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Testing exponential posterior distribution with scale parameter for NBUL class

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Abstract. Recently non-parametric testing using goodness of fit approach for testing standard exponential with scale parameter against the new better than used in the Laplace transform order class (NBUL) has been introduced. In this paper we represent non-parametric testing for NBUL class using goodness of fit approach with λ parameter that λ parameter has prior distribution E (1). The testing Bayesian statistic based on u-statistic, It's provide by departure of nun hypothesis. And also we represent $\hat{\delta}(s,\lambda)$ statistic.

Keywords: Bayesian estimation, NBUL aging class, Posterior distribution, U-statistic, Monte Carlo method.

1. INTRODUCTION

Recently non-parametric testing using goodness of fit approach for testing standard exponential with scale parameter against the new better than used in the Laplace transform order class (NBUL) has been introduced by P. Nasiri, R. Lotfi, F. Shojaat (2014). During the past decards, various classes of life distributions have been proposed in order to model different aspects of ageing. The best known these classes are Increase failure rate (IFR), Increase average failure rate (IFRA), New better than used (NBU), New better than used in failure rate (NBUFR), New better than used in average failure rate (NBUFRA), New better than used in convex order (NBUC), Decrease mean residual life (DMRL), New better than used in expectation (NBUE), Harmonically New better than used in expectation (HNBUE) and Laplace class ($\mathcal{L}$).

The following are the relation between these classes:

$$\mathrm{IFR} \subset \mathrm{IFRA} \subset \mathrm{NBU} \subset \mathrm{NBUFR} \subset \mathrm{NBUFRA}$$
$$\mathrm{NBU} \subset \mathrm{NBUC} \subset \mathrm{NBUE} \subset \mathrm{HNBUE} \subset \mathcal{L} \quad , \quad \mathrm{IFR} \subset \mathrm{DMRL} \subset \mathrm{NBUE}$$

Properties and application of these ageing notation can be found, for instance, in Bryson and Siddiqui [2], Barlow and Proschan [1], Rolski [6], Kelefsjo [4]. And a new class of life distribution New better than used in the Laplace order (NBUL), introduced by Wang [7], L.S. Diab[3] and testing exponential distribution with scale parameter for NBUL class offer by P. Nasiri, R. Lotfi, F. Shojaat (2014)[5].

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In this paper a discussion of testing exponential distribution with λ parameter that λ parameter has prior distribution of standard exponential for $\delta(s,\tilde{\lambda})$ statistic of non-censored data and Bayesian estimation of λ parameter is considered in section 2. In section 3 Monte Carlo null distribution critical points are simulated for sizes 15(1)80. The power estimate for two distributions are also calculated.

2. TESTING FOR NBUL CLASS WITH NON-CENSORED DATA

In the section we test the null hypothesis H_0:F is exponential with λ parameter, that λ parameter has prior distribution of standard exponential distribution against H_1: F is NBUL.
We first find the posterior distribution for λ parameter. And we achieve the Bayesian estimation for λ parameter.

We take
$F(x) = 1 - e^{-\lambda x}$and $\quad \lambda \in e^{-\lambda}$
the posterior distribution for λ parameter will be
$\Pi(\lambda|x) \;\sim\; Gama(2, \bar{x}+1)$
And Bayesian estimation with error square loss function for λ parameter will be

$$\tilde{\lambda} = \frac{2}{\bar{x}+1}$$

According to the Eq.(1) we use a measure of departure from H_0 for finding u-statistic.

$$\delta(s,\tilde{\lambda}) = E\left(\bar{F}(t|x)\int_0^\infty e^{-s\lambda}\bar{F}(\lambda|x)d\lambda - \int_0^\infty e^{-s\lambda}\bar{F}(\lambda|x+t)d\lambda\right) \geq 0 \tag{2}$$

$$\delta(s,\tilde{\lambda}) = \int_0^\infty \left[\bar{F}(t|x)\int_0^\infty e^{-s\lambda}\bar{F}(\lambda|x)d\lambda - \int_0^\infty e^{-s\lambda}\bar{F}(\lambda|x+t)d\lambda\right] dF_0(t|x) \tag{3}$$

That
$F_0(t|x) \;\sim\; Gama(2, t+1)$

Lemma 2.1 Let X be random variable with posterior distribution $F(\lambda|x)$ and let

$$\emptyset(s) = \int_0^\infty e^{-s\lambda}\, dF(\lambda|x) \tag{4}$$

$$\delta(s,\tilde{\lambda}) = \frac{1}{s\left(\frac{2}{\tilde{\lambda}} - s\right)^2}\left[\emptyset\left(\frac{2}{\tilde{\lambda}}\right)\left(\frac{2}{\tilde{\lambda}} - s + s\emptyset(s)\right) + \emptyset(s) - 1\right] \tag{5}$$

Proof. According to the Eq. (3) we have

$$\delta(s,\tilde{\lambda}) = \int_0^\infty \left[\bar{F}(t|x)\int_0^\infty e^{-s\lambda}\bar{F}(\lambda|x)d\lambda - \int_0^\infty e^{-s\lambda}\bar{F}(\lambda|x+t)d\lambda\right] . te^{-t\left(\frac{2}{\tilde{\lambda}}\right)}dt$$

$$= \int_0^\infty t.\bar{F}(t|x)e^{-t\left(\frac{2}{\tilde{\lambda}}\right)}dt \int_0^\infty e^{-s\lambda}\bar{F}(\lambda|x)d\lambda - \int_0^\infty\int_0^\infty t.e^{-s\lambda}.e^{-t\left(\frac{2}{\tilde{\lambda}}\right)}\bar{F}(\lambda|x+t)d\lambda dt$$

Then

$$\int_0^\infty t.\bar{F}(t|x)e^{-t\left(\frac{2}{\tilde{\lambda}}\right)}dt \int_0^\infty e^{-s\lambda}\bar{F}(\lambda|x)d\lambda \tag{*}$$

$$\int_0^\infty \int_0^\infty t.e^{-s\lambda}.e^{-t\left(\frac{2}{\tilde{\lambda}}\right)}\bar{F}(\lambda|x+t)d\lambda dt \qquad (**)$$

We let of Eq.(4) that

$u = e^{-s\lambda} \quad dv = dF(\lambda|x)dx$

And we have

$$\emptyset(s) = \int_0^\infty e^{-s\lambda}\, dF(\lambda|x) = \int_0^\infty de^{-s\lambda}\, F(\lambda|x) + \int_0^\infty sF(\lambda|x)e^{-s\lambda}\, d\lambda$$

$$= s\int_0^\infty e^{-s\lambda}\left(1-\bar{F}(\lambda|x)\right)d\lambda$$

Then we have

$$\int_0^\infty e^{-s\lambda}\,\bar{F}(\lambda|x)d\lambda = \frac{1}{s}\left(1-\emptyset(s)\right)$$

Hence part (*) will equivalent to

$$\int_0^\infty t.\bar{F}(t|x)e^{-t\left(\frac{2}{\tilde{\lambda}}\right)}dt\int_0^\infty e^{-s\lambda}\bar{F}(\lambda|x)d\lambda = \frac{1}{s\left(\frac{2}{\tilde{\lambda}}\right)}\left(\emptyset\left(\frac{2}{\tilde{\lambda}}\right)-1\right)\left(1-\emptyset(s)\right)$$

For part (**) consider

$$\int_0^\infty \int_0^\infty t.e^{-s\lambda}\,e^{-t\left(\frac{2}{\tilde{\lambda}}\right)}\bar{F}(\lambda|x+t)d\lambda dt \xRightarrow[\lambda|x+t=v]{} \int_0^\infty \int_0^\infty t.e^{-su}\,e^{-t\left(\frac{2}{\tilde{\lambda}}-s\right)}\bar{F}(u)dudt$$

$$= \int_0^\infty e^{-su}\bar{F}(u)du\int_0^\infty t.e^{-t\left(\frac{2}{\tilde{\lambda}}-s\right)}dt = \frac{1}{s\left(\frac{2}{\tilde{\lambda}}-s\right)^2}\left(1-\emptyset(s)\right)$$

Hence, u-statistic for NBUL class with Bayesian estimation for λ parameter will be

$$\delta(s,\tilde{\lambda}) = \frac{1}{s\left(\frac{2}{\tilde{\lambda}}-s\right)^2}\left[\emptyset\left(\frac{2}{\tilde{\lambda}}\right)\left(\frac{2}{\tilde{\lambda}}-s+s\emptyset(s)-\emptyset(s)\right)+\emptyset(s)-1\right] \qquad (6)$$

Then the result follows.

To estimate $\delta(s,\tilde{\lambda})$, let $X_1,\dots,X_n$ be a random sample from $F(\lambda|x)$. So the empirical from of $\delta(s,\tilde{\lambda})$ in Eq.(6) is as follow:

$$\hat{\delta}(s,\tilde{\lambda}) = \frac{1}{s\left(\frac{2}{\tilde{\lambda}}-s\right)^2}\left[\sum_i\sum_j e^{-x_i}\left(\frac{2}{\tilde{\lambda}}-s+se^{-sx_j}-e^{-sx_j}\right)+e^{-sx_j}-1\right] \qquad (7)$$

To find the limiting distribution of $\hat{\delta}(s,\tilde{\lambda})$, we make from two observations,

$$\emptyset_{s,\tilde{\lambda}}(X_1,X_2) = e^{-x_1}\left(\frac{2}{\tilde{\lambda}}-s+se^{-sx_2}-e^{-sx_2}\right)+e^{-sx_2} \qquad (8)$$

Then

$$(i)\,\emptyset_{1,s,\tilde{\lambda}}(X_1) = E\left(\emptyset_{s,\tilde{\lambda}}^{(X_1,X_2|X_1)}\right)$$

$$= \int_0^\infty \left[e^{-x_1}\left(\frac{2}{\tilde{\lambda}}-s+se^{-sx_2}-e^{-sx_2}\right)+e^{-sx_2}\right].x_2e^{-x_2\left(\frac{2}{\tilde{\lambda}}\right)}dx_2$$

$$= e^{-x_1}\left(\frac{\tilde{\lambda}^3s^3(\tilde{\lambda}s-1)+4\tilde{\lambda}^2s(\tilde{\lambda}-s)-12\tilde{\lambda}}{4(2+\tilde{\lambda}s)^2}\right)-\frac{\tilde{\lambda}^2}{(2+\tilde{\lambda}s)^2} \qquad (9)$$

$$(ii)\emptyset_{2,s,\tilde{\lambda}}(X_1) = E\left(\emptyset_{s,\tilde{\lambda}}(X_2, X_1|X_1)\right)$$
$$= \int_0^\infty \left[e^{-x_2}\left(\frac{2}{\tilde{\lambda}} - s + se^{-sx_1} - e^{-sx_1}\right) + e^{-sx_1}\right].x_2 e^{-x_2\left(\frac{2}{\tilde{\lambda}}\right)} dx_2$$
$$= \left(\frac{2}{\tilde{\lambda}} - s + se^{-sx_1} - e^{-sx_1}\right)\left(\frac{-\tilde{\lambda}^2}{\left(2+\tilde{\lambda}\right)^2}\right) - \frac{\tilde{\lambda}^2}{4}.e^{-sx_1} \qquad (10)$$

We have
$\psi_{s,\tilde{\lambda}}(X_1) = \emptyset_{1,s,\tilde{\lambda}}(X_1) + \emptyset_{2,s,\tilde{\lambda}}(X_1) \quad \forall\left(s \neq 1, \tilde{\lambda} \neq -2\right)$

Theorem 2.1
As $n \longrightarrow \infty$, $\sqrt{n}\left(\delta\left(s,\tilde{\lambda}\right) - \delta\left(s,\tilde{x}\right)\right)$, is asymptotically normal with mean $\mu_{s,\tilde{\lambda}}$ and variance $\sigma^2_{s,\tilde{\lambda}}$.
That
$\mu_{s,\tilde{\lambda}} = E\left(\psi_{s,\tilde{\lambda}}(X_1)\right)$
$\sigma^2_{s,\tilde{\lambda}} = var\left(\psi_{s,\tilde{\lambda}}(X_1)\right)$

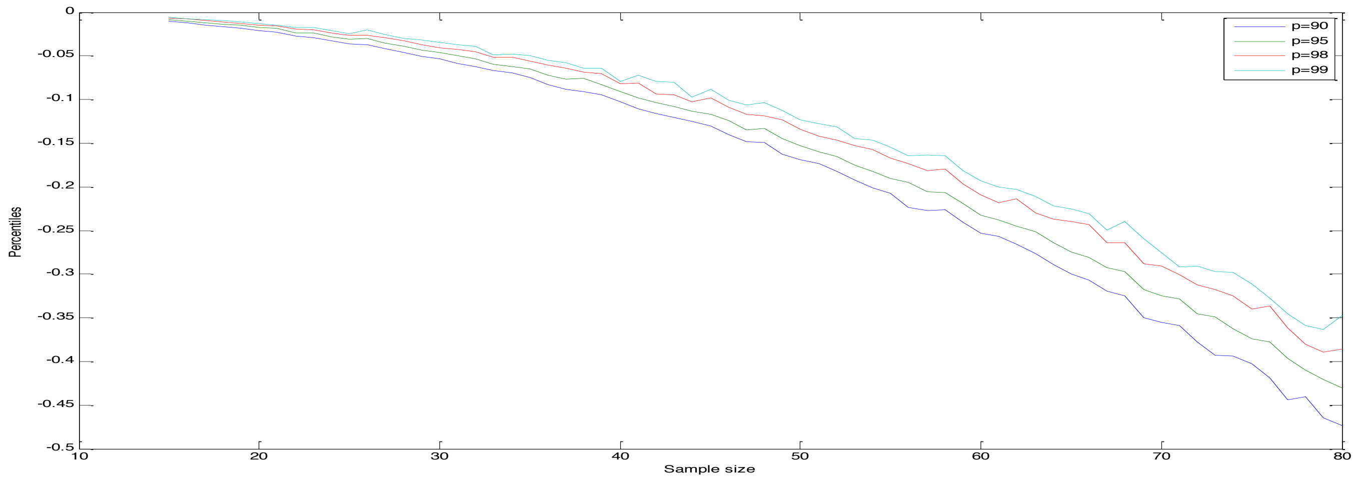
3. MONTE CARLO NULL DISTRIBUTION CRITICAL POINTS

In this section wesimulate the Monte Carlo null distribution critical points for $\widehat{\delta}(s,\tilde{\lambda})$ statistic in Eq.(7). Based on $N = 5000$ simulated sample sizes n=15 (1) 80 from the exponential distribution with scale parameter λ that λ distribution is standard exponential. Table 1 present the upper percentage points of the $\widehat{\delta}(s,\tilde{\lambda})$ statistic and fig present relation between critical values, sample size and confidence levels. Also we perform and estimation of power $\widehat{\delta}(s,\tilde{\lambda})$ statistic with significant level $\alpha = 0/05$, based on $N = 1000$ simulated sample sizes $n = 15, 20, 25, 30$ from posterior distribution with the survival function $1 - e^{-\lambda\left(1+\frac{x^2}{2}\right)-x}$ and posterior distribution with the survival function $1 - e^{-\lambda - x^\lambda}$.Table 2 present an estimation of the power for test statistic $\delta(s,)$.

Table 1. the upper percentages of $\hat{\delta}(s,\lambda)$ with 5000 simulated values of posterior distribution $\Pi(\lambda|x) = Gama(2, \bar{x}+1)$with $\tilde{\lambda} = 0/01$

n	p=90	p=95	p=98	p=99	n	p=90	p=95	p=98	p=99
15	-0.0107	-0.0084	-0.0066	-0.0056	48	-0.1492	-0.1329	-0.1187	-0.1037
16	-0.0124	-0.0101	-0.0080	-0.0073	49	-0.1623	-0.1444	-0.1229	-0.1123
17	-0.0152	-0.0120	-0.0094	-0.0082	50	-0.1686	-0.1524	-0.1339	-0.1228
18	-0.0169	-0.0142	-0.0111	-0.0092	51	-0.1735	-0.1598	-0.1423	-0.1277

19	-0.0184	-0.0150	-0.0130	-0.0114	52	-0.1820	-0.1649	-0.1464	-0.1311
20	-0.0213	-0.0174	-0.0145	-0.0132	53	-0.1918	-0.1749	-0.1525	-0.1450
21	-0.0227	-0.0183	-0.0159	-0.0147	54	-0.2013	-0.1825	-0.1573	-0.1465
22	-0.0274	-0.0234	-0.0191	-0.0171	55	-0.2075	-0.1901	-0.1673	-0.1548
23	-0.0288	-0.0238	-0.0201	-0.0173	56	-0.2233	-0.1946	-0.1732	-0.1648
24	-0.0325	-0.0278	-0.0238	-0.0208	57	-0.2268	-0.2053	-0.1816	-0.1633
25	-0.0359	-0.0308	-0.0267	-0.0251	58	-0.2267	-0.2064	-0.1796	-0.1646
26	-0.0369	-0.0301	-0.0263	-0.0204	59	-0.2410	-0.2189	-0.1971	-0.1811
27	-0.0417	-0.0353	-0.0295	-0.0254	60	-0.2532	-0.2321	-0.2089	-0.1933
28	-0.0461	-0.0392	-0.0328	-0.0302	61	-0.2569	-0.2375	-0.2178	-0.2003
29	-0.0508	-0.0438	-0.0370	-0.0314	62	-0.2660	-0.2449	-0.2138	-0.2027
30	-0.0533	-0.0464	-0.0410	-0.0349	63	-0.2761	-0.2511	-0.2301	-0.2114
31	-0.0591	-0.0496	-0.0422	-0.0371	64	-0.2886	-0.2638	-0.2370	-0.2221
32	-0.0618	-0.0535	-0.0454	-0.0386	65	-0.3001	-0.2745	-0.2395	-0.2255
33	-0.0671	-0.0594	-0.0515	-0.0490	66	-0.3072	-0.2809	-0.2432	-0.2310
34	-0.0698	-0.0621	-0.0518	-0.0476	67	-0.3198	-0.2922	-0.2636	-0.2497
35	-0.0748	-0.0650	-0.0559	-0.0501	68	-0.3249	-0.2971	-0.2638	-0.2398
36	-0.0825	-0.0724	-0.0606	-0.0549	69	-0.3495	-0.3178	-0.2885	-0.2591
37	-0.0886	-0.0769	-0.0644	-0.0582	70	-0.3551	-0.3250	-0.2909	-0.2751
38	-0.0913	-0.0757	-0.0685	-0.0639	71	-0.3588	-0.3288	-0.3006	-0.2918
39	-0.0946	-0.0832	-0.0704	-0.0643	72	-0.3776	-0.3451	-0.3122	-0.2904
40	-0.1025	-0.0910	-0.0821	-0.0795	73	-0.3931	-0.3486	-0.3181	-0.2967
41	-0.1107	-0.0982	-0.0815	-0.0724	74	-0.3940	-0.3621	-0.3244	-0.2982
42	-0.1160	-0.1032	-0.0936	-0.0794	75	-0.4031	-0.3737	-0.3400	-0.3112
43	-0.1206	-0.1076	-0.0941	-0.0802	76	-0.4187	-0.3777	-0.3361	-0.3272
44	-0.1248	-0.1131	-0.1025	-0.0971	77	-0.4436	-0.3962	-0.3618	-0.3454
45	-0.1301	-0.1170	-0.0979	-0.0884	78	-0.4404	-0.4101	-0.3804	-0.3586
46	-0.1402	-0.1240	-0.1090	-0.1010	79	-0.4642	-0.4207	-0.3891	-0.3632
47	-0.1485	-0.1349	-0.1166	-0.1065	80	-0.4739	-0.4303	-0.3857	-0.3470



It is noted from table 1 and fig that the critical values are increasing as the confidence level increasing and decreasing as the sample size increasing.

Table 2. power estimate for posterior distributions:

$(1)\bar{F} = (\lambda|x) = 1 - e^{-\lambda\left(1+\frac{x^2}{2}\right)-x}$

$\tilde{\lambda}$	10	15	20	25	30
2	1	1	1	1	1
3	1	1	1	1	1
4	1	1	1	1	1
5	1	1	1	1	1
6	1	1	1	1	1

$(2)\bar{F} = (\lambda|x) = \ 1 - e^{-\lambda-x^{\lambda}}$

$\tilde{\lambda}$	10	15	20	25	30
2	1	1	1	1	1
3	1	1	1	1	1
4	1	1	1	1	1
5	1	0.9	1	1	1
6	0.9 9	1	1	1	1

It is clear from the above table that our test has good power.

4. CONCLUSIONS

In this paper we have introduce u-statistic based on a goodness of fit approach for testing exponentially with scale parameter λ that λprior distribution is standard exponential. Against the new better than used in the Laplace transform (NBUL) order. We provide critical values and power for posterior distributions with non-censored data.

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