



## SUPRA $bT$ -SET CONNECTED FUNCTIONS IN SUPRA TOPOLOGICAL SPACES

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**ABSTRACT.** In this paper, we came out with the new approach of functions called  $bT^\mu$ -set connected functions and studied its properties. Also, we have retrieve separation axioms using  $bT^\mu$ -set connected functions in supra topological spaces.

### 1. INTRODUCTION

Function play an necessary role in Mathematics. The latest growth of simulating and universality of continuous function, compactness, disconnectedness and separation axioms etc., are used by many topologist. The idea of clopen functions was introduced by Reilly and Vamanamurthy in the year 1983([14]). In the year 1999, Dontchev, Ganster and Reilly came out with a new function called regular set- connected([3]). The concept of regular set-connected functions is extended to almost clopen functions by Ekici in the year 2005[5]. The concept of supra topology were introduced by Mashhour et.al.([13]) and also studied  $S$ -continuous maps and  $S^*$ - continuous maps in Supra topological spaces. Also Mashhour et.al ([13]) discussed that many results of topological spaces, whereas some become false. Also the authors remarked that the intersection of two supra open sets need not be supra open and also the intersection of an open set and supra open set need not be supra open. They were also introduced  $S-T_0, S-T_1, S-T_2, S-T_2^*$  spaces and discussed their relationship with the topological spaces  $T_0, T_1, T_2, T_2^*$  spaces. In 2008, R.Devi, S.Sampath kumar and M. Caldas([2]) introduced supra  $\alpha$ -open sets and  $s\alpha$ -continuous functions and investigate some of the basic properties of this function. Sayed and Takashi Noiri([15]) studied the approach of  $b$ -open set and supra  $b$  continuity in supra topological space in 2010 and also discussed about the

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relation between supra b-continuous maps and supra b-open maps. E. Ekici and M. Caldas([4]) introduced and discussed some properties of slightly- $\gamma$ continuous functions. Ganes M. Pandya, C. Janaki and I. Arockiarani([6]) introduced a new class of set connected functions called  $\pi$ -set connected and also investigated the relationship between  $\pi$ -set connected function, separation axioms and covering properties. Supra b-compact and supra b-Lindelof spaces were introduced by Jamal M. Mustafa.et.al([7]) and also discussed about the compactness and Lindelof spaces. The authors([8]) came out with a new concept of supra bT closed set and studied some of its properties.

The idea of this paper is to come out with the new theory of  $bT^\mu$ -set connected functions and also to bring out new separation axioms utilized  $bT^\mu$ -set connected functions in supra topological spaces. We define this class of functions by the requirement that the inverse image of each supra clopen (that is, supra open and supra closed) set in the codomain is  $bT^\mu$ -clopen (that is,  $bT^\mu$ -open and  $bT^\mu$ -closed) in the domain. Also we investigate the fundamental properties of  $bT^\mu$ -set connected functions.

In the present paper,  $(X, \tau)$  and  $(Y, \sigma)$  represent supra topological spaces on which no separation axioms are assumed unless otherwise mentioned. Let  $A$  be a subset of  $X$ . We denote the supra interior and supra closure of a set by  $int^\mu(A)$  and  $cl^\mu(A)$  respectively.

## 2. PRELIMINARIES

**Definition 2.1.** [13, 15] *A subfamily of  $\mu$  of  $X$  is said to be a supra topology on  $X$ , if*

- (i)  $X, \phi \in \mu$
- (ii) if  $A_i \in \mu$  for all  $i \in J$  then  $\cup A_i \in \mu$ .

*The pair  $(X, \mu)$  is called supra topological space. The elements of  $\mu$  are called supra open sets in  $(X, \mu)$  and complement of a supra open set is called a supra closed set.*

**Definition 2.2.** [13, 15]

- (i) *The supra closure of a set  $A$  is denoted by  $cl^\mu(A)$  and is defined as  $cl^\mu(A) = \cap\{B : B \text{ is a supra closed set and } A \subseteq B\}$ .*
- (ii) *The supra interior of a set  $A$  is denoted by  $int^\mu(A)$  and defined as  $int^\mu(A) = \cup\{B : B \text{ is a supra open set and } A \supseteq B\}$ .*

**Definition 2.3.** [8] *A subset  $A$  of a supra topological space  $(X, \tau)$  is called  $bT^\mu$ -closed set if  $bcl^\mu(A) \subset U$  whenever  $A \subset U$  and  $U$  is  $T^\mu$ -open in  $(X, \tau)$ .*

**Definition 2.4.** [8] *Let  $(X, \tau)$  and  $(Y, \sigma)$  be two supra topological spaces. A function  $f:(X, \tau) \rightarrow (Y, \sigma)$  is called  $bT^\mu$ -continuous if  $f^{-1}(V)$  is  $bT^\mu$ -closed in  $(X, \tau)$  for every supra closed set  $V$  of  $(Y, \sigma)$ .*

**Definition 2.5.** [11] A function  $f:(X, \tau) \rightarrow (Y, \sigma)$  is called contra  $bT^\mu$ -continuous if  $f^{-1}(V)$  is  $bT^\mu$ -closed in  $(X, \tau)$  for every supra open set  $V$  of  $(Y, \sigma)$ .

**Definition 2.6.** [10] A function  $f:(X, \tau) \rightarrow (Y, \sigma)$  is called perfect  $bT^\mu$ -continuous if the inverse image of every  $bT^\mu$  - closed in  $(Y, \sigma)$  is both supra closed and supra open set  $V$  of  $(X, \tau)$ .

**Definition 2.7.** [13] A function  $f:(X, \tau) \rightarrow (Y, \sigma)$  is called  $S^*$ -continuous if  $f^{-1}(V)$  is supra closed in  $(X, \tau)$  for every supra closed set  $V$  of  $(Y, \sigma)$ .

**Definition 2.8.** [9] A map  $f:(X, \tau) \rightarrow (Y, \sigma)$  is said to be a  $bT^\mu$ -closed map ( $bT^\mu$ -open map) if the image  $f(A)$  is  $bT^\mu$ -closed( $bT^\mu$  -open) in  $(Y, \sigma)$  for each supra closed (supra open) set  $A$  in  $(X, \tau)$ .

**Definition 2.9.** [10] Let  $(X, \tau)$  and  $(Y, \sigma)$  be two supra topological spaces. A function  $f:(X, \tau) \rightarrow (Y, \sigma)$  is called Strongly  $bT^\mu$ -continuous if  $f^{-1}(V)$  is supra closed in  $(X, \tau)$  for every  $bT^\mu$ -closed set  $V$  of  $(Y, \sigma)$ .

**Definition 2.10.** [12] A space  $X$  is called supra normal (resp. mildly normal) if for any pair of disjoint supra closed (resp. supra regular closed) subsets  $A$  and  $B$  of  $X$ , then there exist disjoint supra open sets  $U$  and  $V$  such that  $A \subset U, B \subset V$ .

**Definition 2.11.** [12] A space  $X$  is called almost supra normal if for each disjoint supra closed set  $A$  and supra regular closed set  $B$  of  $X$ , there exist disjoint supra open sets  $U$  and  $V$  such that  $A \subset U, B \subset V$ .

**Definition 2.12.** [10] Let  $(X, \tau)$  and  $(Y, \sigma)$  be two supra topological spaces. A function  $f:(X, \tau) \rightarrow (Y, \sigma)$  is called perfectly  $bT^\mu$ -continuous if the inverse image of every  $bT^\mu$ -closed in  $Y$  is both supra closed and supra open in  $X$ .

**Definition 2.13.** [4] A supra topological space  $X$  is said to be supra clopen  $T_1$  space if for each pair of distinct points  $x$  and  $y$  of  $X$ , there exist supraopen sets  $U$  and  $V$  containing  $x$  and  $y$  respectively such that  $x \in U, y \notin U$  and  $x \notin V, y \in V$ .

**Definition 2.14.** [4] A supra topological space  $X$  is said to be supra ultra Hausdroff space if every two distinct points of  $X$  can be separated by disjoint supra clopen sets.

**Definition 2.15.** [4] A supra topological space  $X$  is said to be supra clopen  $T_0$  if for each pair of distinct points in  $X$ , there exist a supra clopen set containing one point but not the other.

**Definition 2.16.** [4] A supra topological space  $X$  is said to be supra ultra regular if for each supra closed set  $F$  of  $X$  and each  $x \notin F$ , there exist disjoint supra clopen sets  $U$  and  $V$  such that  $F \subset U$  and  $x \in V$ .

**Definition 2.17.** [5] A space  $X$  is said to be

- (i) clopen  $T_1$ , if for each pair of distinct points  $x$  and  $y$  of  $X$ , there exist clopen sets  $U$  and  $V$  containing  $x$  and  $y$  respectively such that  $y \notin U$  and  $x \notin V$ .

(ii) clopen  $T_2$ , if for each pair of distinct points  $x$  and  $y$  of  $X$ , there exist disjoint clopen sets  $U$  and  $V$  in  $X$  such that  $x \in U$  and  $y \in V$ .

**Definition 2.18.** [12] A space  $X$  is said to be strongly  $bT^\mu$ -normal if for every pair of disjoint  $bT^\mu$ -closed sets  $A$  and  $B$ , then there exist disjoint  $bT^\mu$ -open sets  $U$  and  $V$  such that  $A \subset U$  and  $B \subset V$ .

**Definition 2.19.** [6] A function  $f:(X,\tau) \rightarrow (Y, \sigma)$  is said to be set connected if  $f^{-1}(V)$  is clopen in  $X$  for every  $V \in CO(Y)$ .

### 3. $bT^\mu$ - SET CONNECTED FUNCTION

**Definition 3.1.** A function  $f:(X,\tau) \rightarrow (Y, \sigma)$  is called supra set connected function if  $f^{-1}(V)$  is supra clopen in  $(X,\tau)$  for every  $V \in$  supra clopen  $(Y, \sigma)$ .

**Definition 3.2.** A function  $f:(X,\tau) \rightarrow (Y, \sigma)$  is called  $bT^\mu$ -set connected function if  $f^{-1}(V)$  is  $bT^\mu$  clopen in  $(X,\tau)$  for every supra clopen set  $V$  in  $(Y, \sigma)$ .

**Example 3.3.** Let  $X = Y = \{a, b, c\}$  with  $\tau = \{X, \phi, \{a\}, \{a, c\}, \{b, c\}\}$  and  $\sigma = \{Y, \phi, \{b\}, \{b, c\}, \{a, c\}\}$ . A function  $f:(X,\tau) \rightarrow (Y,\sigma)$  is defined as  $f(a) = b$ ,  $f(b) = a$ ,  $f(c) = c$ . The supra clopen set of  $(Y,\sigma)$  are  $\{Y, \phi, \{b\}, \{a, c\}\}$  and the  $bT^\mu$  clopen of  $(X,\tau)$  are  $\{X, \phi, \{a\}, \{c\}, \{a, b\}, \{b, c\}\}$ . Here  $f$  is  $bT^\mu$ -set connected function.

**Theorem 3.4.** A function  $f:(X,\tau) \rightarrow (Y, \sigma)$  is perfectly  $bT^\mu$ -continuous function then  $f$  is  $bT^\mu$ -set connected functions.

*Proof.* Let  $V$  be supra clopen in  $(Y,\sigma)$ . Since  $f$  is perfectly  $bT^\mu$ -continuous function,  $f^{-1}(V)$  is supra clopen in  $(X,\tau)$ . We know that every supra clopen set is  $bT^\mu$  clopen. Therefore  $f^{-1}(V)$  is  $bT^\mu$  clopen in  $(X,\tau)$ . Hence  $f$  is  $bT^\mu$ -set connected functions.  $\square$

**Definition 3.5.** A supra topological space  $(X,\tau)$  is said to be  $bT^\mu$  locally indiscrete if every  $bT^\mu$  open set of  $(X,\tau)$  is  $bT^\mu$  closed set in  $(X,\tau)$ .

**Theorem 3.6.** For a function  $f:(X,\tau) \rightarrow (Y, \sigma)$  is perfectly  $bT^\mu$ -continuous function and  $(X,\tau)$  is  $bT^\mu$  locally indiscrete then  $f$  is  $bT^\mu$ -set connected function.

*Proof.* Let  $V$  be supra clopen in  $(Y,\sigma)$ . Since  $f$  is perfectly  $bT^\mu$ -continuous and  $(X,\tau)$  is  $bT^\mu$  locally indiscrete,  $f^{-1}(V)$  is  $bT^\mu$ -open and  $bT^\mu$ -closed in  $(X,\tau)$ . Therefore  $f^{-1}(V)$  is  $bT^\mu$  clopen in  $(X,\tau)$ . Hence  $f$  is  $bT^\mu$ -set connected functions.  $\square$

**Theorem 3.7.** Let  $f:(X,\tau) \rightarrow (Y, \sigma)$  and  $g:(Y,\sigma) \rightarrow (Z, \gamma)$ . The following properties hold.

- (i) If  $f$  and  $g$  are  $bT^\mu$ -set connected functions then  $g \circ f:(X,\tau) \rightarrow (Z,\gamma)$  is  $bT^\mu$ -set connected functions.
- (ii) If  $f$  is  $bT^\mu$ -set connected functions and  $g$  is perfectly  $bT^\mu$ -continuous function then  $g \circ f:(X,\tau) \rightarrow (Z, \gamma)$  is  $bT^\mu$ -set connected functions.
- (iii) If  $f$  is supra set connected and  $g$  is  $bT^\mu$ -set connected functions then  $g \circ f:(X,\tau) \rightarrow (Z, \gamma)$  is  $bT^\mu$ -set connected functions.

*Proof.* (i) Let  $V$  is supra clopen in  $(Z, \gamma)$ . By hypothesis,  $g^{-1}(V)$  is  $bT^\mu$  clopen in  $(Y, \sigma)$ . Since,  $f$  is  $bT^\mu$ -set connected functions,  $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$  is  $bT^\mu$  clopen . Therefore  $g \circ f$  is  $bT^\mu$ -set connected functions.

(ii) Let  $V$  is supra clopen in  $(Z, \gamma)$ . By hypothesis,  $g^{-1}(V)$  is supra clopen in  $(Y, \sigma)$ . Since,  $f$  is  $bT^\mu$ -set connected functions,  $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$  is  $bT^\mu$  clopen . Therefore  $g \circ f$  is co  $bT^\mu$  set connected functions.

(iii) Let  $V$  is supra clopen in  $(Z, \gamma)$ . By hypothesis,  $g^{-1}(V)$  is  $bT^\mu$  clopen in  $(Y, \sigma)$ . Since,  $f$  is supra set connected functions,  $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$  is supra clopen. Since, we know that every supra clopen set is  $bT^\mu$  clopen. Therefore  $g \circ f$  is  $bT^\mu$ -set connected functions.  $\square$

**Theorem 3.8.** *If  $f: (X, \tau) \rightarrow (Y, \sigma)$  is a surjective  $bT^\mu$ -open and  $bT^\mu$ -closed function and  $g: (Y, \sigma) \rightarrow (Z, \gamma)$  is a function such that  $g \circ f : (X, \tau) \rightarrow (Z, \gamma)$  is  $bT^\mu$ -set connected , then  $g$  is  $bT^\mu$ -set connected functions.*

*Proof.* Let  $V$  be a supra clopen in  $(Z, \gamma)$ . By hypothesis,  $(g \circ f)^{-1}(V)$  is  $bT^\mu$  clopen in  $(X, \tau)$ ,  $f^{-1}(g^{-1}(V))$  is  $bT^\mu$  clopen in  $(X, \tau)$ . Since,  $f$  is surjective  $bT^\mu$ -open and  $bT^\mu$ -closed.  $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$  is  $bT^\mu$  clopen. Therefore  $g$  is  $bT^\mu$ -set connected functions.  $\square$

**Definition 3.9.** *A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called co  $bT^\mu$ -irresolute function if the inverse image of each  $bT^\mu$  clopen set in  $Y$  is a  $bT^\mu$  clopen set in  $(X, \tau)$ .*

**Example 3.10.** *Let  $X = Y = \{a, b, c\}$  with  $\tau = \{X, \phi, \{a\}, \{a, c\}, \{b, c\}\}$  and  $\sigma = \{Y, \phi, \{b\}, \{b, c\}, \{a, c\}\}$ . A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is defined as  $f(a) = b, f(b) = a, f(c) = c$ . The  $bT^\mu$  clopen set of  $(Y, \sigma)$  are  $\{Y, \phi, \{b\}, \{c\}, \{a, c\}, \{a, b\}, \}$  and the  $bT^\mu$  clopen of  $(X, \tau)$  are  $\{X, \phi, \{a\}, \{c\}, \{a, b\}, \{b, c\}\}$ . Here  $f$  is co  $bT^\mu$ -irresolute function.*

**Theorem 3.11.** *Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  and  $g: (Y, \sigma) \rightarrow (Z, \gamma)$  be a function. Then the following properties hold.*

- (i) *If  $f$  is co  $bT^\mu$ -irresolute and  $g$  is  $bT^\mu$ -set connected, then  $g \circ f$  is  $bT^\mu$ -set connected .*
- (ii) *If  $f$  is  $bT^\mu$ -set connected and  $g$  is co  $bT^\mu$ -irresolute, then  $g \circ f$  is perfectly  $bT^\mu$ -continuous.*

*Proof.* (i) Let  $V$  be any supra clopen set in  $(Z, \gamma)$ . Since,  $g$  is  $bT^\mu$ -set connected,  $g^{-1}(V)$  is  $bT^\mu$  clopen. Since,  $f$  is co  $bT^\mu$ -irresolute,  $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$  is  $bT^\mu$  clopen. Therefore  $g \circ f$  is  $bT^\mu$ -set connected .

(ii) Let  $V$  be any supra clopen set in  $(Z, \gamma)$ . By the co  $bT^\mu$ -irresolute of  $g$ ,  $g^{-1}(V)$  is  $bT^\mu$  clopen. Since,  $f$  is  $bT^\mu$ -set connected ,  $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$  is  $bT^\mu$  clopen. Since we know that every  $bT^\mu$  clopen set is  $bT^\mu$ -closed. Therefore  $g \circ f$  is perfectly  $bT^\mu$ -continuous.  $\square$

## 4. SEPARATION AXIOMS

**Definition 4.1.** A supra topological space  $X$  is said to be  $co\ bT^\mu\ T_1$  if for each pair of distinct points  $x$  and  $y$  of  $X$ , there exist  $bT^\mu$  clopen sets  $U$  and  $V$  containing  $x$  and  $y$  respectively such that  $x \in U$ ,  $y \notin U$  and  $x \notin V$ ,  $y \in V$ .

**Example 4.2.** Let  $X = \{a, b, c, d\}$  with  $\tau = \{X, \phi, \{a\}, \{c\}, \{a, c\}, \{a, d\}, \{a, c, d\}\}$ . The  $bT^\mu$  clopen sets are  $\{X, \phi, \{c\}, \{a, d\}, \{b, c\}, \{a, b, d\}, \}$ . Let  $U = \{a, b, d\}$  and  $V = \{b, c\}$ . Let  $a$  and  $c$  be the point in  $X$  and  $a \in U$ ,  $c \notin U$  and  $c \in V$ ,  $a \notin V$ . Hence  $X$  is  $co\ bT^\mu\ T_1$  space.

**Theorem 4.3.** If  $f:(X,\tau) \rightarrow (Y, \sigma)$  is  $bT^\mu$ -set connected injective and  $Y$  is supra clopen  $T_1$  space, then  $X$  is  $co\ bT^\mu\ T_1$  space.

*Proof.* Let  $x$  and  $y$  be any two distinct points in  $(X,\tau)$ . Since  $f$  is injective, we have  $f(x)$  and  $f(y)$  such that  $f(x) \neq f(y)$ . Since  $(Y,\sigma)$  is supra clopen  $T_1$ , there exist a supra clopen sets  $U$  and  $V$  in  $(Y,\sigma)$  such that  $f(x) \in U$ ,  $f(y) \notin U$ ,  $f(y) \in V$  and  $f(x) \notin V$ . Therefore we have  $x \in f^{-1}(U)$ ,  $y \notin f^{-1}(U)$ ,  $y \in f^{-1}(V)$  and  $x \notin f^{-1}(V)$ , where  $f^{-1}(U)$  and  $f^{-1}(V)$  are  $bT^\mu$  clopen subsets of  $(X,\tau)$  because,  $f$  is  $bT^\mu$ -set connected functions. This shows that  $(X,\tau)$  is  $co\ bT^\mu\ T_1$  space.  $\square$

**Definition 4.4.** A supra topological space  $(X,\tau)$  is said to be  $co\ bT^\mu\ T_0$  if for each pair of distinct points in  $(X,\tau)$ , there exist a  $bT^\mu$  clopen set containing one point but not the other.

**Example 4.5.** Let  $X = \{a, b, c, d\}$  with  $\tau = \{X, \phi, \{a\}, \{c\}, \{a, c\}, \{a, d\}, \{a, c, d\}\}$ . The  $bT^\mu$  clopen sets are  $\{X, \phi, \{c\}, \{a, d\}, \{b, c\}, \{a, b, d\}\}$ . Let the distinct point be  $a$  and  $c$  in  $(X,\tau)$ . Let the  $bT^\mu$  clopen set  $\{a, b, d\}$  containing the point  $a$  but not the point  $c$ . Hence  $X$  is  $co\ bT^\mu\ T_0$  space.

**Definition 4.6.** A supra topological space  $(X,\tau)$  is said to be  $co\ bT^\mu\ T_2$  space if every two distinct points of  $(X,\tau)$  can be separated by disjoint  $bT^\mu$  clopen sets.

**Example 4.7.** Let  $X = \{a, b, c, d\}$  with  $\tau = \{X, \phi, \{a\}, \{c\}, \{a, c\}, \{a, d\}, \{a, c, d\}\}$ . The  $bT^\mu$  clopen sets are  $\{X, \phi, \{c\}, \{a, d\}, \{b, c\}, \{a, b, d\}, \}$ . Let  $a$  and  $c$  be the two distinct point in  $(X,\tau)$ . Let  $U = \{a, d\}$  and  $V = \{b, c\}$ ,  $a \in U$ ,  $c \in V$  and  $U \cap V = \phi$ . Hence  $(X,\tau)$  is a  $co\ bT^\mu\ T_2$  space.

**Theorem 4.8.** If  $f:(X,\tau) \rightarrow (Y,\sigma)$  is  $bT^\mu$ -set connected injection on  $Y$  is clopen  $T_0$ , then  $(X,\tau)$  is  $co\ bT^\mu\ T_2$  space.

*Proof.* Let  $x$  and  $y$  be any pair of distinct points of  $(X,\tau)$  and  $f$  be injective. Then  $f(x) \neq f(y)$  in  $(Y,\sigma)$ . Since  $(Y,\sigma)$  is clopen  $T_0$ , there exist a supra clopen set containing  $f(x)$  but not  $f(y)$ . Then we have  $x \in f^{-1}(U)$  and  $y \notin f^{-1}(U)$ . Since  $f$  is  $bT^\mu$ -set connected,  $f^{-1}(U)$  is  $bT^\mu$  clopen in  $(X,\tau)$ . Also  $x \in f^{-1}(U)$  and  $y \in X - f^{-1}(U)$ . This implies every pair of distinct points of  $(X,\tau)$  can be separated by disjoint  $bT^\mu$  clopen sets in  $(X,\tau)$ . Therefore  $(X,\tau)$  is  $co\ bT^\mu\ T_2$  space.  $\square$

**Theorem 4.9.** *If  $f:(X,\tau) \rightarrow(Y, \sigma)$  is perfectly  $bT^\mu$ -continuous injection on  $(Y,\sigma)$  is co  $bT^\mu T_0$ , then  $X$  is supra ultra Hausdorff space.*

*Proof.* Let  $a$  and  $b$  be any pair of distinct points of  $(X,\tau)$  and  $f$  be injective. Then  $f(a) \neq f(b)$  in  $(Y,\sigma)$ . Since  $(Y,\sigma)$  is co  $bT^\mu T_0$ , there exist a  $bT^\mu$  clopen set containing say  $f(a)$  but not  $f(b)$ . Then we have  $a \in f^{-1}(U)$  and  $b \notin f^{-1}(U)$ . Since  $f$  is perfectly  $bT^\mu$ -continuous,  $f^{-1}(U)$  is supra clopen in  $(X,\tau)$ . Also  $a \in f^{-1}(U)$  and  $b \in X-f^{-1}(U)$ . This implies every pair of distinct points of  $X$  can be separated by disjoint supra clopen sets in  $(X,\tau)$ . Therefore  $(X,\tau)$  is supra ultra Hausdorff.  $\square$

**Theorem 4.10.** *If  $f:(X,\tau) \rightarrow(Y, \sigma)$  is  $bT^\mu$ -set connected injection on  $(Y,\sigma)$  is supra ultra Hausdorff space, then  $X$  is co  $bT^\mu T_2$  space.*

*Proof.* Let  $c$  and  $d$  be any pair of distinct points of  $(X,\tau)$  and  $f$  be injective. Then  $f(c) \neq f(d)$  in  $(Y,\sigma)$ . Since  $(Y,\sigma)$  is supra ultra Hausdorff space, there exist supra clopen sets  $U$  and  $V$  in  $(Y,\sigma)$  such that  $f(c) \in U$  and  $f(d) \in V$  and  $U \cap V = \phi$ . This implies  $c \in f^{-1}(U)$  and  $d \in f^{-1}(V)$ . Since  $f$  is  $bT^\mu$ -set connected,  $f^{-1}(U)$  and  $f^{-1}(V)$  is  $bT^\mu$  clopen in  $(X,\tau)$ . Also  $f^{-1}(U) \cap f^{-1}(V) = f^{-1}(U \cap V) = \phi$ . Thus every two distinct points of  $(X,\tau)$  can be separated by disjoint  $bT^\mu$  clopen sets. Therefore  $(X,\tau)$  is co  $bT^\mu T_2$  space.  $\square$

**Theorem 4.11.** *If  $f:(X,\tau) \rightarrow(Y, \sigma)$  is perfectly  $bT^\mu$ -continuous injection on  $(Y,\sigma)$  is co  $bT^\mu T_2$  space, then  $X$  is supra ultra Hausdorff space.*

*Proof.* Let  $a$  and  $b$  be any pair of distinct points of  $(X,\tau)$  and  $f$  be injective. Then  $f(a) \neq f(b)$  in  $(Y,\sigma)$ . Since  $(Y,\sigma)$  is co  $bT^\mu T_2$  space, there exist  $bT^\mu$  clopen sets  $U$  and  $V$  in  $(Y,\sigma)$  such that  $f(a) \in U$  and  $f(b) \in V$  and  $U \cap V = \phi$ . This implies  $a \in f^{-1}(U)$  and  $b \in f^{-1}(V)$ . Since  $f$  is perfectly  $bT^\mu$ -continuous,  $f^{-1}(U)$  and  $f^{-1}(V)$  is  $bT^\mu$  clopen in  $(X,\tau)$ . Also  $f^{-1}(U) \cap f^{-1}(V) = f^{-1}(U \cap V) = \phi$ . Thus every two distinct points of  $(X,\tau)$  can be separated by disjoint supra clopen sets. Therefore  $(X,\tau)$  is supra ultra Hausdorff.  $\square$

**Theorem 4.12.** *Let  $(X,\tau)$  be a space and  $(Y,\sigma)$  be co  $bT^\mu T_2$  space. If  $f:(X,\tau) \rightarrow(Y, \sigma)$  is co  $bT^\mu$ -irresolute injective, then  $(X,\tau)$  is co  $bT^\mu T_2$ .*

*Proof.* Let  $x$  and  $y$  be any two distinct points of  $(X,\tau)$ . Then,  $f(x)$  and  $f(y)$  are distinct points of  $(Y,\sigma)$ , because  $f$  is injective. Since  $(Y,\sigma)$  is co  $bT^\mu T_2$  space, there are disjoint  $bT^\mu$  clopen sets  $U$  and  $V$  in  $(Y,\sigma)$  containing  $f(x)$  and  $f(y)$  respectively. Since,  $f$  is co  $bT^\mu$ -irresolute and  $U \cap V = \phi$ , we have  $f^{-1}(U)$  and  $f^{-1}(V)$  are disjoint  $bT^\mu$  clopen sets in  $(X,\tau)$  such that  $x \in f^{-1}(U)$  and  $y \in f^{-1}(V)$ . Hence  $(X,\tau)$  is co  $bT^\mu T_2$  space.  $\square$

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