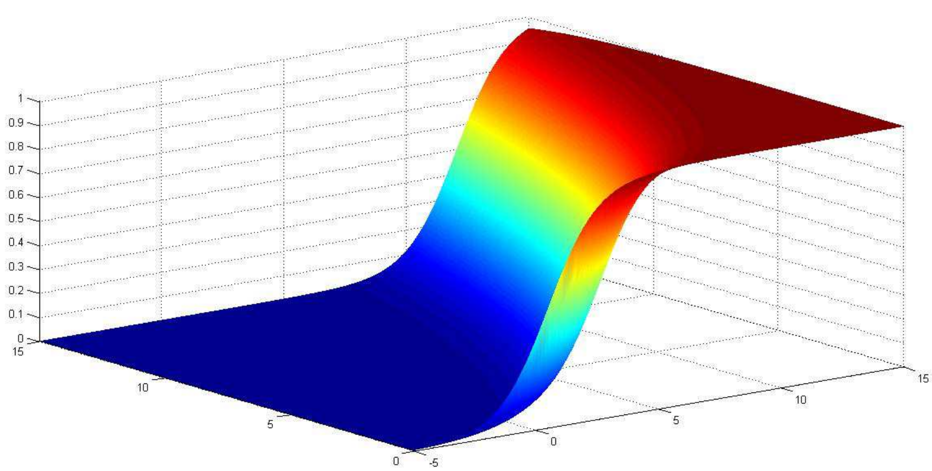

Figure 3. 3D plot of the solitary wave solutions $u_1(x, 1, t)$ of Eq.(4.1) .