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# Study of Kinematic, Dynamic and Jocobian Analysis of Electro- pneumatic Stewart Parallel Robot (6dof) and Provide a Comprehensive Stat- Space Models for Control Purposes

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**Abstract.** In recent years, parallel robots have become the focus of research since, as compared with similar serial robots, these robots possess many superior features. Upon a brief introduction of the Stewart parallel robot as a robot with the most industrial applications, its many advantages over similar serial robots were studied and a complete kinematic and dynamic analysis of this robot presented. Subsequently, to obtain an integrated system for control applications, a comprehensive model of this robot's state- space was introduced through formulating the explicit governing equations for the Stewart system and its associated drive/actuator system.

**Keywords:** Stewart Parallel Robot, Kinematics, Dynamics, Jacobian Analysis, State- Space Model.

## 1. INTRODUCTION

As compared with their serial counterparts, parallel-linked robotic arms, better known as the Stewart Platform, have demonstrated greater advantages including higher stiffness and hardness, higher load-to-weight ratio, and greater precision; features that are now recognized by many researchers around the world. The parallel mechanism was first used for testing vehicle wheels by Gaff, a member of Britain's ruling party. Later on, Dr. Stewart developed the parallel mechanism as a flight simulation system. His innovation found many useful and diverse applications in the subsequent years. Today, the generalized Stewart Platform is used in such applications as automation, defensive and security, transportation, and development of machine tools in shipbuilding industries [1].

Pneumatic drives are still very widely used in automation processes. As actuators, these drives are implemented in light-duty equipment with relatively high load-to-weight indexes. On the average, pneumatic actuators are preferred to hydraulic and electric actuators due to their approximately 20% lower cost manufacturing technology. The most significant advantages of

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pneumatic systems are simple installation and maintenance, wide range of accessibility, clean and non-inflammable operation, and lower sensitivity to temperature change [2, 5].

In spite of the fact that the Stewart parallel robot has been researched on a wide scale, there are few published works on the dynamics of this robot's drive system and the friction in its joint space. Various methods have been proposed for kinematic and dynamic analysis of this type of robot and the deriving of its governing equations. This article presents a systematic approach for obtaining these equations. In the first section, the direct and inverse kinematics of the mechanism are systematically analyzed. Upon completion of the kinematic analysis, the relevant equations were derived and their time derivatives obtained to yield the relations, in the form of velocity and Jacobian terms that existed between various components of the robot. The second section focuses on the dynamic analysis of the robotic system and presents a comprehensive model of the mechanism with regard to its actuator as well as its rigid-body dynamic. There are various methods for dynamic analysis of the Stewart mechanism. In the present study, the Lagrange method was used to extract the dynamic equations. In this method, the dynamic equations are obtained by considering the work/energy stored in the system so as to automatically eliminate the burdensome constraint forces [3]. Finally, in the last section, the general governing equations are appropriately formulated to achieve an integrated system.

## 2. SYSTEM MODELING

### 2.1. Kinematics Analysis

In this section, the kinematics of the Gough- Stewart presented. In Figure 1, an example of this mechanism is shown.

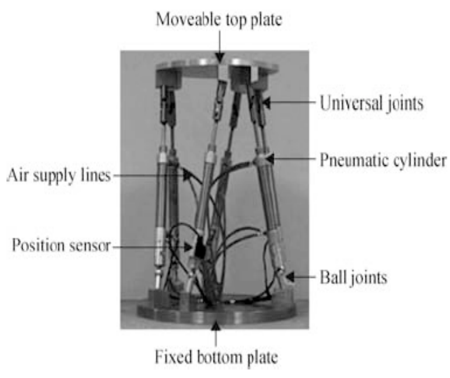


**Figure 1.** Stewart electro- pneumatic mechanism [4].

#### 2.1.1. Inverse Kinematics Analysis

In this section, the six-degree-of-freedom Goff-Stewart mechanism is studied. As was mentioned in the Introduction, the inverse kinematic analysis involves selecting an arbitrary path for the end effector to calculate the motion of all the existing links in the mechanism. This system consists of two planes A and B. Plane A is fixed whereas Plane B moves. These planes are pinned together at points Ai and Bi (rotational axes) via six bases fitted with lifting jacks. The movable plane can be moved via jack piston strokes. Figure 2, shows the components of the Stewart mechanism [1, 3].

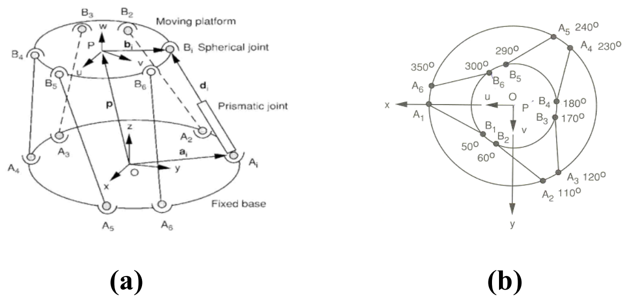


**Figure 2.** Top and beside view of a Stewart platform [6].

In Figure 2, Plane A is the base (fixed) platform, and Plane B the moving platform. Figure 3 demonstrates the mass (center of gravity) coordinate system of the end effector (movable platform).

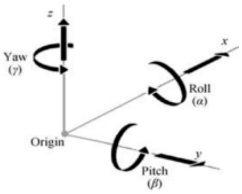


**Figure 3.** Linear and angular motions [4].

In this system, (x,y,z,α,β,γ) is the coordinates of the movable platform center of gravity. The closed- loop kinematic equations can be expressed as [3, 4]:

$$a_i + D_i s_i = p + b_i \tag{1}$$

In Equation 1, we have:

$$a_i = \begin{bmatrix} r_A \cos\lambda_i \\ r_A \sin\lambda_i \\ 0 \end{bmatrix} = \begin{bmatrix} a_{ix} & a_{iy} & 0 \end{bmatrix}^T \tag{2}$$

Equation 2 determines the position vector A relative to the fixed plane. The above vector's parameters can be calculated for the following relations:

$$\lambda_i = \frac{i\pi}{3} - \frac{\theta_A}{2} \xrightarrow{for} i = 1,3,5 \tag{3}$$

$$\lambda_i = \lambda_{i-1} + \theta_{Ai} \xrightarrow{for} i = 2,4,6 \tag{4}$$

$$\theta_{Ai} = \cos^{-1} \frac{d_i n_{Ai}}{|d_i|} \le \theta_{A,\max} \tag{5}$$

In Equations 1 to 5, di, θA, θA,max and nAi represent the length of mechanism base vectors, rotation angles of the joints, maximum angle of each joint (for the joints installed on the base plane), and the unit vector along the rotational joints in the basic coordinate system. Equation 1 also expresses the position vector Bi relative to the movable plane as:

$$b_i{}^B = \begin{bmatrix} r_B \cos\varphi_i \\ r_B \sin\varphi_i \\ 0 \end{bmatrix} = \begin{bmatrix} b_{iu} & b_{iv} & 0 \end{bmatrix}^T \tag{6}$$

So that the above vector's parameters can be computed from the following relations:

$$\varphi_i = \frac{i\pi}{3} - \frac{\theta_B}{2} \xrightarrow{for} i = 1,3,5 \tag{7}$$

$$\varphi_i = \varphi_{i-1} + \theta_{Bi} \xrightarrow{for} i = 2,4,6 \tag{8}$$

$$\theta_{Bi} = \cos^{-1} \frac{d_i R_B^A n_{Bi}}{|d_i|} \le \theta_{B,\max} \tag{9}$$

In Equations 6 to 9, θB, max and nBi are the maximum joint angle (for the joints installed on the movable plane) and the unit vector along the rotational joints of the moving platform coordinates. Also, $b_i = \begin{bmatrix} b_{ix} & b_{iy} & b_{iz} \end{bmatrix}^T$ expresses the position of vector $b_i^B$ relative to the fixed plane. The unit vectors along each platform base can be obtained as:

$$s_i = (p + b_i - a_i) / D_i \tag{10}$$

In Equation 10, the coordinates of the moving platform (p) center of gravity (c.g.) is demonstrated as:

$$p = [x, y, z]^T \tag{11}$$

And each actuator length vector is calculated from the following relation:

$$D_i = p + R_B^A \cdot B_{b_i} - a_i \xrightarrow{for} i = 1,2,\ldots,6 \tag{12}$$

If Di represents the lengths of the Stewart mechanism bases, then:

$$if : d_i = \left\| D \right\| \quad , \qquad d_{\min} \le d \le d_{\max}$$

$$\Rightarrow d_i = \pm \left\{ \left[ p + R_B^A b_i^B - a_i \right]^T \left[ p + R_B^A b_i^B - a_i \right] \right\}^{\frac{1}{2}} \tag{13}$$

Based on Equation 13, we can argue that at each instant, the status information of the moving platform can be accessed. Also, as negative length does not exist, only the positive part of the above relation shall be acceptable. Now, by substituting the relevant parameters in Equation 13, we can rewrite Di as:

$$\begin{aligned} \mathrm{d}_i^2 &= x^2 + y^2 + z^2 + r_A^2 + r_B^2 + 2\left(r_{11} b_{ix} + r_{12} b_{iy}\right)\left(x - a_{ix}\right) \\ &+ 2\left(r_{21} b_{ix} + r_{22} b_{iy}\right)\left(y - a_{iy}\right) \\ &+ 2\left(r_{31} b_{ix} + r_{32} b_{iy}\right)z - 2\left(x a_{ix} + y a_{iy}\right) \end{aligned} \tag{14}$$

In Equation 14, rA and rB are the radii of the fixed and moving planes respectively. Also, the rotation matrix $R_B^A$ can be calculated from the following relation:

$$\begin{aligned} R_B^A &= \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \\ &= \begin{bmatrix} c\alpha c\beta & c\alpha s\beta s\gamma - s\alpha c\gamma & c\alpha s\beta c\gamma + s\alpha s\gamma \\ s\alpha c\beta & s\alpha s\beta s\gamma + c\alpha c\gamma & s\alpha s\beta c\gamma - c\alpha s\gamma \\ -s\beta & c\beta s\gamma & c\beta c\gamma \end{bmatrix} \end{aligned} \tag{15}$$

In Equation 15, we have: s=sin, c=cos

### 2.1.2. Forward Kinematics Analysis

As was pointed out in the Introduction, the purpose of analyzing the system in this section is to calculate the coordinates of the moving platform c.g. using the existing joints in the mechanism. The relation between the c.g. coordinates and the joint coordinates in the mechanism can be rewritten as [1]:

$$
\begin{aligned}
F_i(x,y,z,\alpha,\beta,\gamma) &= x^2 + y^2 + z^2 + r_A^2 + r_B^2 + 2\left(r_{11}b_{ix} + r_{12}b_{iy}\right)(x - a_{ix}) \\
&+2\left(r_{21}b_{ix} + r_{22}b_{iy}\right)(y - a_{iy}) + 2\left(r_{31}b_{ix} + r_{32}b_{iy}\right)z \\
&-2\left(xa_{ix} + ya_{iy}\right) - d_i^2 = 0 \xrightarrow{for} i = 1,2,\ldots,6
\end{aligned} \tag{16}
$$

### 2.2. JACOBIAN ANALYSIS

The Jacobian of a system actually describes the relation between dependent and independent variables in that system. As shall be defined later, the momentum of a system is a function of the change in the stroke of the pneumatic cylinders. Therefore, the Jacobian matrix (the magnitude of which represents the length of the bases in the mechanism) can be calculated from the following relation [4, 5]:

$$\Delta\dot{q} = [\dot{d}_1, \dot{d}_2, \dot{d}_3, \dot{d}_4, \dot{d}_5, \dot{d}_6] \tag{17}$$

$$\dot{X} = [v_p, w_p]^T \tag{18}$$

In Equations 17 and 18, $\Delta\dot{q}$ and $\dot{X}$ are the output vector and the momentum respectively. The Jacobian equation can be extracted from the closed-loop equation of the ith member (i.e., the velocity of that member). Considering the physical structure of the system in Fig. 2, we can write:

$$\overline{op} + \overline{pb_i} = \overline{oA_i} + \overline{A_iB_i} \tag{19}$$

Taking the time derivative of Equation 19, wee obtain:

$$v_p + w_p \times b_i = d_i w_i \times s_i + s_i d_i \tag{20}$$

So that in Equation 20, we have:

$$s_i = \overrightarrow{A_iB_i} \tag{21}$$

$$b_i = \overrightarrow{pB_i} \tag{22}$$

In the above relation, wi, vp, and wp represent the angular velocity of the i-th member relative to the fixed plane, the linear velocity of Vector P, and the angular velocity of Vector P respectively. Multiplying both sides of the Equation 20 by si, we obtain [6]:

$$s_i \cdot v_p + (p_i \times s_i) \cdot p_i = \dot{d}_i \tag{23}$$

$$J_x \dot{X} = J_q \dot{q} \tag{24}$$

$$J = J_q^{-1} \times J_x = \begin{bmatrix} s_1^T & (b_1 \times s_1)^T \\ s_2^T & (b_2 \times s_2)^T \\ s_3^T & (b_3 \times s_3)^T \\ s_4^T & (b_4 \times s_4)^T \\ s_5^T & (b_5 \times s_5)^T \\ s_6^T & (b_6 \times s_6)^T \end{bmatrix} \tag{25}$$

## 2.3. DYNAMICS ANALYSIS

In this section, the Lagrangian (energy) method was used to extract the kinetic equations. In this method, the kinetic equations are obtained via the work/energy stored in the system so that the bothersome constraint forces can be eliminated automatically. The Lagrangian equation is expressed as [1, 6]:

$$L = T - U \tag{26}$$

Where in equation (26), T is the kinetic energy and U is the potential energy of the system calculated from the following relationship:

$$T = \frac{1}{2} m v^2 + \frac{1}{2} I w^2 \tag{27}$$

$$U = mgh \tag{28}$$

$$\frac{d}{dt}\left(\frac{\partial l}{\partial \dot{q}}\right) - \frac{\partial l}{\partial q} + \frac{\partial D \cdot E}{\partial \dot{q}} = Q \tag{29}$$

Where in equation (29), the parameters q, D.E, and Q are the vector of generalized coordinates, friction and damper forces vector, and the generalized forces vector respectively, which are calculated using the principle of virtual work. By placing Lagrangian equation into motion equation of a dynamic system, dynamic equation can be expressed in the following form:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + F_{fr}(\dot{q}) = \tau \tag{30}$$

Where in equation (30), the parameters M, C, G, Ffr, and τ indicate positive definitive mass matrix with the order of 6*1, forces and torques generated by the vector provider of centrifugal forces and Carioles acceleration with the order of 6*1, vector provider of torque caused by the gravitational pull with the order of 6*1, vector provider of torque caused by the friction forces with the order of 6*1, and the vector of generalized forces applied with the order of 6*1, respectively. In equation (5), generalized torques vector can be rewritten in terms of applied forces of the mechanism drives [1]:

$$\mathrm{M} = \begin{bmatrix} \mathrm{m} & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathrm{m} & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathrm{m} & 0 & 0 & 0 \\ 0 & 0 & 0 & M_{44} & M_{45} & M_{46} \\ 0 & 0 & 0 & M_{54} & M_{55} & 0 \\ 0 & 0 & 0 & M_{64} & 0 & M_{66} \end{bmatrix} \tag{31}$$

As element of the inertia matrix (M) are calculated according to the following equations [6]:

$$I = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix} \tag{32}$$

$$\begin{cases} M_{44} = I_x C_\beta^2 C_\gamma^2 + I_y C_\beta^2 S_\gamma^2 + I_z S_\beta^2 \\ M_{45} = M_{54} = (I_x - I_y) C_\beta C_\gamma S_\gamma \\ M_{46} = M_{64} = I_z S_\beta \\ M_{55} = I_x S_\gamma^2 + I_y C_\gamma^2 \\ M_{66} = I_z \end{cases} \tag{33}$$

$$C = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & c_{22} \end{bmatrix} \tag{34}$$

$$C_{22} = \begin{bmatrix} -k_1\dot{\beta} - k_2\dot{\gamma} & -k_1\dot{\alpha} - k_3\dot{\beta} + k_4\dot{\gamma} & -k_2\dot{\alpha} + k_4\dot{\beta} \\ k_1\dot{\alpha} + k_4\dot{\gamma} & k_5\dot{\gamma} & k_4\dot{\alpha} + k_5\dot{\beta} \\ k_2\dot{\alpha} - k_4\dot{\beta} & -k_4\dot{\alpha} - k_5\dot{\beta} & 0 \end{bmatrix} \tag{35}$$

$$k_1 = C_\beta S_\beta (C_\gamma^2 I_x + S_\gamma^2 I_y - I_z) \tag{36}$$

$$k_2 = C_\beta^2 C_\gamma S_\gamma (I_x - I_y) \tag{37}$$

$$k_3 = C_\gamma S_\gamma^2 (I_x - I_y) \tag{38}$$

$$k_4 = \frac{1}{2} C_\beta (C_\gamma - S_\gamma)(C_\gamma + S_\gamma)(I_x - I_y) \tag{39}$$

$$k_5 = C_\gamma S_\gamma (I_x - I_y) \tag{40}$$

$$G_k(q) = \begin{bmatrix} -m_k g \\ 0 \end{bmatrix} \tag{41}$$

In Eq. (41) Also, the platform mass and g is the gravitational acceleration equal to 8.9 N/m2. In equation (30), the torque vector can be rewritten in terms of the forces driving mechanism:

$$\tau = J^T F_p \tag{42}$$

Where in equation (42), parameters J and Fp are Jacobian matrix of the system with the order of 6*6 and vector of driving forces with the order of 6*6, respectively. The driving force vector is defined as:

$$F_p = [F_{p,1}, F_{p,2}, ..., F_{p,6}] \tag{43}$$

Using inverse kinematics mechanism and the equation (43), we have [5]:

$$M^*(q)\ddot{d} + C^*(q,\dot{q})\dot{d} + G^*(q) + F_{fr}^*(\dot{d}) = F_p \tag{44}$$

Where in equation (44), parameters d, M*(q), C*(q, $\dot{q}$ ), G*(q), and $F_{fr}^*(\dot{d})$ are the vector of mechanism lengths drivers, definitive mass matrix with the order of 6*6, vector provider of centrifugal forces and carioles with the order of 6*1, vector provider of torque caused by the gravitational pull with the order of 6*1, vector provider of friction forces in the joint space with the order of 6*1, respectively. In equation (8), the terms M*(q), $C^*(q,\dot{q})$ and G*(q) can be calculated using the following relationship:

$$M^*(q) = \left[J(q)^T\right]^{-1} M(q) J(q)^{-1} \tag{45}$$

$$C^*(q,\dot{q}) = \left[J(q)^T\right]^{-1} \left[C(q,\dot{q}) - M(q)\dot{J}(q,\dot{q}) \cdot \dot{q}\right] \tag{46}$$

$$G^*(q) = \left[J(q)^T\right]^{-1} G(q) \tag{47}$$

There are many ways to obtain the friction vector, so that in this paper, the calculation method is applicable to the following expression [5]:

$$\mathrm{F}_{\mathrm{fr}}^{*}(\dot{d}) = F_{v}^{*}(\dot{d}) + F_{c}^{*}(\dot{d}) + F_{s}^{*}(\dot{d}) \tag{48}$$

As in equation (48) parameters $F_v^*$, $F_c^*$ and $F_s^*$, respectively sticking friction force, Coulomb friction force and friction force are static. Each element of the friction force is calculated from the following relationship:

$$F_{v,j}^{*}(\dot{d}) = \begin{cases} 0 & \dot{d}_j = 0, j - 1,2,\ldots,6 \\ b_j \dot{d}_j & \dot{d}_j \neq 0, j = 1,2,\ldots,6 \end{cases} \tag{49}$$

$$F_{c,j}^{*}(\dot{d}) = \begin{cases} 0 & \dot{d}_j = 0, j - 1,2,\ldots,6 \\ F_{c_{0.j}} \operatorname{sgn}(\dot{d}_j) & \dot{d}_j \neq 0, j = 1,2,\ldots,6 \end{cases} \tag{50}$$

$$F_{s,j}^{*}(\dot{d}) = \begin{cases} F_{ext,j} & \left|F_{ext,j}\right| \langle F_{c_{0,j}}, \dot{d}_j = 0, \\ & \ddot{d}_j = 0, j = 1,2,\ldots,6 \\ F_{s_{0,j}} \operatorname{sgn}(F_{ext,j}) & \left|F_{ext,j}\right| \rangle F_{c_{0,j}}, \dot{d}_j = 0, \\ & \ddot{d}_j \neq 0, j = 1,2,\ldots,6 \\ 0 & \dot{d}_j = 0, j = 1,2,\ldots,6 \end{cases} \tag{51}$$

That the relations (49), (50) and (51), Parameter bj, j-th element parameter friction sticking and the parameter Fext, j, j-th element external force and Fs0, j, jth element and volatile energy Fc0, j, jth element of the parameter is the Coulomb friction. And the sign function is expressed as follows:

$$\operatorname{sgn}(F_{ext,j}) = \begin{cases} +1 & \dot{d}_j \rangle 0 \\ 0 & \dot{d}_j = 0 \xrightarrow{for} j = 1,2,\ldots,6 \\ -1 & \dot{d}_j \langle 0 \end{cases} \tag{52}$$

## 3. PNUEMATIC SYSTEM DYNAMICS

In this section, the pneumatic system dynamic model will be derived. Figure 4 depicts the pneumatic system along with its components.

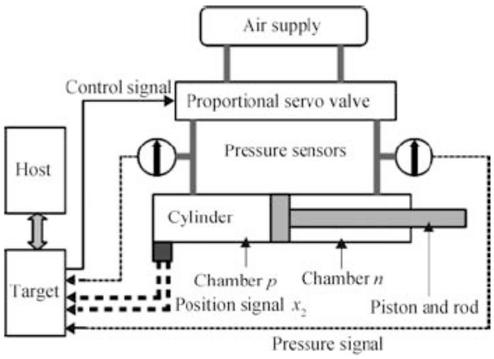


**Figure 4.** Pneumatic system [3].

The governing equations for the pneumatic system can be written as [3]:

$$\dot{p}_n = \frac{\gamma_1 A_n p_n}{v_n}\dot{x}_{pos} - k_v \frac{\gamma_1 R_g T_s}{v_n} V \tag{53}$$

$$\dot{p}_p = \frac{-\gamma_1 A_p p_p}{v_p}\dot{x}_{pos} + k_v \frac{\gamma_1 R_g T_s}{vp} V \tag{54}$$

$$\ddot{x}_{pos} = \frac{A_p p_p - A_n p_n}{M_p} - \frac{F_{fr}}{M_p}\ddot{x}_{pos} \tag{55}$$

The parameters $T_s$, $\gamma_1$, $F_f$, $R_g$, $K_v$, $M$, $A_p$, $A_n$, $P_p$, $P_n$, $V_p$, and $V_n$ are the working temperature, specific temperature coefficient, friction force, the universal gas constant, servo-valve constant, piston mass, piston cross sectional area (region p), piston cross sectional area (region n), nominal pressure in region p, nominal pressure in region n, nominal volume in region p, and nominal volume in region n respectively.

## 4. STATE- SPACE MODEL

For control purposes, it is necessary to write the Stewart Platform system and the pneumatic system equations in the integrated form so that the overall input and output of the system can be determined. Since the system is multi-variable, it is better to write its equations in the state-space form. For this, the state-space equations for each system must be separately extracted. For simplicity, we omit the * superscript from the parameters in the final equations [8].

### 4.1. Stewart Constract State-Space Model

Now, we can rearrange the terms of Equation 41 in terms of the second time derivative of the actuator lengths:

$$\ddot{d} = M^{-1}(C + F_{fr})\dot{d} - M^{-1}\left[G(q) - F_p\right] \tag{56}$$

$$\begin{bmatrix} \ddot{d}_1 \\ \ddot{d}_2 \\ \ddot{d}_3 \\ \ddot{d}_4 \\ \ddot{d}_5 \\ \ddot{d}_6 \end{bmatrix} = M^{-1}(C + F_{fr}) \begin{bmatrix} \dot{d}_1 \\ \dot{d}_2 \\ \dot{d}_3 \\ \dot{d}_4 \\ \dot{d}_5 \\ \dot{d}_6 \end{bmatrix} - M^{-1}\left[G(q) - F_p\right] \tag{57}$$

The state equations of the system are defined as:

$$if \begin{cases} z_1 = y \\ \dot{z}_1 = z_2 \\ \ddot{z}_2 = \ddot{z}_1 \end{cases} \Rightarrow \begin{cases} \dot{z} = Az + Bu \\ y = Cz + Du \end{cases} \tag{58}$$

Therefore, the state-space equation for the Stewart Platform system is obtained as:

$$\begin{cases} \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} 0 & I \\ 0 & -M^{-1}(C + F_{fr}) \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \\ \qquad + \begin{bmatrix} 0 \\ -M^{-1} \end{bmatrix} \left[G(q) + F_p\right] \\ y = \begin{bmatrix} I & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \end{cases} \tag{59}$$

### 4.2. State-Space Equation of Pneumatic System

Assuming that the cross sections of the two internal sections in the cylinder are equal and that the pressure difference between these two cylinder sections is Pd, we obtain:

$$if \begin{cases} A_p = A_n = A_{pn} \\ p_p - p_n = p_d \end{cases} \tag{60}$$

Thus, the state equation for the pneumatic system is:

$$\begin{cases} \begin{bmatrix} \dot{p}_d \\ \dot{x}_{pos} \\ \ddot{x}_{pos} \end{bmatrix} = \begin{bmatrix} 0 & 0 & a-b \\ 0 & 0 & I \\ \dfrac{A_{pn}}{M_p} & 0 & \dfrac{-F_{fr}}{M_p} \end{bmatrix} \begin{bmatrix} p_d \\ x_{pos} \\ \dot{x}_{pos} \end{bmatrix} + \begin{bmatrix} c-d \\ 0 \\ 0 \end{bmatrix} V \\ y_{pos} = \begin{bmatrix} 0 & I & 0 \end{bmatrix} \begin{bmatrix} p_d \\ x_{pos} \\ \dot{x}_{pos} \end{bmatrix}, D = 0 \end{cases} \tag{61}$$

In Equation 61, we have:

$$a = -\frac{\gamma_1 A_{pn} p_p}{v_p}, \quad b = \frac{\gamma_1 A_{pn} p_n}{v_n}, \quad c = k_v \frac{\gamma_1 R_g T_s}{v_p}, \quad d = -k_v \frac{\gamma_1 R_g T_s}{v_n}$$

### 4.3. System State-Space Model

In this section, we obtain the overall integrated system equations through selecting suitable state variables. To this end, we must first change the state variables in the pneumatic system to the following form:

$$\begin{cases} x_1 = p_d \\ x_2 = x_{pos} \\ x_3 = \dot{x}_{pos} \\ x_4 = \dot{x}_3 = \ddot{x}_{pos} \\ y_{pos} = x_2 \end{cases} \tag{62}$$

Taking the derivative of Equation 62, we obtain:

$$\begin{cases} \dot{x}_1 = \dot{p}_d = (a-b)x_3 + (c-d)V \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = \dfrac{A_{pn}}{M_p} x_1 - \dfrac{F_{fr}}{M_p} x_3 \\ \dot{x}_4 = \ddot{x}_3 = \dfrac{A_{pn}}{M_p} \dot{x}_1 - \dfrac{F_{fr}}{M_p} \dot{x}_3 \end{cases} \tag{63}$$

And placing $\dot{x}_1$ and $\dot{x}_3$ into $\dot{x}_4$ relation, thus:

$$\begin{aligned}\dot{x}_4 &= \frac{A_{pn}}{M_p}(a-b)x_3 + \frac{A_{pn}}{M_p}(c-d)V - \frac{F_{fr}A_{pn}}{M_p^2}x_1 + \left(\frac{F_{fr}}{M_p}\right)^2 x_3 \\ &= -\frac{F_{fr}A_{pn}}{M_p^2}x_1 + \left\{\frac{A_{pn}}{M_p}(a-b) + \left(\frac{F_{fr}}{M_p}\right)^2\right\}x_3 + \frac{A_{pn}}{M_p}(c-d)V\end{aligned} \tag{64}$$

Thus, the pneumatic system state-space equations can be rewritten in matrix form as:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix} 0 & 0 & (a-b) & 0 \\ 0 & 0 & I & 0 \\ \frac{A_{pn}}{M_p} & 0 & \frac{-F_{fr}}{M_p} & 0 \\ \frac{-F_{fr}A_{pn}}{M_p^2} & 0 & \frac{A_{pn}}{m_p}(a-b) + \left(\frac{F_{fr}}{M_p}\right)^2 & 0 \end{bmatrix} \begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}(c-d)\\ 0\\ 0\\ \frac{A_{pn}}{M_p}(c-d)\end{bmatrix} V \tag{65}$$

$$F_p = \begin{bmatrix}0 & 0 & 0 & M_p\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} \tag{66}$$

Now, we rewrite the Stewart Platform variables in terms of the new state variables as:

$$\dot{x}_5 = x_6 \tag{67}$$

$$\dot{x}_6 = -M^{-1}(c+F_{fr})x_6 - M^{-1}[G(q) - F_p] \tag{68}$$

Therefore, the state-space equations for the integrated system are:

$$\begin{cases}\dot{x} = Ax + BV \\ y = Cx\end{cases} \tag{69}$$

$$\left\{\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\\ \dot{x}_5\\ \dot{x}_6\end{bmatrix} = \begin{bmatrix} 0 & 0 & (a-b) & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 & 0 \\ \frac{A_{pn}}{M_p} & 0 & \frac{-F_{fr}}{M_p} & 0 & 0 & 0 \\ \frac{-A_{pn}F_{fr}}{M_p^2} & 0 & \frac{A_{pn}}{M_p}(a-b) + \left(\frac{F_{fr}}{M_p}\right)^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I \\ 0 & 0 & 0 & -M^{-1} & 0 & -M^{-1}(c+F_{fr}) \end{bmatrix} \begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\\ x_5\\ x_6\end{bmatrix} + \begin{bmatrix}(c-d)\\ 0\\ 0\\ \frac{A_{pn}}{M_p}(c-d)\\ 0\\ 0\end{bmatrix} V\right.$$

$$y = \begin{bmatrix}0 & 0 & 0 & 0 & I & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\\ x_5\\ x_6\end{bmatrix} \tag{70}$$

## 5. CONCLUSIONS

In this paper, a systematic approach was followed for the kinematic analysis of the Stewart mechanism. In the proposed method, the inverse kinematics of the system was appropriately analyzed using the Stewart parallel robot geometry, and subsequently, the direct kinematics of the mechanism was extracted based on the findings obtained from the inverse kinematics approach. Upon completion of the kinematic analysis (where the behavior of the system was described regardless of the causes (forces) that gave rise to displacement, velocity, and acceleration), the relations between various system components were extracte by using kinematic time derivatives in the form of velocity and Jacobian terms. Theen, the dynamic analysis was conducted on the system to obtain a comprehensive dynamic model through considering actuator dynamics and the existing friction forces at the joints as well as the rigid body dynamics of the system as a whole. The dynamic analysis of the Stewart mechanism was conducted through the Lagrangial method (for obtaining kinetic equations of the system). In this method, the equations are obtained by taking into account the work/energy stored in the system so as to eliminate the undesirable constraint forces. Ultimately, to achieve an integrated system for control purposes, the equations for the whole system were duly formulated in the form of state- space equations.

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